Central Bank Digital Currency Design: Implications for Market Composition and Monetary Policy*

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Abstract

We explore the implications of introducing a central bank digital currency (CBDC) through commercial banks that differ in size. We focus on two design features of CBDCs: the interest rate and payment convenience. These features correspond to the "store of value" and "medium of exchange" properties of currencies. Payment convenience is an under-explored aspect of CBDC design which may substitute for or complement the monetary benefits of interest payments. Recognition of the broader aspects of CBDC design are important, not only for understanding consumer adoption of CBDC, but also its effects on broader market outcomes.

Keywords: central bank digital currency, interest on (excess) reserves, deposit interest rates, bank lending

JEL Classification: E42, G21, G28, L11, L15

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"If all a CBDC did was to substitute for cash – if it bore no interest and came without any of the extra services we get with bank accounts – people would probably still want to keep most of their money in commercial banks."
—Ben Broadbent, Deputy Governor of the Bank of England, in a 2016 speech

1 Introduction

A central bank digital currency (CBDC) "is a digital payment instrument, denominated in the national unit of account, that is a direct liability of the central bank" (BIS 2020). In recent years, central banks and the broader market have increasingly viewed CBDCs as a potentially helpful innovation that can improve the payment system, promote financial inclusion, enhance monetary policy transmission, and reduce systemic risk (BIS 2020). But what design features would help the CBDC achieve these objectives?

In this paper, we explore the implications of introducing a CBDC that has two salient features: interest-bearing and payment convenience. Interest-bearing is related to the "store of value" property of currencies, and payment convenience is related to the "medium of exchange" property. These two dimensions are aptly reflected in the quote above by Ben Broadbent in 2016 and also cover most of the features proposed by the Committee on Payments and Market Infrastructures (CPMI, 2018).[1]

If the CBDC is meant to offer a solution, then it is useful to first identify the problems. There are at least two market imperfections that a CBDC may help mitigate. The first is that interest rate pass-through in the economy is far from complete. Drechsler, Savov, and Schnabl (2017) find that "[f]or every 100 bps increase in the Fed funds rate, the spread between the Fed funds rate and the deposit rate increases by 54 bps." Duffie and Krishnamurthy (2016) document a sizable dispersion of a broad range of money market interest rates, which widened as the Fed raised its interest on reserves. Both papers point out that imperfect competition in the U.S. deposit market is partially responsible. If the CBDC is accessible to households and pays interest directly, interest rate pass-through in the economy could be improved. Hence, we consider the interest-bearing feature.

A second area where a CBDC can lead to improvements is the payment system. The ability to pay is a fundamental need of citizens and underlies almost all economic activity. A publicly provided CBDC could be less expensive to use and more widely accessible than existing, privately offered payment methods, and it could offer access to new platforms. This

[1] They are 24/7 availability, anonymity, transfer mechanism, interest-bearing, and limits or caps.

could be the case, for example, if the CBDC was programmable and allowed consumers to purchase goods or services on permissioned DLT platforms. A CBDC could also provide privacy. Policy makers have argued that the central bank is specially positioned to provide privacy in payments because the central bank does not have a profit motive to exploit consumer payment data (See Lagarde, 2018). For these reasons we consider the convenience value feature.

In our model, the CBDC is offered through commercial banks. While CBDC balances are the direct liability of the central bank, we envision that commercial banks will act as the central bank's agents to conduct KYC (Know Your Customer) and AML (Anti-Money Laundering) as well as offering various payment services such as money transfer. This "tiered" design of CBDC is consistent with the recent pilot of E-Krona in Sweden, the CBDC experiment in China, and the Banking for All Act in the United States.[2] Even within the tiered design framework we are considering, the central bank could implement schemes to protect user privacy in ways that differentiate CBDC from other electronic payments.[3]

A key contribution of our analysis is to provide a framework for the evaluation of CBDC that captures salient aspects of the U.S. banking system. Banks engage in deposit creation to make loans. There are a large amount of reserves in the banking system and the central bank pays interest rate on reserves (IOR), denoted f, to commercial banks.[4] Banks differ in terms of size and this impacts the desirability of their deposit accounts and their customer demand for loans. More specifically, the commercial banking sector in our model consists of a large bank and a small bank. The large bank's deposit offers a higher convenience value than the small bank's deposit. For example, a large bank could have a more expansive network of branches and ATMs, a better mobile App, or a wider range of other (unmodeled) services. We normalize the small bank's convenience value to be zero, and model the large bank's convenience value as a nonnegative random variable, with a probability distribution across a continuum of agents who deposit in either bank. The higher convenience value of its deposit gives the large bank an advantage in the deposit market. As a result, the large bank offers a lower deposit interest rate than the small bank in equilibrium and yet has a

[2]The Act argues that Digital Dollar Wallets should provide a number of auxiliary services including debit cards, online account access, automatic bill-pay and mobile banking. These features (in particular mobile banking which could give access to a variety of platforms that customers of a particular bank might otherwise not have access to) could result in a CBDC with its own convenience value.

[3]A programmable CBDC could employ zero-knowledge proofs.

[4]In the United States, IOR has been paid since October 2008. Since the financial crisis of 2008-09, interest on excess reserves (IOER) has become the Federal Reserve's main policy tool to adjust interest rate. In July 2021, the Federal Reserve renamed IOER to interest on reserve balances (IORB), as required reserves are currently zero. For simplicity, we will use the term interest on reserves.

larger market share.

Deposit market share is important not only because it determines profits on deposits, but also because it impacts the cost of lending. The two commercial banks in our model lend to entrepreneurs who take on risky projects that differ in their quality (i.e., expected returns). When a loan is made by either bank, the bank creates a new deposit in the name of the entrepreneur as a new liability, which is exactly balanced by the new loan as a new asset. The entrepreneur immediately pays a randomly selected worker, and the deposit may flow out of the original bank to the other. For example, if the entrepreneur originally takes a loan from the small bank but the worker she hired has a high enough value for the convenience of the large bank, the deposit flows from the small bank to the large one, resulting in a flow of central bank reserves in the same direction. Because the large bank has an inherent advantage in the deposit market, the large bank's newly created deposit in the lending process has a high probability of staying at the large bank, meaning the creation of the loan as no associated loss of reserves. In contrast, the small bank is likely to lose the newly created deposit and the associated interest on the accompanying reserves. Holding fixed the quality of the entrepreneur's project, the greater ability to retain reserves means the large bank earns a higher total profit by lending than the small bank, and hence the large bank is willing to lend to lower-quality projects due to the higher expected profit of earning interest on reserves. This is a mechanism in which a deposit market advantage translates into a lending market advantage.

To analyze the impact of the CBDC, we perform multiple comparative static exercises. First, we increase the CBDC interest rate while holding the interest rate on reserves and the CBDC convenience value fixed. This analysis mimics the type of testing that was done with Overnight Reverse Repurchase rates when that facility was first introduced by the Federal Reserve in 2013. Under certain conditions, an increase in the CBDC interest rate raises the deposit interest rates of both banks and also their weighted average, that is, monetary policy is enhanced. Setting the CBDC interest rate equal to the interest rate on reserves would imply full monetary policy pass-through. However, by forcing both banks to raise interest rates, a higher CBDC interest rate makes it more difficult for the small bank to compete with the large bank by offering higher deposit interest rate. Thus, a higher CBDC interest rate reduces the market share of the small bank in deposit and lending markets, further widening the large bank-small bank gap.

Second, we vary the CBDC convenience value, holding the IOR rate and CBDC interest rate constant. Making the CBDC more convenient weakens the market power of the large bank by narrowing the convenience gap between the two banks. For example, by hosting a convenient CBDC, a small community bank partially "catches up" with large global banks in offering payment functionalities. The most immediate implication is that a convenient CBDC results in a lower deposit rate at the small bank, because the small bank does not have to compensate depositors as much for forgoing the large bank's convenience. At the same time, deposit interest rates at the large bank may remain unchanged or increase, depending on whether the the equilibrium is constrained or not. In the range of low convenience values, increasing convenience weakens the transmission of monetary policy through IOR to the deposit market. However, once the convenience value of the CBDC reaches a certain level that the large bank's interest rate is no longer at the lower bound, a further increase in CBDC convenience increases monetary policy transmission by increasing the weighted average deposit interest rate.

In terms of market composition, the overall impact of increasing CBDC convenience is that the deposit interest rates, deposit market shares, lending standards, and loan volume of the small bank and the large bank start to converge. In other words, a convenient CBDC "levels the playing field" for bank competition by chipping away the convenience advantage of the large bank.

Our final contribution is to examine various combinations of CBDC interest rates and convenience values. Rather than assuming a central bank objective function, we rank the interest rate-convenience value combinations from the perspectives of banks (collectively) and depositors (collectively). We show that a higher CBDC interest rate benefits depositors but reduces total bank profits; that is, depositors and banks cannot agree on the CBDC interest rate. However, in the constrained equilibrium, where the large bank's deposit interest rate is equal to the CBDC interest rate, both depositors and banks prefer a higher CBDC convenience value.[5] This analysis enables us to identify a set of Pareto-dominated points in interest rate-convenience space. By focusing on the boundary of this set we can evaluate a variety of monetary policy and financial stability objectives solely in terms of the CBDC interest rate choice.

2 Literature

Our work builds on previous literature that has modelled deposit and lending markets in the current regime of large excess reserves. In Martin, McAndrews and Skeie (2013), a loan is

[5] This agreement about convenience value does not arise in the unconstrained equilibrium.

made if its return exceeds the marginal opportunity cost of reserves, which can be either the federal funds rate or the IOR rate, depending on the regime. Our model differs in that we have multiple banks and hence lent money may return to the same bank as new deposits. Hence, our opportunity cost of lending is lower. Nevertheless, we share the conclusion that the aggregate level of bank reserves does not determine the level of bank lending.

There is now a growing literature that seeks to examine the impact of CBDC on deposit and lending markets. The conclusions vary and depend upon the level of competition, the interest rate on the CBDC, and other features (e.g., liquidity properties of CBDC and reserve requirements). Keister and Sanches (2020) consider a competitive banking environment in which deposit interest rates are determined jointly by the transactions demand for deposits and the supply of investment projects. If the CBDC serves as a substitute for bank deposits, then its introduction causes deposit interest rates to rise, and the levels of deposits and bank lending to fall.

In contrast, if banks have market power in the deposit market, the introduction of a CBDC does not disintermediate banks, as banks can prevent consumers from holding the CBDC by matching its interest rate. This lowers their profit margin, but does not lower the level of deposits, and may even increase it. This is true in the model proposed by Andolfatto (2020), where the bank is a monopolist. In that paper, an interest bearing CBDC causes deposit interest rates to rise and the level of deposits to increase. Likewise, in that paper, banks have monopoly power in the lending market, and, as in Martin, McAndrews and Skeie (2013), lending is not tied directly to the level of deposits, Hence, a CBDC does not impact the interest rate on bank lending or the level of investment.

Chiu et al. (2019) also consider banks with market power and show that an interest-bearing CBDC can lead to more, fewer or no change in deposits, depending on the level of the CBDC interest rate. In an intermediate range of rates, the CBDC impacts the deposit market in a manner similar to Andolfatto (2020) in that banks offer higher deposit interest rates and increase deposits. Since, similar to Keister and Sanches (2020), lending is tied to the level of deposits, adding the CBDC results in increased lending.

Our work is closest to Andolfatto (2020). We do not specify the overlapping generations framework that he uses to make money essential. However, like Andolfatto (2020), in our model, reserves are abundant, lending is determined by a performance threshold, and banks have monopoly power in lending market. Hence, lending is determined not by deposit levels, but instead by the opportunity cost of funds. In our model, this opportunity cost is lower than the IOR rate, since we allow for the realistic feature that reserves come back to the lending bank with a probability that depends on the deposit market share. Unlike Andolfato (2020), and the other works mentioned above, we incorporate two key design aspects of CBDC, interest rate and convenience value, and we examine the combined impact these features have on market outcomes in an environment with heterogeneous banks.

The impact of adding a CBDC can be richer in the presence of other frictions. In a model with real goods and competitive banks, Piazzesi and Schneider (2020) find that the introduction of CBDC is beneficial if all payments are made through deposits and the central bank has a lower cost in offering deposits. However, they also find that the CBDC can be harmful if the payer prefers to use a commercial bank credit line, but the receiver prefers central bank money. Parlour, Rajan, and Walden (2021) argue that a wholesale CBDC that enhances the efficiency of interbank settlement system could exacerbate the asymmetry between banks if the CBDC does not distinguish net-paying and net-receiving banks. Agur et al. (2019) consider an environment where households suffer disutility from using a payment instrument that is not commonly used. They examine trade-offs faced by the central bank in preserving variety in payment instruments and show that the adverse effects of CBDC on financial intermediation are harder to overcome with a non-interest-bearing CBDC.

Fernández-Villaverde et al. (2020) extend the analysis of CBDC to a Diamond and Dybvig (1983) environment in which banks are prone to bank runs. In this setting, the fact that the central bank may offer more rigid deposit contracts allows it to prevent runs. Since commercial banks cannot commit to the same contract, the central bank becomes a deposit monopolist. Provided that the central bank does not exploit this monopoly power, the first-best amount of maturity transformation in the economy is still achieved.

Brunnermeier and Niepelt (2019) and Fernández-Villaverde et al. (2020) derive conditions under which the addition of a CBDC does not affect equilibrium outcomes. Key to their result is the central bank's active role in providing funding to commercial banks in order to neutralize the CBDC's impact on their deposits.

3 Model and Equilibrium

3.1 Setup

The economy has a large bank (L) and a small bank (S). There are $X = X_S + X_L$ reserves in the banking system, where X_S denotes the reserve holding of the small bank and X_L

denotes the reserve holding of the large bank.[6] For simplicity, the banks start off holding reserves as their only asset, balanced by exactly the same amount of deposits. Following Martin, McAndrews and Skeie (2013), we assume that the level of reserves X is exogenously determined by the central bank and is assumed to be large. The central bank pays the two commercial banks an exogenously determined interest rate f on their reserve holdings, which is called interest on reserves (IOR). The large and small banks pay depositors endogenously determined deposit interest rates r_L and r_S, respectively. Thus, if nothing else happens, bank j's total profit would be $X_j(f - r_j)$.

Commercial bank deposits are valuable not just for the interest they pay, but also for payment convenience such as direct debit and deposit. The convenience value of the small bank's deposits is normalized to be zero, the convenience value of the large bank's deposits is a random variable $\delta \geq 0$ that has the cumulative distribution function G that is twice differentiable. As we discuss shortly, each depositor in the economy draws their large-bank convenience value δ independently from the distribution G.

The central bank offers a "retail" CBDC that is universally available. The CBDC has two features: it pays an interest rate of s to depositors who use it and it provides a per-dollar convenience value $v \geq 0$ to users that is the same across all depositors. The convenience value can be interpreted as a benefit that depositors receive from transacting using central bank money. This benefit can include access to platforms on which CBDC can be spent, aspects of the mobile user interface (app features) or any other account services that are associated with central bank accounts. We assume that the CBDC is offered via commercial banks, so that money can be transferred seamlessly between a depositor's deposit account at a commercial bank and their CBDC account offered via the commercial bank. Therefore, a depositor who has an account at the large bank can use the deposit account or CBDC account, whichever has the greater convenience value. That is, the depositor of the large bank receives the convenience value $\max(\delta, v)$, and a depositor who has an account at the small bank receives convenience value $\max(0, v) = v$. Here, we used $\max(\cdot)$ to capture the general idea that the CBDC's convenience value is more useful for small bank depositors than large bank depositors.

There is a unit mass of agents, and each potentially plays three roles: entrepreneur (borrower), worker, and depositor. The main heterogeneity among the agents is their convenience value for large bank deposits.

[6] We normalize the size of an individual loan to be \$1, so reserves are in units of the standard loan size. For example, if a loan size is \$1 million and the actual reserve is \$1 trillion, then in our model, X is interpreted as $\$1\,trillion/\$1\,million = 10^6$.

The model has four periods. At $t = 0$, the commercial banks set the deposit interest rates r_L and r_S. The central bank sets the interest on reserves rate f, the CBDC interest rate s, and the CBDC convenience value v. In the model, f, s, and v are exogenous, and r_L and r_S are endogenous. At the start of the model, a fraction m_L of agents have existing deposits at the large bank and a fraction $m_S = 1 - m_L$ of agents have existing deposits at the small bank. The amount of deposits per capita across agents is identical. This means $m_L = X_L/X$ and $m_S = X_S/X$. As discussed above, because the CBDC is offered via commercial banks, a CBDC account offered via a commercial bank has the same convenience value as the deposit account of that commercial bank. For this reason, the CBDC interest rate s is a lower bound on banks' deposit interest rates, i.e., $r_L \geq s$ and $r_S \geq s$.

At $t = 1$, the agents act as entrepreneurs and workers. Each agent is endowed with a project, and each project requires \$1 of investment and pays $A > 1$ with probability q_i and zero with probability of $1 - q_i$, where q_i has the distribution function Q and A is a commonly known constant. The expected payoff per dollar invested is thus $q_i A$. Each agent can only borrow from the bank where she keeps her deposit (the "relationship" bank). The bank prices the loan as a monopolist. If the loan is granted, the entrepreneur pays \$1 to a randomly selected agent from the same population. The selected agent plays the role as a worker and completes the project. The main point of introducing workers is to generate some money flow in the economy.

At $t = 2$, agents play the role as depositors. Workers who receive wages choose where to deposit the wage. The depositor can pick either the large bank or the small bank, and within a bank, the depositor can pick either the bank's own deposit account or the CBDC account. These choices are made after considering the depositor's own convenience value for large bank deposits, the convenience value of the CBDC, and all relevant interest rates.

At $t = 3$, the projects succeeds or fails.The banks earn interest on reserves and pays depositors according to their deposit holdings and the deposit interest rates.

3.2 Bank deposit creation

For the purpose of illustration it is convenient to illustrate the deposit creation process by considering a discrete set-up, in which we characterize the bank's decision to make a single loan. The condition on bank lending that we derive will be applicable to the continuum model in which borrowers (i.e., the entrepreneurs) are infinitesimal.

The tables below show the sequence of changes in the large bank's balance sheet in the loan process. The changes in the small bank's balance sheet in the loan process are entirely analogous.

1. Before lending, the large bank starts with X_L reserves. Its balance sheet looks like:

Asset	Liability
Reserves X_L	Deposits X_L

2. If the large bank makes a loan of \$1, it immediately creates deposit of \$1 in the name of the entrepreneur. The balance sheet of the bank becomes:

Asset	Liability
Reserves X_L	Deposits X_L
Loans 1	New Deposits 1

3. Eventually, the entrepreneur will spend her money to pay a worker. The large bank anticipates that, in expectation, a fraction α_S of the \$1 new deposit will be transferred to the small bank, leading to a reduction of reserves by the same amount. The fraction α_L remains in the bank because the worker has an account with the same bank. The bank's balance sheet becomes:

Asset	Liability
Reserves $X_L - \alpha_S$	Deposits X_L
Loans 1	New Deposits α_L

If the large bank makes the \$1 loan to entrepreneur i, and charges interest rate R_i, its total expected profit, by counting all items in the balance sheet, will be

$$\underbrace{(X_L - \alpha_S)f}_{\text{Interest on reserves}} + \underbrace{[q_i(1+R_i) - 1]}_{\text{Gross profit on the loan}} - \underbrace{(X_L + \alpha_L)r_L}_{\text{Cost of deposits}}. \tag{1}$$

If the large bank does not make the loan, then its total profit will be

$$X_L(f - r_L). \tag{2}$$

The large bank's marginal profit from making the loan, compared to not making it, is

$$\pi_i = \underbrace{q_i(1+R_i) - (1+f)}_{\text{Net profit on the loan}} + \underbrace{\alpha_L(f - r_L).}_{\text{Profit on deposit}} \tag{3}$$

In the expression of π_i, the net profit on the loan reflects the true opportunity cost of capital. Besides the usual profit on the loan, the large bank makes an additional profit equal to $\alpha_L(f - r_L)$. This is because each \$1 lent out stays with the large bank with probability α_L and earns the bank the IOR-deposit spread of $f - r_L$. The corresponding term for the small bank's marginal profit of lending is $\alpha_S(f - r_S)$. In the equilibrium we characterize, it will be the case that $\alpha_L(f - r_L) > \alpha_S(f - r_S)$, i.e., the large bank's convenience value of deposits translates into an advantage in the lending market. Such feature would not be present if banks were homogeneous.

3.3 Equilibrium

We solve the model backward in time.

Deposit market at $t = 2$. A depositor with a large-bank convenience value of δ faces four choices:

	Large bank		Small bank	
	Deposit	CBDC	Deposit	CBDC
Convenience value	$\max(\delta, v)$	$\max(\delta, v)$	v	v
Interest rate	r_L	s	r_S	s

Obviously, the small bank attracts no depositors if $r_S < r_L$. So $r_S \geq r_L$ in equilibrium. For technical simplicity, whenever a depositor is indifference between two choices, their preference is the small bank, the large bank, and finally the CBDC, in this order.[7]

We will characterize parameter conditions under which $r_S > r_L$. This implies that a depositor with convenience value δ chooses the large bank if and only if

$$\delta > v \text{ and } r_L + \delta > r_S + v \Rightarrow \delta > r_S - r_L + v. \tag{4}$$

Therefore, the eventual market shares of the banks in the newly created deposits are

$$\alpha_L = 1 - G(r_S - r_L + v), \tag{5}$$

$$\alpha_S = G(r_S - r_L + v). \tag{6}$$

[7]The tie-breaking rule between commercial banks and the CBDC is without loss of generality because a commercial bank can always offer ϵ above s so that depositors strictly prefer commercial bank deposits to the CBDC. The tie-breaking rule also preserves continuity in the fractions of depositors as parameters change to make depositors indifferent.

Loan market at $t = 1$. In the previous section we derived the marginal profit of a bank from making a loan. While the entrepreneur is infinitesimal here, expression (3) still applies.

The monopolist position of each bank in the lending market implies that a bank can make a take-it-or-leave-it offer to the entrepreneur. The bank's optimal interest rate quote would be $R_i = A - 1$ (or just tiny amount below), and the entrepreneur, who has no alternative source of funds, would accept. The lending bank takes the full surplus.

Hence, the large bank makes the loan if and only if

$$q_i A - (1 + f) + \alpha_L (f - r_L) > 0, \tag{7}$$

or

$$q_i > q_L^* = \frac{1 + f - \alpha_L (f - r_L)}{A}. \tag{8}$$

Exactly the same calculation for the small bank yields the comparable investment threshold

$$q_S^* = \frac{1 + f - \alpha_S (f - r_S)}{A}. \tag{9}$$

Choice of deposit interest rates at $t = 0$. Again, we start with the large bank. The large bank makes profits in two ways. Because the large bank is a monopolist when lending to its customers, its first source of profit is on the loans, $m_L \int_{q_l^*} (qA - 1 - f) dQ(q)$. The second source of the large bank's profit is on the interest rate spread. The existing deposit in the banking sytem is $X = X_L + X_S$. As discussed above, the lending process also creates new deposits. The amount of new deposit created by the large bank is $m_L(1 - Q(q_L^*))$, by the normalization that each loan is of \$1. Likewise, the small bank creates new deposit $m_S(1 - Q(q_S^*))$. When the two banks compete for depositors by setting the deposit interest rates r_L and r_S, we already show above that a fraction $\alpha_L = 1 - G(r_S - r_L + v)$ of total deposits end up with the large bank, enabling the large bank to collect a spread of $f - r_L$ per unit of deposit held.

Adding up the two components, we can write the large bank's total profit as

$$\begin{aligned} \Pi_L &= m_L \int_{q_L^*}^1 (qA - 1 - f) dQ(q) + [X_L + X_S + m_L(1 - Q(q_L^*)) + m_S(1 - Q(q_S^*))] \alpha_L (f - r_L) \\ &= m_L \int_{q_L^*}^1 [qA - (1 + f) + \alpha_L (f - r_L)] dQ(q) + [X_L + X_S + m_S(1 - Q(q_S^*))] \alpha_L (f - r_L). \end{aligned} \tag{10}$$

Likewise, the small bank's total profit is

$$\Pi_S = m_S \int_{q_S^*}^{1} [qA - (1+f) + \alpha_S(f - r_S)]dQ(q) + [X_L + X_S + m_L(1 - Q(q_L^*))]\alpha_S(f - r_S). \tag{11}$$

As discussed before, the CBDC interest rate puts a lower bound on commercial banks' deposit interest rates, i.e., $r_L \geq s$, and $r_S \geq s$. There are two cases. The first is that $r_L > s$, so that the CBDC interest rate does not constrain the commercial banks' deposit interest rates. We call the first case the *unconstrained equilibrium.* The second case is that $r_L = s$, i.e., the CBDC interest rate binds the large bank's deposit interest rate. We call the second case the *constrained equilibrium.*

Unconstrained equilibrium. Assuming that Π_L is strictly quasi-concave in r_L, the sufficient condition for a unique maximum of the function Π_L with respect to r_L is

$$\begin{aligned}
\frac{d\Pi_L}{dr_L} &= m_L(1 - Q(q_L^*))\frac{d[\alpha_L(f - r_L)]}{dr_L} - m_L \underbrace{[q_L^* A - (1+f) + \alpha_L(f - r_L)]}_{=0} \frac{dq_L^*}{dr_L} \\
&\quad + [X_L + X_S + m_S(1 - Q(q_S^*))]\frac{d[\alpha_L(f - r_L)]}{dr_L} - m_S\alpha_L(f - r_L)Q'(q_S^*)\frac{dq_S^*}{dr_L} \\
&= [X_L + X_S + m_L(1 - Q(q_L^*)) + m_S(1 - Q(q_S^*))] \cdot [(f - r_L)G'(r_S - r_L + v) - 1 + G(r_S - r_L + v)] \\
&\quad - m_S\alpha_L(f - r_L)Q'(q_S^*)\frac{(f - r_S)G'(r_S - r_L + v)}{A}.
\end{aligned} \tag{12}$$

Likewise, the first-order condition of the small bank is

$$\begin{aligned}
\frac{d\Pi_S}{dr_S} &= [X_L + X_S + m_L(1 - Q(q_L^*)) + m_S(1 - Q(q_S^*))] \cdot [(f - r_S)G'(r_S - r_L + v) - G(r_S - r_L + v)] \\
&\quad - m_L\alpha_S(f - r_S)Q'(q_L^*)\frac{(f - r_L)G'(r_S - r_L + v)}{A}.
\end{aligned} \tag{13}$$

For simplicity, let $Q(\cdot)$ be the uniform distribution on $[0, 1]$. And further impose a "stationarity" condition that the market shares of deposit $\{\alpha_j\}$ are identical to the starting market share $\{m_j\}$. The first-order conditions are simplified to

$$0 = \frac{d\Pi_L}{dr_L} = [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - r_L)G'(r_S - r_L + v) - 1 + G(r_S - r_L + v)] \\ - \frac{1}{A}\alpha_S\alpha_L(f - r_L)(f - r_S)G'(r_S - r_L + v), \quad (14)$$

$$0 = \frac{d\Pi_S}{dr_S} = [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - r_S)G'(r_S - r_L + v) - G(r_S - r_L + v)] \\ - \frac{1}{A}\alpha_L\alpha_S(f - r_L)(f - r_S)G'(r_S - r_L + v). \quad (15)$$

From the above conditions we derive

$$(r_S - r_L)G'(r_S - r_L + v) + 2G(r_S - r_L + v) = 1. \quad (16)$$

Proposition 1. *Suppose that the profit function Π_j is quasi-concave in r_j, $j \in \{L, S\}$. Also suppose that $G(v) < 0.5$. Let r_L and r_S solve equations (14)–(15). If $r_L > s$ and $r_S > s$, then it is an unconstrained equilibrium that the banks set r_L and r_S as their deposit interest rates. In this equilibrium:*

1. *The large bank sets a lower deposit interest rate ($r_L < r_S < f$) and has a larger market share ($\alpha_L > \alpha_S$) than the small bank.*

2. *The large bank uses a looser lending standard than the small bank does ($q_L^* < q_S^*$).*

Proofs are in the Appendix.

The condition $G(v) < 0.5$ ensures that the CBDC does not increase the market share of the small bank so much that it fully eliminates the large bank's convenience value advantage in its deposits. The small bank still needs to compete by offering a higher deposit interest rate than the large bank.

Further intuition of the equilibrium may be gained by considering an example. Suppose that $G(\delta) = \delta/\Delta$, where $\delta \in [0, \Delta]$ for a sufficiently large Δ. Then $G'(\cdot) = 1/\Delta$. The two first-order conditions reduce to

$$\frac{f - r_L}{\Delta} = 1 - \frac{r_S - r_L + v}{\Delta} + B, \quad (17)$$

$$\frac{f - r_S}{\Delta} = \frac{r_S - r_L + v}{\Delta} + B, \quad (18)$$

where

$$B \equiv \frac{\frac{1}{A\Delta}\alpha_L \alpha_S (f - r_L)(f - r_S)}{X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)} > 0. \tag{19}$$

As the total reserve X becomes large, B becomes close to zero. So the equilibrium deposit interest rates of the two banks become approximately $r_L \approx f - \frac{2}{3}\Delta + \frac{1}{3}v$ and $r_S \approx f - \frac{1}{3}\Delta - \frac{1}{3}v$. This shows directly how an increase in convenience reduces the spread between deposit rates.

Constrained equilibrium. The second case of the equilibrium is if the CBDC interest rate s becomes binding for the large bank. Recall the tie-breaking rule that at $r_L = s$, depositors use the large bank.

The small bank's profit function and first-order condition are as before:

$$\begin{aligned} 0 = \frac{d\Pi_S}{dr_S} &= [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - r_S)G'(r_S - s + v) - G(r_S - s + v)] \\ &\quad - \frac{1}{A}\alpha_L \alpha_S (f - s)(f - r_S)G'(r_S - s + v). \end{aligned} \tag{20}$$

By contrast, the large bank's first order condition takes an inequality because the conjectured optimal solution is at the left corner:

$$\begin{aligned} 0 > \left.\frac{d\Pi_L}{dr_L}\right|_{r_L \downarrow s} &= [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - s)G'(r_S - s + v) - 1 + G(r_S - s + v)] \\ &\quad - \frac{1}{A}\alpha_S \alpha_L (f - s)(f - r_S)G'(r_S - s + v). \end{aligned} \tag{21}$$

Proposition 2. *Suppose that the profit function Π_j is quasi-concave in r_j, $j \in \{L, S\}$. Suppose that $v < \bar{v}$ for some $\bar{v}$ that may depend on f and s. Let r_S solve equation (20). If, at r_S, equation (21) also holds, then it is a constrained equilibrium that the large bank sets s and the small bank sets r_S as their deposit interest rates. In this equilibrium:*

1. *The large bank sets a lower deposit interest rate ($s < r_S$) and has a larger market share ($\alpha_L > \alpha_S$) than the small bank.*

2. *The large bank uses a looser lending standard than the small bank does ($q_L^* < q_S^*$).*

Like Proposition 1, the condition that v cannot be too high guarantees that the small bank still wishes to compete by offering a higher deposit interest rate.

Parlour, Rajan, and Walden (2021) also analyze asymmetries in the banking sector and their consequences. In their model, a bank that is a net payer incurs an additional settlement cost and hence reduces lending, compared to net-receiving bank. In this sense, the net payer bank in their model looks like the small bank in ours. Despite similar predictions on lending, the two models are driven by different mechanisms. In our model, there is no exogenous cost associated with interbank settlement; rather, the main advantage of the large bank in lending is a higher likelihood that a lent dollar stays with the large bank and earns interest on reserves from the central bank. Moreover, the size of the large bank's advantage depends on the interest rate paid on reserves and the CBDC design, including its interest rate and convenience value, as we see in the next section.

Using confidential FedWire transaction data, Li and Li (2021) calculate the volatility of daily net payments as a fraction of daily gross payments for various banks. They find that banks with higher payment volatility pay a higher deposit interest rate and have lower loan volume growth, controlling for a set of observables. While our model does not have payment volatility, our predictions are consistent with the negative cross-sectional correlation they compute between the deposit interest rate and lending.

4 Impact of CBDC Interest Rate and Convenience Value

In this section, we begin to discuss the consequences of varying the CBDC interest rate s and convenience value v. We start off by looking at the implications of varying each variable in isolation, holding the other constant. The results are characterized in Propositions 3 and 4.

4.1 Impact of CBDC interest rate s

When the federal reserve introduced the overnight reverse repo program (ON RRP) as a temporary facility to support its IOR policy, it began by testing the facility by varying the ONRRP rate between 1 basis point and 10 basis points, while holding the IOR rate fixed at 25 basis points. Here we examine how market outcomes change as s varies from a rate of 0 to f, while holding f fixed.

We focus on the case where, given a fixed value of v, f is sufficiently low that the constrained equilibrium applies. This case is most relevant to the current economic environment in the United States. (In the unconstrained equilibrium, market outcomes are invariant to the CBDC interest rate s by definition.)

Before a formal statement of comparative statics, it is useful to illustrate the impact of CBDC interest rate s in an example. The top row of Figure 1 plots the behavior in the deposit markets as the CBDC interest rate rises from 0 to $f = 2\%$. The charts are computed numerically using a uniform distribution for G and a zero CBDC convenience value ($v = 0$). As we see in the top left plot, raising the CBDC interest rate increases the deposit interest rates of both banks as well as the weighted average deposit interest rates. The top right plot shows the corresponding changes in deposit market shares α_j, $j = L, S$, which are easily computed from (5) and (6). Since the large bank's deposit rate rises faster than the small bank's, the large bank gains market share from the small bank. Intuitively, the small bank competes with the large bank primarily by offering a higher deposit interest rate. As s increases, the maximum spread $f - s$ shrinks, limiting the small bank's ability to compete with its interest rate choice. Once the deposit rates are equal at f, the large bank obtains the entire market share of depositors, given the higher convenience value of its deposits.

The bottom row of Figure 1 illustrates the impact raising the CBDC interest rate has on the lending market. Raising the CBDC interest rate changes the incentives to make loans via the expected profit on the interest rate spread, $\alpha_j(f - r_j)$. Because, as shown above, both α_S and $f - r_S$ decrease in s, so does $\alpha_S(f - r_S)$. Thus, the small bank's loan quality threshold, $q_S^* = \frac{1+f-\alpha_S(f-r_S)}{A}$, increases in s, and its loan volume, $\alpha_S(1 - q_S^*)$, decreases in s. In this example, the large bank's loan quality threshold, $q_L^* = \frac{1+f-\alpha_L(f-r_L)}{A}$, increases in s, and its loan volume $\alpha_L(1-q_L^*)$, also increases due to its larger market share. In this example, the total loan volume declines in s.

The following proposition characterizes the impact of CBDC interest rate s on the deposit and lending markets in more general cases.

Proposition 3. *Suppose that $G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f - s + v]$. Then, for a sufficiently large X, increasing the CBDC interest rate in a constrained equilibrium has the following impact on the deposit and lending markets:*

	As s increases	
	Large	*Small*
Deposit interest rates r_L and r_S	↑	↑
Deposit market shares α_L and α_S	↑	↓
Weighted average deposit interest rate	↑	
Loan quality thresholds q_L^* and q_S^*	↑	↑
Loan volume $\alpha_L(1 - Q(q_L^*))$ and $\alpha_S(1 - Q(q_S^*))$	*unclear*	↓
Total loan volume, i.e., total deposit created	↓ *if* $G'' \leq 0$	

Most of the numerical results illustrated in Figure 1 turn out to be analytically proven in Proposition 3 under some conditions. The only theoretically ambiguous result is that the

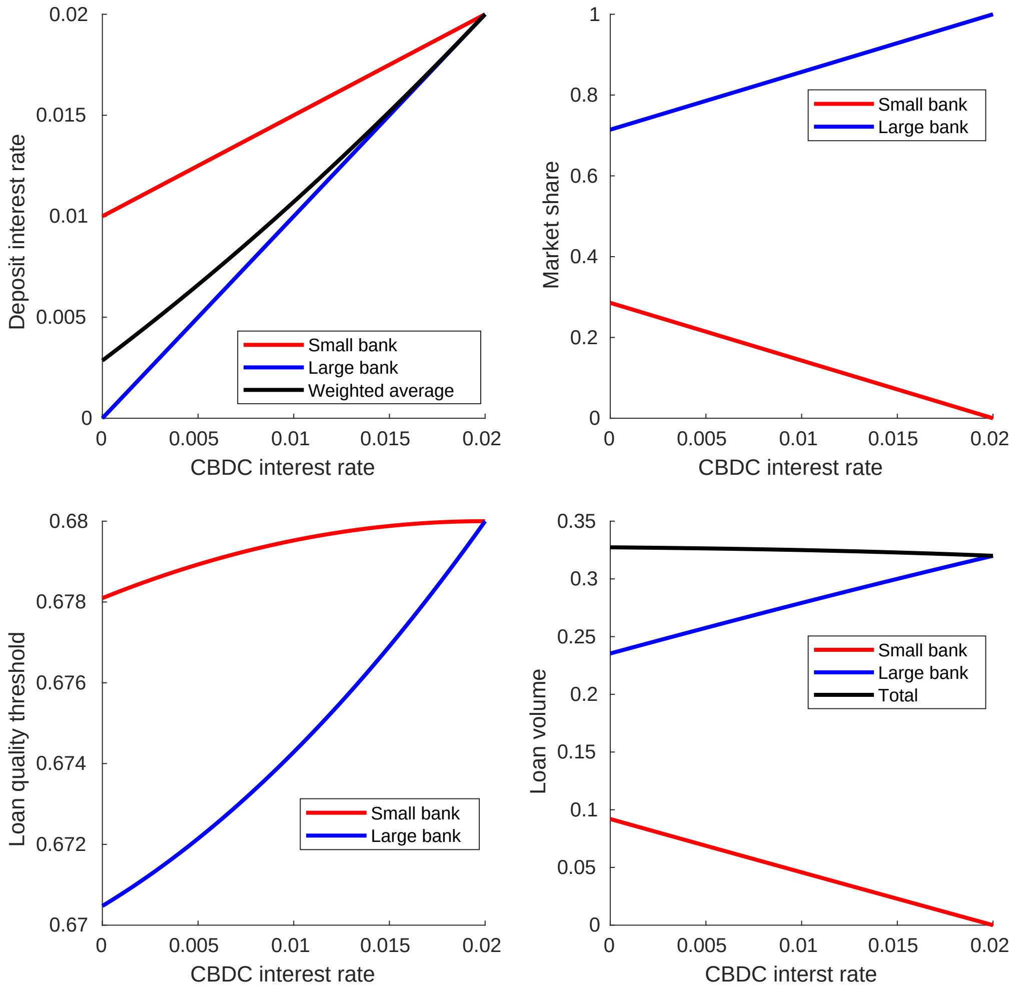

Figure 1: Impact of CBDC interest rate on deposit and lending markets. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02, v = 0$.

large bank's loan quality threshold may go up or down in s. The condition that $G''(\cdot)$ is not too large puts a bound on the mass of agents who value large bank's deposits highly, and ensures the small bank will compete to win additional depositors when s rises by raising its deposit interest rate. That is, the condition on $G''(\cdot)$ implies that $dr_S/ds > 0$. Recall that the main advantage of the large bank in the lending market is the spillover from the deposit market, that is, lent money stays with the large bank with a higher probability. The condition that the total deposit X is sufficiently large guarantees that the total profits earned from interest rate spread are large enough and thus heterogeneous enough between the two banks.

4.2 Impact of CBDC convenient value v

A convenient CBDC reduces the large bank's convenience advantage and hence has an impact even if its interest rate is zero. We illustrate the impact of a convenient CBDC by considering this polar case in Figure 2. The top row shows the outcomes in the deposit market. As v rises, the inconvenience disadvantage of the small bank shrinks. As long as the large bank's deposit interest rate remains at the floor rate, the small bank can afford to lower its interest rate and still capture a growing market share. Once v get large enough, the large bank responds by raising its interest rate; however, the small bank can still afford to continue lowering its deposit interest rate for the same reason that the convenience gap between the two banks continues to shrink. Throughout this process the large bank loses market share and the small bank gains market share, albeit at a slower rate once the large bank is no longer constrained. The overall impact of increasing the CBDC convenience value is a "convergence" of the two banks in terms of deposit interest rates and market shares. Note that in Propositions 1 and 2, as long as v is not too high, we have $r_S > r_L$.

One interesting implication is that a modest CBDC convenience value is enough to fully level the playing field. Under the uniform distribution of large-bank preference δ, when v rises to the point $v = \Delta/2$, depositors with $\delta > \Delta/2$ strictly prefer the large bank, and depositors with $\delta < \Delta/2$ strictly prefers the small bank. That is, deposit market shares become equal. So do the deposit interest rates, loan quality thresholds, and loan volume.

Another interesting observation is that the CBDC convenience value has a nuanced impact on the transmission of monetary policy. In a constrained equilibrium, a higher v results in a lower weighted average deposit interest rate when the large bank's deposit interest rate is at the lower bound, that is, a convenient CBDC weakens the transmission of monetary policy to the deposit market through IOR. Once the economy transitions to an unconstrained equilibrium with a sufficiently high v, however, a higher CBDC convenience value increases the average deposit interest rate, speeding up the transition of monetary policy.

The bottom row of Figure 2 shows the outcomes in the lending market. Because the two deposit interest rates and the deposit market shares get closer to each other as v rises, it is unsurprising that the loan quality thresholds and loan volume of the two banks are also getting closer to each other. In this example, the total loan volume is almost invariant to v, and the most salient effect is the reallocation of loans from the large bank to the small one.

Figure 3 below further exploits the impact of CBDC convenience value on loan volume, using a different parametrization of $f = 3\%$ and $s = 1.25\%$. These parameters lead to only constrained equilibrium, with $r_L = s$. In this example, the total lending volume (left axis) is

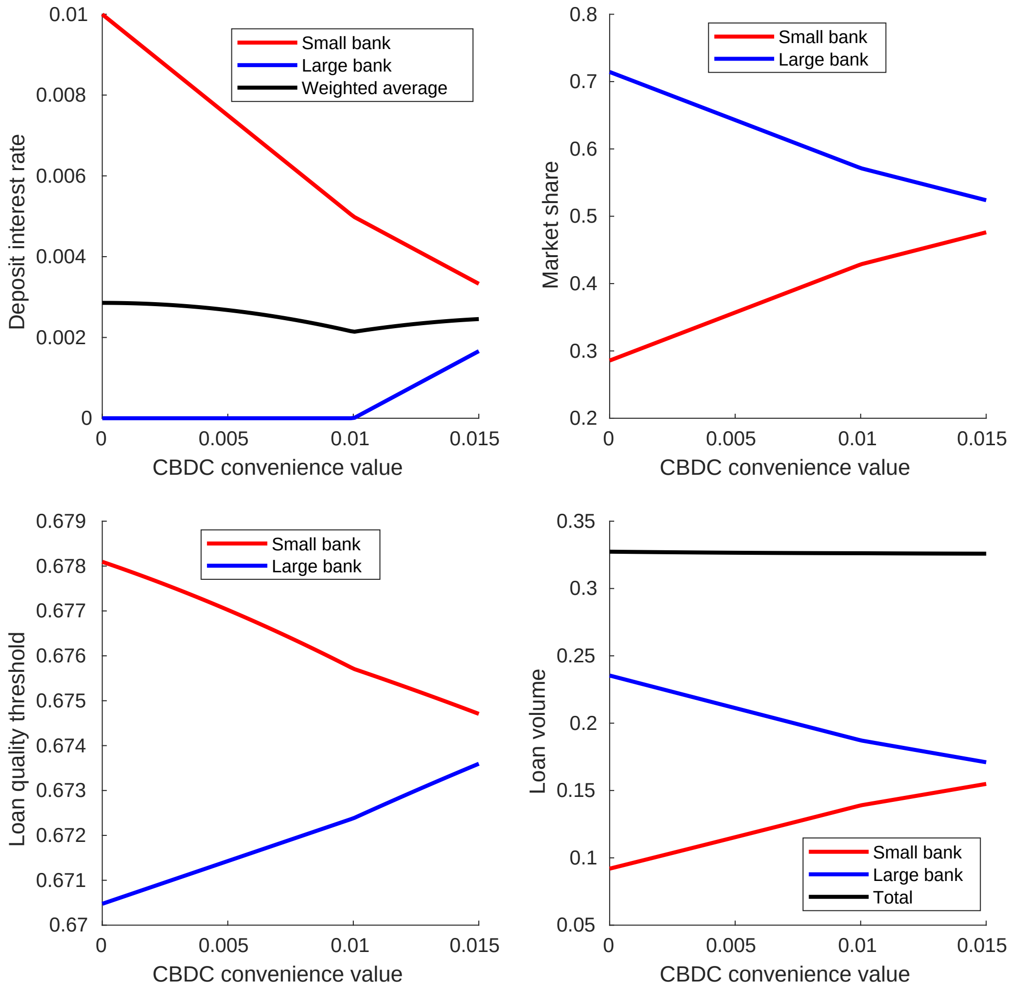

Figure 2: Impact of CBDC convenience value on deposit and lending markets. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02, s = 0$.

first decreasing in v and then increasing in v. The magnitude of the axes suggests that the more salient action is, again, the shift of lending from the large bank to the small one.

The next proposition summarizes the comparative statics with respect to v.

Proposition 4. *Suppose that G satisfies $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f - s + v]$. For a sufficiently large X, the impact of increasing v is given in the following table:*

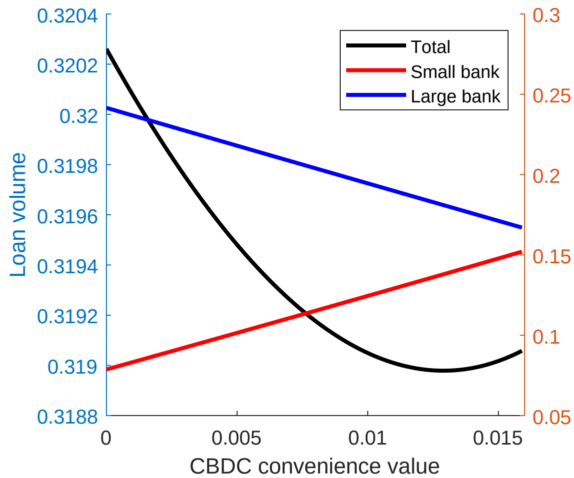

Figure 3: CBDC convenience value and loan volume. Total loan volume in on left axis. Loan volume of the two banks are on right axis. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.03, s = 0.0125$.

As v increases	*Constrained*		*Unconstrained*	
	Large	*Small*	*Large*	*Small*
Deposit interest rates r_L and r_S	*Flat(=s)*	$\downarrow$	$\uparrow$	$\downarrow$
Deposit market shares α_L and α_S	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Weighted average deposit interest rate	$\downarrow$ *if* $G'' \leq 0$		$\uparrow$ *if* $G'' \geq 0$	
Loan quality thresholds q_L^* and q_S^*	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
Loan volume $\alpha_L(1-q_L^*)$ and $\alpha_S(1-q_S^*)$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Total loan volume, i.e., total deposit created	*unclear*		$\downarrow$ *if* $G'' \geq 0$	

As before, most of the comparative statics illustrated in Figure 2 are proven analytically in Proposition 4 under some conditions. Here, the sufficient condition requires that $G(\cdot)$ cannot be too convex or too concave. The intuition is that as long as there is not too much mass of agents with very high or low large-bank convenience values, both banks would prefer competing with each other to specializing in serving distinct subsets of the population.

The impact on the weighted average interest rate, $\alpha_S r_S + \alpha_L r_L$, is nuanced. An increase in v shifts market share to the small bank and reduces the small bank's deposit interest rate, but the small bank has a higher deposit interest rate to start with, so the overall effect can be ambiguous. In the constrained equilibrium, a concave G means that relatively more depositors have a weak (but still positive) preference for large bank's deposits, so a higher v quickly eliminates the large bank's advantage. As a result, the small bank can afford to reduce its deposit interest rate quickly, leading to a lower weighted average $\alpha_S r_S + \alpha_L r_L$.

In the unconstrained equilibrium, r_L increases in v. A convex G means that relative more depositors have a strong (but still positive) preference for large bank's deposits, so the large bank raises r_L aggressively compared to the reduction in r_S, leading to a higher $\alpha_S r_S + \alpha_L r_L$.

Finally, total loan volume decreases in v in the unconstrained equilibrium if G is weakly convex. Intuitively, the weighted average deposit interest rate increases in v, so the IOR-deposit interest rate spread is compressed, which discourages lending. But in the constrained equilibrium, the impact of v on total lending can be negative or positive, as shown in Figure 3.

5 The Trade-off Between Interest and Convenience

Central banks must evaluate how the introduction of the CBDC might affect various stakeholders. In our model, there are two groups of stakeholders: households and banks. Households in the model acts as entrepreneurs and depositors. Because we have assumed that the banks take all of the surplus in the loan market, household welfare in the model is only about depositor welfare, including the interest income received on deposits and the payment convenience value of deposits. The banks, collectively, make profits by taking the profits in the lending market and paying depositors interest rate that is below the interest on reserves.

We start this analysis by studying the impact of a higher interest rate s and a higher convenience value v on depositors' welfare. Their utility, per dollar of deposit, is:

$$W = \alpha_S r_S + \alpha_L r_L + vG(r_S - r_L + v) + \int_{\delta=r_S-r_L+v}^{\infty} \delta dG(\delta). \tag{22}$$

Because the existing stock of deposit is X and the newly created deposit is $\alpha_L^*(1-q_L^*) + \alpha_S(1-q_S^*)$, the total welfare of depositors would be W scaled by the total deposits. For a sufficiently large X, the welfare received from the stock of existing deposits dominates that from the newly created deposits. Therefore, performing comparative statics with respect to W is almost without loss of generality.

The following result shows that, unsurprisingly, depositors in the economy prefer a higher CBDC interest rate and a higher CBDC convenience value.

Proposition 5. *For a sufficiently large X:*

1. *W increases in s in the constrained equilibrium if $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f-s+v]$;*

2. *W increases in v in the constrained equilibrium if* $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ *for any* $\delta \in [0, f - s + v]$*; and*

3. *W increases in v in the unconstrained equilibrium if* $0 \le G''(\delta) < G'(\delta)/f$ *for any* $\delta \in [0, f - s + v]$.

The conditions on primitive model parameters such as G mirror those in Propositions 3 and 4. These conditions primarily ensure that the two banks want to compete with each other for depositors. The implications for depositor welfare largely follows the weighted average interest rate.

An intuitive way to illustrate the depositor welfare result—and the trade-off between the CBDC interest rate and convenience value—is to plot the indifference curves of depositors. Figure 4, shows depositor indifference curves assuming a uniform distribution for G. Green dots represents the unconstrained equilibrium ($r_L > s$) and blue dots represents the constrained one ($r_L = s$). Recall that the equilibrium we characterized have $r_S > r_L$, guaranteed by the condition that v is not too high.[8] The CBDC interest rate has no impact on depositor welfare in the unconstrained equilibrium, hence the indifference curves are flat in this region. In the unconstrained equilibrium, the indifference curves generally have a downward slope. Consistent with Proposition 5, depositor welfare increases in the northeast direction.

Next, we study the impact of a higher CBDC interest rate s and a higher convenience value v on bank profits. Each bank's profit comes from two sources: loans and interest-rate spread. They sum up to

$$\Pi_L = \alpha_L \left[\frac{A}{2}(1 - q_L^{*2}) - (1+f)(1 - q_L^*) \right] + \left[X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*) \right] \alpha_L (f - r_L), \tag{23}$$

$$\Pi_S = \alpha_S \left[\frac{A}{2}(1 - q_S^{*2}) - (1+f)(1 - q_S^*) \right] + \left[X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*) \right] \alpha_S (f - r_S). \tag{24}$$

In both expressions, the term $\alpha_j[\frac{A}{2}(1 - q_j^{*2}) - (1+f)(1 - q_j^*)]$ is the profit on loans, and the other term is the profit from the interest-rate spread.

The following proposition summarizes how bank profits vary in s and v.

[8] Numerically, if a (s, v) pair produces a solution $r_L \le r_S$ in the equations of Proposition 1 or Proposition 2, we do not plot that point.

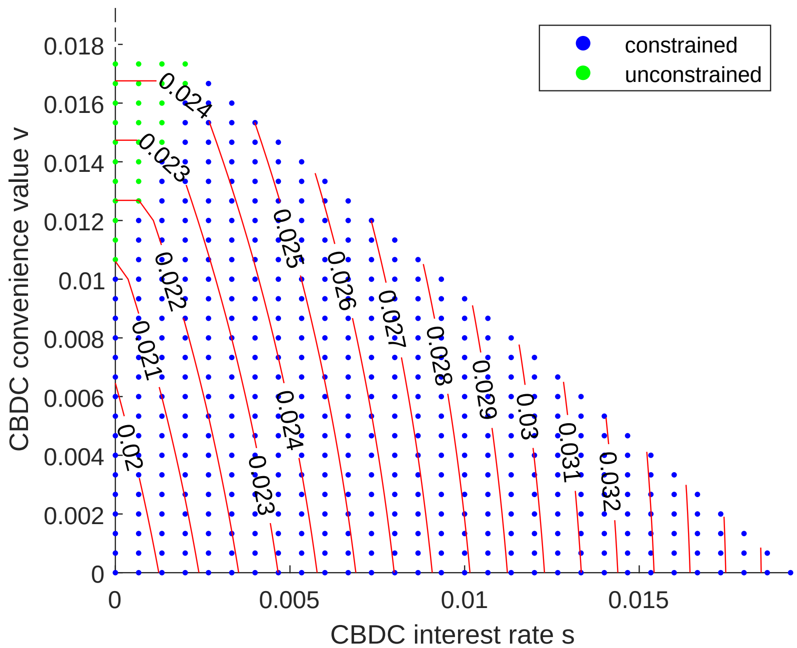

Figure 4: Interest rate-convenience indifference curves for depositors. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02$.

Proposition 6. *Suppose that* $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ *for any* $\delta \in [0, f - s + v]$*. Then, for a sufficiently large* X*, increasing the CBDC interest rate or convenience value has the following impact on bank profits:*

	As s increases	*As v increases*	
	Constrained	*Constrained*	*Unconstrained*
Total profit of both banks	$\downarrow$	$\uparrow$ *if* $G'' \leq 0$	$\downarrow$ *if* $G'' \geq 0$
Small bank's profit Π_S	$\downarrow$	$\uparrow$	$\uparrow$
Large bank's profit Π_L	$\downarrow$	$\downarrow$	$\downarrow$

Moreover, the small bank's total profit is lower than the large bank's, i.e., $\Pi_S < \Pi_L$*.*

Proposition 6 shows that both banks prefer a lower CBDC interest rate, but they differ in their preference regarding CBDC convenience value. The small bank's profit increases in v, whereas the large bank's profit decreases in v. Moreover, we are focusing on the situation with large reserves X, which means that the impact on interest rate spread is the dominant determinant of bank profits. If G is concave, total bank profit increases in CBDC convenience value v in the constrained equilibrium. This seemingly counter-intuitive result maps directly to the earlier result in Proposition 4 that the weighted average interest rate decreases in v in the constrained equilibrium under the same condition.

Figure 5 illustrates the indifference curves of the banking sector over (s, v) pairs that can sustain an equilibrium. For our purposes, it is reasonable to think about the banking sector as a whole because the higher total profit could be shared between the large bank and the small bank through other arrangements. As mentioned, an intriguing result is that the total profit of the two banks increases in v in the constrained equilibrium, whereas it declines in v in the unconstrained equilibrium. Therefore, in the constrained region, bank profit increase in the northwest direction. Moreover, while the more profitable bank (the large one) suffers from a higher v, the less profitable bank (the small one) benefits from it. Hence, financial stability, as measured by profit level of the least profitable bank, is enhanced.

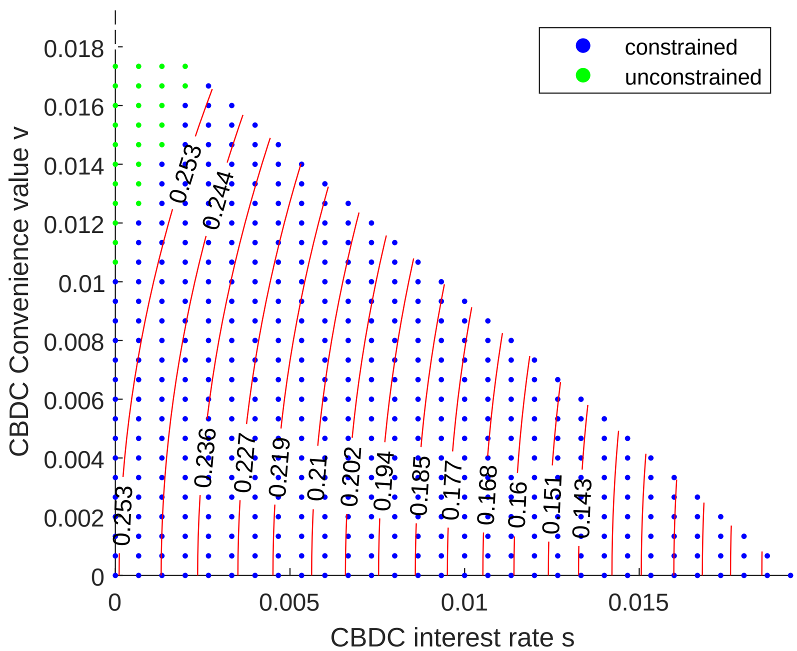

Figure 5: The interest rate-convenience trade-off for banks. Each curve represents a level of total profit of both banks. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02$.

An immediate implication of Proposition 5 and Proposition 6 is that under the general conditions of the Propositions, and focusing on the case where G is weakly concave, increasing v in the constrained equilibrium is a Pareto improvement for depositors and banks—if we view all depositors collectively as one party and the two banks collectively as the other. Pareto improvements continue up to the boundary of the constrained equilibrium. Figure 6 graphically illustrates the locations of Pareto-dominated pairs of (s, v) for uniform G; any point below the orange boundary is dominated by a point on the boundary.

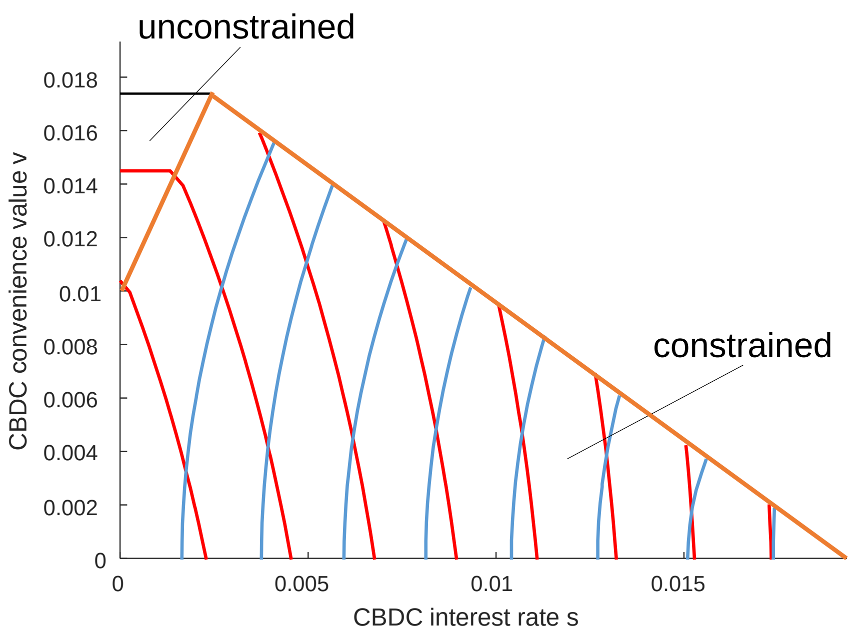

Figure 6: The interest rate-convenience value trade-off for CBDC design. Depositors' indifference curves in red increase in northeast direction ($\nearrow$), and banks' iso-profit lines increase in northwest ($\nwarrow$). Any point below the boundary (the orange line) is dominated by a point on the boundary for banks and households. Indifference curves of depositors are in red. Indifference curves of banks are in blue. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02$.

Above the orange boundary in Figure 6 there are two regions to consider. At (s, v) pairs above and to the left of the orange boundary, but below the gray line, the equilibrium is unconstrained and the surpluses of banks and depositors move in opposite directions. Hence, there is no Pareto ranking of these points. At (s, v) pairs above the upper envelope created by the orange boundary and the gray line, we are not able to characterize the equilibrium, although we can show that there does not exist a pure strategy equilibrium.[9]

Exactly which point in the design space the central bank should pick will depend on its broader monetary policy and financial stability objectives. We illustrate the trade-offs

[9] We know that $r_S < r_L$ cannot be an equilibrium as the small bank would have zero market share. So any pure strategy equilibrium must have $r_L = r_S$. If $r_L = r_L < f$, then for a sufficiently large CBDC convenience value v, a significant fraction of households would choose a bank based only on the deposit interest rates, which encourages the two banks to beat each other's interest rate by a tiny amount. If $r_L = r_L = f$, then the banks make zero interest rate spread, and the large bank would deviate by reducing the deposit interest rate by a tiny amount to make a positive profit on a fraction $1 - G(v)$ of depositors.

by considering movements along the undominated points on the design boundary shown in Figure 6. Figure 7 plots five possible central bank objectives as functions of s, along the design boundary. That is, the v used for every s is taken from the boundary of the constrained equilibrium. As shown in the left plot, a central bank that wishes to maximize monetary policy pass-through or depositor welfare would set $s = f$ and $v = 0$. In contrast, a central bank that wishes to maximize total commercial bank profit would set, in this numerical example, $s = 0$ and $v \approx 0.01$. While setting $s = f$ to maximize monetary policy pass-through barely needs a model, setting a nonzero v to maximize total bank profit does require a modeling framework like ours.

Other central bank objectives lead to subtler and possibly more interesting choices, as shown in the right plot of Figure 7. The objective of leveling playing field can be interpreted as maximizing the small bank's market share, and it has an interior solution. In this example with $G(\delta) = \delta/0.035$ and $f = 2\%$, the optimal design has $s \approx 0.0025$ and $v \approx 0.0175$, which corresponds to the "tip" of the boundary. This CBDC pays a small interest rate of 25 bps and delivers a fairly high convenience value that is equivalent to 1.75% annual interest rate. The same design also delivers the highest profit to the small bank and, to the extent that the small bank is the "weakest link" of the economy, it can be interpreted as maximizing financial stability.

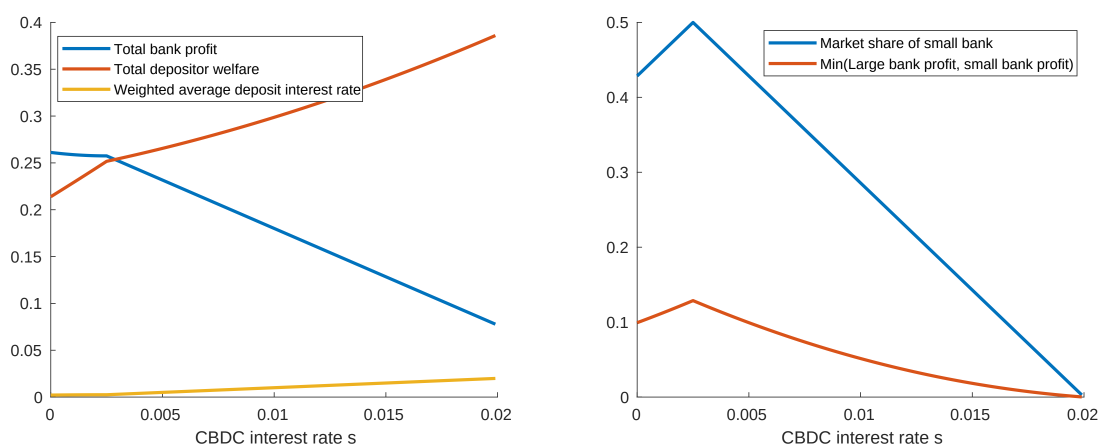

Figure 7: Central bank objectives along the CBDC design boundary. The five outcomes are plotted as functions of s, using the value of v on the boundary of the constrained equilibrium. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02$.

6 Concluding Remarks

Our objective in this study is to understand how CBDC designs impact deposit and lending markets. Our institutional setup is most suited for the current environment in the United States, where there are large quantities of excess reserves and an administered rate, the interest on reserves, that determines banks' opportunity cost of lending. We construct a model that describes how banks set deposit interest rates and decide on the project quality threshold for lending. In this regard we believe it is crucial to consider multiple, heterogeneous banks. Heterogeneous banks are required to capture differences in depositor preferences for banks of different sizes, where size is proxy for an array of preferred services and economies of scale provided by larger banks. It turns out that this modelling feature is key to understanding the impact of CBDCs on the competition and composition of the banking sector.

A CBDC could be designed to carry an interest rate and a convenience value, such as payment functionalities. These design features fit the CBDC proposals or proofs-of-concept in various countries, including the Banking for All Act that was introduced into the U.S. Senate in March of 2020. Here we identified an interesting trade-off. A higher interest rate on the CBDC enhances the pass-through of monetary policy to deposit interest rates, but it shrinks the market share of small banks. By contrast, a higher CBDC convenience value levels the playing field for competition between banks, but in doing so can weaken the transmission of monetary policy through the interest rate on reserves. Somewhat surprisingly, a convenient CBDC can strengthen the transmission of monetary policy if the CBDC convenience value is sufficiently high. Building on this analysis, we consider a two-dimensional design space in which we are able to identify the direction of improvement of CBDC design configurations and hence, the set of dominated designs.

An interesting aspect of our results is that the provision of CBDC impacts equilibrium outcomes even though the currency is not held in equilibrium. This is also true in Chiu et al. (2019) and Garratt and Lee (2020), where the option to use CBDC changes the equilibrium outcome even it is not exercised. An exception is Keister and Sanches (2020), where the CBDC has specific liquidity benefits that leads to its use. The idea that a central bank introduces a program to influence market rates by increasing the bargaining power of lenders is not new. Early descriptions of the overnight reverse repurchase agreement facility that the Federal Reserve Bank of New York began testing in September 2013 indicated that "the option to invest in ON RRPs [Overnight Reverse Repurchase Agreement Facility] also would provide bargaining power to investors in their negotiations with borrowers in money markets, so even if actual ON RRP take-up is not very large, such a facility would help provide a floor on short-term interest rates..." (Frost et al. 2015, p.7).

The results of our paper could be extended in multiple directions. One possible extension is to add short-term investment vehicles such as money market mutual funds and repurchase agreements that typically pay higher interest rates than bank deposits but cannot be easily used for processing payments. If the CBDC pays a sufficiently high interest rate, it is possible that money would flow out of these short-term investment vehicles into the CBDC, i.e., investors would earn returns from the Fed rather than short-term Treasury Bills. This additional channel is unlikely to affect lending because money market investors do not make loans anyway. Another possibility is to consider heterogeneous CBDC interest rates paid to banks of different sizes, which adds yet another degree of freedom in the central bank's toolkit. In particular, the central bank could use heterogenous CBDC interest rates to fine-tune the competitive positions of large and small banks. These extensions are left for future research.

Appendix: Proofs

Proof of Proposition 1

Taking the difference of the two FOCs, we have $(r_S - r_L)G'(r_S - r_L + v) = 1 - 2G(r_S - r_L + v)$. If $r_S \le r_L$, then the left-hand side is non-positive but the right-hand side is $1 - 2G(r_S - r_L + v) \ge 1 - 2G(v) > 0$, a contradiction. So $r_S > r_L$. This implies that $G(r_S - r_L + v) < 0.5$ in equilibrium, i.e., $\alpha_L > \alpha_S$.

Let

$$B \equiv \frac{\frac{1}{A}\alpha_L \alpha_S (f - r_L)(f - r_S)G'(r_S - r_L + v)}{X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)} > 0. \tag{25}$$

The two FOCs are separately written as

$$(f - r_L)G'(r_S - r_L + v) = \alpha_L + B, \tag{26}$$

$$(f - r_S)G'(r_S - r_L + v) = \alpha_S + B. \tag{27}$$

So both r_L and r_S are below f. Take the ratio:

$$\frac{f - r_L}{f - r_S} = \frac{\alpha_L + B}{\alpha_S + B} > 1 > \frac{\alpha_S}{\alpha_L}. \tag{28}$$

Hence, $(f - r_L)\alpha_L > (f - r_S)\alpha_S$, and $q_L^* < q_S^*$.

Proof of Proposition 2

By the assumption that the function Π_S is well behaved to admit a unique global maximum, the derivative $d\Pi_S/dr_S$ should be strictly decreasing in r_S. To show that $r_S > s$, it is sufficient that the right-hand side of (20) is positive at $r_S = s$, i.e.,

$$[X+\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*)][(f-s)G'(v)-G(v)]-\frac{1}{A}G(v)(1-G(v))(f-s)^2G'(v)>0. \quad (29)$$

Clearly, the above equation holds at $v = 0$. By continuity, it also holds if v is below a cutoff, say $\bar{v}$. If $v \in [0, \bar{v})$, we have $r_S > s = r_L$.

Let $B \equiv \frac{\frac{1}{A}\alpha_L\alpha_S(f-s)(f-r_S)G'(r_S-s+v)}{X+\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*)} > 0$. The two FOCs are separately written as

$$(f-s)G'(r_S-s+v) < \alpha_L + B, \quad (30)$$

$$(f-r_S)G'(r_S-s+v) = \alpha_S + B. \quad (31)$$

Since $B > 0$, $G'(r_S - s + v) > 0$, we know $r_S < f$. Take the difference, we have $0 < (r_S - s)G'(r_S - s + v) < \alpha_L - \alpha_S$. That is, $\alpha_L > \alpha_S$. It follows that $(f - s)\alpha_L > (f - r_S)\alpha_S$ and $q_L^* < q_S^*$.

Proof of Proposition 3

Since the large bank is constrained by the lower bound, its deposit rate rises in step with the CBDC interest rate s. Meanwhile, the small bank adjusts its equilibrium deposit interest rate at a slower pace, continuing to balance its ability to maintain depositors while its profit margin shrinks. To see how r_S is affected by s, let $\Gamma_S = d\Pi_S/dr_S$, $\Gamma_L = d\Pi_L/dr_L$, and start with the expression

$$0 = \frac{\partial \Gamma_S}{\partial s} + \frac{\partial \Gamma_S}{\partial r_S}\frac{dr_S}{ds}. \quad (32)$$

Because $\partial\Gamma_S/\partial r_S < 0$, a sufficient condition for $dr_S/ds > 0$ is $\partial\Gamma_S/\partial s > 0$. Writing total loan volume as $V = \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$, we have

$$\begin{aligned}\frac{\partial \Gamma_S}{\partial s} &= \left(-\alpha_L\frac{\alpha_L}{A}\right)[(f-r_S)G'(r_S-s+v)-G(r_S-s+v)]\\ &\quad + (X+V)[-(f-r_S)G''(r_S-s+v)+G'(r_S-s+v)]\\ &\quad + \frac{1}{A}(f-r_S)\frac{\partial}{\partial s}[\alpha_L\alpha_S(f-s)G'(r_S-s+v)]. \quad (33)\end{aligned}$$

On any closed region of f and s, the first and third term are bounded, by G being twice-differentiable. So if X is sufficiently large, the second term dominates. Under the assumption that $G''(\delta) < G'(\delta)/f$ for $\delta \in [0, f-s+v]$, we have $-(f-r_S)G''(r_S-s+v)+G'(r_S-s+v) > 0$, so a sufficiently large X would imply that $\partial\Gamma_S/\partial s > 0$, and so is dr_S/ds.

Next, we show that $r_S - s$ decreases in s. We have

$$\begin{aligned}\frac{\partial\Gamma_S}{\partial r_S} &= X\frac{\partial}{\partial r_S}[(f-r_S)G'(r_S-s+v) - G(r_S-s+v)] \\ &+ \frac{\partial\Gamma_s}{\partial r_S}\{[\alpha_L(1-q_L^*) + \alpha_S(1-q_S^*)]\cdot[(f-r_S)G'(r_S-s+v) - G(r_S-s+v)]\} \\ &- \frac{\partial\Gamma_s}{\partial r_S}\left[\frac{1}{A}\alpha_L\alpha_S(f-s)(f-r_S)G'(r_S-s+v)\right]. \end{aligned} \tag{34}$$

The second and the third term are bounded on any closed region of r_S. The first term equals $X[(f-r_S)G''(r_S-s+v) - 2G(r_S-s+v)]$. Hence,

$$\frac{dr_S}{ds} = -\frac{\partial\Gamma_S/\partial s}{\partial\Gamma_S/\partial r_S} \to \frac{(f-r_S)G''(r_S-s+v) - G'(r_S-s+v)}{(f-r_S)G''(r_S-s+v) - 2G'(r_S-s+v)}, \tag{35}$$

as X becomes sufficiently large. We also have $d(r_S-s)/ds = dr_S/ds-1 = \frac{G'(r_S-s+v)}{(f-r_S)G''(r_S-s+v)-2G'(r_S-s+v)}$, whose denomintor is negative under the condition that $G''(\delta) < G'(\delta)/f$ Hence, $d(r_S - s)/ds < 0$. This implies that $\alpha_S = G(r_S-s+v)$ is decreasing in s, and α_L is increasing in s.

The weighted average interest rate is $\alpha_S r_S + \alpha_L s$. Its derivative with respect to s is

$$\frac{d(\alpha_S r_S + \alpha_L s)}{ds} = \frac{d\alpha_S}{ds}r_S + \alpha_S\frac{dr_S}{ds} + \frac{d\alpha_L}{ds}s + \alpha_L = [(r_S-s)G'(r_S-s+v)+\alpha_S]\left(\frac{dr_S}{ds}-1\right)+1. \tag{36}$$

By the calculation earlier, as X becomes large, $\frac{dr_S}{ds} - 1 \to \frac{G'(r_S-s)}{(f-r_S)G''(r_S-s+v)-2G'(r_S-s+v)} > -\frac{f}{f+r_S}$, where the inequality follows from $G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f-s+v]$. So, as X becomes large,

$$\frac{d(\alpha_S r_S + \alpha_L s)}{ds} > 1 - \frac{f}{f+r_S}[(r_S-s)G'(r_S-s+v) + \alpha_S] \geq 1 - \frac{f}{f+r_S} > 0, \tag{37}$$

where the second last inequality follows from the large bank's FOC that, $\lim_{X\to\infty}(f - s)G'(r_S-s+v) + G(r_S-s+v) \leq 1$.

Now we turn to loan market outcomes. Since α_S decreases in s and r_S increases in s, $\alpha_S(f-r_S)$ is decreasing in s and q_S^* is increasing in s. The small bank's loan volume, $\alpha_S(1-q_S^*)$, is then decreasing in s.

For the large bank's loan quality q_L^*, we have

$$\frac{dq_L^*}{ds} = -\frac{1}{A}\Big[G'(r_S - s + v)(1 - \frac{dr_S}{ds})(f - s) - 1 + G(r_S - s + v)\Big]. \tag{38}$$

For the first term in the brackets, we know that $G'(r_S - s + v)(1 - \frac{dr_S}{ds})(f - s) < (f - s)G'(r_S - s + v)$, since $dr_S/ds > 0$. Also, from the large bank's optimality condition, as X is sufficiently large, we know that $(f - s)G'(r_S - s + v) - 1 + G(r_S - s + v) \leq 0$. That means $dq_L^*/ds > 0$ and q_L^* is increasing in s. However, the impact of s on the large bank's loan volume $\alpha_L(1 - q_L^*)$ is ambiguous.

The total loan volume is $\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$. Its derivative with respect to s is

$$\frac{1}{A}[2\alpha_S(f - r_S) - 2\alpha_L(f - s)]G'(r_S - s + v)\left(\frac{dr_S}{ds} - 1\right) - \frac{1}{A}\alpha_L^2 - \frac{1}{A}\alpha_S^2\frac{dr_S}{ds}. \tag{39}$$

While the first term is positive, the last two terms are negative. It is, however, possible to show that this derivative is negative if $G''(\delta) \leq 0$ and X is sufficiently large. As X becomes large, the two first-order conditions imply that

$$\begin{aligned} \lim_{X\to\infty}(f - s)G'(r_S - s + v) - \underbrace{(1 - G(r_S - s + v))}_{\alpha_L} &\leq 0, \\ \lim_{X\to\infty}(f - r_S)G'(r_S - s + v) - \underbrace{G(r_S - s + v)}_{\alpha_S} &= 0. \end{aligned} \tag{40}$$

Multiplying α_L to the first equation and α_S to the second equation, we have

$$\begin{aligned} \lim_{X\to\infty}\alpha_L(f - s)G'(r_S - s + v) - \alpha_L^2 &\leq 0, \\ \lim_{X\to\infty}\alpha_S(f - r_S)G'(r_S - s + v) - \alpha_S^2 &= 0. \end{aligned} \tag{41}$$

Plugging these in Equation (39), we have, as X becomes large,

$$\begin{aligned} \lim_{X\to\infty}\frac{d}{ds}(\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)) &\leq \frac{1}{A}(2\alpha_L^2 - 2\alpha_S^2)\left(1 - \frac{dr_S}{ds}\right) - \frac{1}{A}\alpha_L^2 - \frac{1}{A}\alpha_S^2\frac{dr_S}{ds} \\ &= \frac{1}{A}\left[\left(1 - 2\frac{dr_S}{ds}\right)\alpha_L^2 + \left(\frac{dr_S}{ds} - 2\right)\alpha_S^2\right], \end{aligned} \tag{42}$$

Because $\frac{dr_S}{ds} < 1$, $(\frac{dr_S}{ds} - 2)\alpha_S^2 < 0$. If $G''(\delta) \leq 0$ and X is sufficiently large, we know from the expression of $\frac{dr_S}{ds}$ above that $\frac{dr_S}{ds} \geq \frac{1}{2}$. That means $(1 - 2\frac{dr_S}{ds})\alpha_L^2 \leq 0$ as well. So the total loan is decreasing in s in the limit. Because the limit is strictly negative, it is also negative for finite but large enough X.

Proof of Proposition 4

First we consider the unconstrained equilibrium and then the constrained one.

The unconstrained equilibrium

We know that $r_L < r_S < f$, and $\alpha_S < \frac{1}{2} < \alpha_L$. Let $\Gamma_S = d\Pi_S/dr_S$, $\Gamma_L = d\Pi_L/dr_L$. To calculate how r_L and r_S are affected by v, we take derivative of Γ_L and Γ_S at the equilibrium values and obtain

$$0 = \frac{\partial \Gamma_L}{\partial v} + \frac{\partial \Gamma_L}{\partial r_L}\frac{dr_L}{dv}, \tag{43}$$

$$0 = \frac{\partial \Gamma_S}{\partial v} + \frac{\partial \Gamma_S}{\partial r_S}\frac{dr_S}{dv}. \tag{44}$$

The second-order condition implies that $\partial\Gamma_j/\partial r_j < 0$. When X is sufficiently large, the term $X[(f - r_L)G''(r_S - r_L + v) + G'(r_S - r_L + v)]$ dominates $\frac{\partial \Gamma_L}{\partial v}$. And the term $X[(f-r_S)G''(r_S-r_L+v)-G'(r_S-r_L+v)]$ dominates $\frac{\partial \Gamma_S}{\partial v}$. Since $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$, $(f-r_L)G''(r_S-r_L+v)+G'(r_S-r_L+v)$ is positive, so $\frac{dr_L}{dv} > 0$. Also, $(f-r_S)G''(r_S-r_L+v) - G'(r_S-r_L+v)$ is negative, so $\frac{dr_S}{dv} < 0$. So r_L is increasing and r_S is decreasing in v.

For deposit market share $\alpha_S = G(r_S - r_L + v)$, we take the difference of the two FOCs, and have

$$(r_S - r_L)G'(r_S - r_L + v) + 2G(r_S - r_L + v) = 1. \tag{45}$$

Write $y = r_S - r_L + v$, and take derivative of the above equation with respect to v, then we have

$$[3G'(y) + (r_S - r_L)G''(y)]\frac{dy}{dv} - G'(y) = 0 \tag{46}$$

Since $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$, we know that $3G'(y)+(r_S-r_L)G''(y) > 0$, hence $\frac{dy}{dv} > 0$. So α_S is increasing in v, and α_L is decreasing in v.

The weighted average deposit interest rate is $\alpha_S r_S + \alpha_L r_L = \alpha_S(r_S - r_L) + r_L$. Its derivative with respect to v is

$$\begin{aligned}\frac{d(\alpha_S r_S+\alpha_L r_L)}{dv} &= \frac{d\alpha_S}{dv}(r_S-r_L)+\alpha_S\frac{d(r_S-r_L)}{dv}+\frac{dr_L}{dv} \qquad (47)\\ &> \frac{d\alpha_S}{dv}(r_S-r_L)+\frac{1}{2}\frac{d(r_S-r_L)}{dv}+\frac{dr_L}{dv}=\underbrace{\frac{d\alpha_S}{dv}}_{>0}(r_S-r_L)+\frac{1}{2}\frac{d(r_L+r_S)}{dv},\end{aligned}$$

where the inequality follows from $\alpha_S<\frac{1}{2}$ and r_S-r_L decreasing in v. As X becomes large,

$$\begin{aligned}\frac{d(r_S+r_L)}{dv} &\to \frac{(f-r_L)G''(y)+G'(y)}{(f-r_L)G''(y)+2G'(y)}-\frac{(f-r_S)G''(y)-G'(y)}{(f-r_S)G''(y)-2G'(y)}\\ &= \frac{(r_L+r_S-2f)G''(y)G'(y)}{[(f-r_L)G''(y)+2G'(y)][(f-r_S)G''(y)-2G'(y)]}. \qquad (48)\end{aligned}$$

The denominator is negative as $-G'(\delta)/f<G''(\delta)<G'(\delta)/f$, and $r_L+r_S-2f<0$. So if $G''(\delta)\geq 0$, we have $\frac{d(r_L+r_S)}{dv}\geq 0$, and hence $\alpha_S r_S+\alpha_L r_L$ increases in v.

For loan quality thresholds, since α_L is decreasing in v and r_L is increasing in v, q_L^* is increasing in v. Since α_S is increasing in v and r_S is decreasing in v, q_S^* decreasing in v.

For loan volumes, $\alpha_L(1-q_L^*)$ is decreasing in v, since α_L is decreasing and q_L^* is increasing. Similarly, $\alpha_S(1-q_S^*)$ is increasing in v.

Total loan volume equals $\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*)=1-\frac{1+f}{A}+\frac{\alpha_L^2(f-r_L)+\alpha_S^2(f-r_S)}{A}$. Its derivative with respect to v is

$$\frac{1}{A}\left\{[2\alpha_S(f-r_S)-2\alpha_L(f-r_L)]\frac{d\alpha_S}{dv}-\alpha_L^2\frac{dr_L}{dv}-\alpha_S^2\frac{dr_S}{dv}\right\}. \qquad (49)$$

where $2\alpha_S(f-r_S)-2\alpha_L(f-r_L)<0$, $\frac{d\alpha_S}{dv}>0$, $\frac{dr_L}{dv}>0$, and $\frac{dr_S}{dv}<0$. We know $-\alpha_L^2\frac{dr_L}{dv}-\alpha_S^2\frac{dr_S}{dv}<-\alpha_S^2\frac{d(r_L+r_S)}{dv}$. If $G''(\delta)\geq 0$, we know from above that $\frac{d(r_L+r_S)}{dv}\geq 0$, so $-\alpha_L^2\frac{dr_L}{dv}-\alpha_S^2\frac{dr_S}{dv}\leq 0$, and so Equation (49) is negative. If $G''(\delta)<0$, however, the sign of the equation is ambiguous.

The constrained equilibrium

To calculate how r_S is affected by v, we take derivative of Γ_S at the equilibrium values and obtain

$$0=\frac{\partial\Gamma_S}{\partial v}+\frac{\partial\Gamma_S}{\partial r_S}\frac{dr_S}{dv}. \qquad (50)$$

When X is sufficiently large, the term $X[(f-s)G''(r_S-s+v)-G'(r_S-s+v)]$ dominates $\frac{\partial\Gamma_S}{\partial v}$. Since $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$, we know that $\frac{\partial\Gamma_S}{\partial v} < 0$. The second-order condition implies that $\partial\Gamma_S/\partial r_S < 0$. Hence $\frac{dr_S}{dv} < 0$, i.e., r_S is decreasing in v.

For deposit market share $\alpha_S = G(r_S - s + v)$, when X becomes sufficiently large, we have

$$\frac{dr_S}{dv} = -\frac{\partial\Gamma_S}{\partial v}/\frac{\partial\Gamma_S}{\partial r_S} \to -\frac{(f-r_S)G''(r_S-s+v)-G'(r_S-s+v)}{(f-r_S)G''(r_S-s+v)-2G'(r_S-s+v)}. \tag{51}$$

Hence,

$$\frac{d(r_S-s+v)}{dv} = \frac{dr_S}{dv}+1 = \frac{-G'(r_S-s+v)}{(f-r_S)G''(r_S-s+v)-2G'(r_s-s+v)}, \tag{52}$$

where the numerator and the denominator are both negative. So $\frac{d(r_S-s+v)}{dv} > 0$, i.e., α_S is increasing in v and α_L is decreasing in v.

For weighted average deposit interest rate, take derivative with respect to v:

$$\begin{aligned}\frac{d}{dv}(\alpha_L s + \alpha_S r_S) &= \frac{d\alpha_S}{dv}(r_S - s) + \alpha_S\frac{dr_S}{dv}\\ &= \frac{(f-s)dr_S/dv + r_S - s}{f-r_S}\alpha_S\end{aligned} \tag{53}$$

where the second equality uses $d\alpha_S/dv \to \frac{\alpha_S}{(f-r_S)}(dr_S/dv+1)$, implied by the small bank's FOC when X is sufficiently large. The derivative is negative if and only if $(f-s)dr_S/dv+r_S-s<0$. Write $y = r_S-r_L+v$. Plugging in $\frac{dr_S}{dv} = -\frac{(f-r_S)G''(y)-G'(y)}{(f-r_S)G''(y)-2G'(y)}$, the derivative is negative if and only if $G''(r_S-s+v) \leq \frac{f+s-2r_S}{(f-r_S)^2}G'(r_S-s+v)$. We now show that $f+s-2r_S \geq 0$. We know that r_S is decreasing in v. So we only need to show, given s, $f+s-2r_S \geq 0$ when $v=0$. Let $l(x) = (f-x)G'(x-s)-G(x-s)$, then $l(r_S)=0$, and $\frac{dl(x)}{dx} = (f-x)G''(x-s)-2G'(x-s) < 0$ under the condition that $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f-s+v]$. To show that $f+s-2r_S \geq 0$, we only need $l(\frac{1}{2}(f+s)) \leq 0$. That is, $\frac{f-s}{2}G'(\frac{f-s}{2}) - G(\frac{f-s}{2}) \leq 0$. This is true because if we let $m(x) = xG'(x)-G(x)$, then $\frac{dm(x)}{dx} = xG''(x) \leq 0$. And since $m(0)=0$, we have $m(\frac{f-s}{2}) \leq 0$.

For loan quality thresholds and individual banks' loan volumes, the same proofs for the unconstrained equilibrium apply and are omitted.

Total loan volume equals $\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*) = 1-\frac{1+f}{A}+\frac{\alpha_L^2(f-s)+\alpha_S^2(f-r_S)}{A}$.

Its derivative with respect to v is

$$\frac{1}{A}\left\{[2\alpha_S(f-r_S)-2\alpha_L(f-s)]\frac{d\alpha_S}{dv}-\alpha_S^2\frac{dr_S}{dv}\right\}. \tag{54}$$

Its sign is ambiguous because while the first term in the brackets is negative, the second term is positive.

Proof of Proposition 5

We know from Proposition 3 that weighted average deposit interest rate $(\alpha_S r_S + \alpha_L s)$ is increasing in s. To show that welfare W is increasing in s, we only need to show that $vG(r_S - s + v) + \int_{\delta=r_S-r_L+v}^{\infty} \delta dG(\delta)$ is increasing in s. Take its derivative with respect to s, we have

$$\begin{aligned}
&vG'(r_S-s+v)\frac{d(r_S-s)}{ds}-(r_S-s+v)G'(r_S-s+v)\frac{d(r_S-s)}{ds}\\
&=(r_S-s)G'(r_S-s+v)(1-\frac{dr_S}{ds})
\end{aligned} \tag{55}$$

For a sufficiently large X, and for G that satisfies $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f-s+v]$, we have $1 - dr_S/ds > 0$. Since $r_S - s > 0$, the derivative is positive.

In the constrained equilibrium, write $y = r_S - s + v$. Take derivative of W with respect to v:

$$\begin{aligned}
\frac{dW}{dv} &= \frac{d(\alpha_S y)}{dv}+\frac{d}{dv}\int_{\delta=r_S-r_L+v}^{\infty}\delta dG(\delta)\\
&=G'(y)\frac{dy}{dv}y+\alpha_S\frac{dy}{dv}-yG'\frac{dy}{dv}=\alpha_S\frac{dy}{dv}.
\end{aligned} \tag{56}$$

From Proposition 4 we know that $dy/dv > 0$. So $dW/dv > 0$, and welfare W is increasing in v.

In the unconstrained equilibrium, write $y = r_S - r_L + v$. Take derivative of W with respect to v:

$$\frac{dW}{dv}=\frac{d(\alpha_S r_S+\alpha_L r_L)}{dv}+G(y)+vG'(y)\frac{dy}{dv}-yG'(y)\frac{dy}{dv}. \tag{57}$$

Since $\alpha_S < \frac{1}{2} < \alpha_L$, and $d(r_S - r_L)/dv < 0$ (because r_S is decreasing in v and r_L is increasing), we have

$$\begin{aligned}\frac{d(\alpha_S r_S + \alpha_L r_L)}{dv} &= \frac{d\alpha_S}{dv}(r_S - r_L) + \alpha_S \frac{d(r_S - r_L)}{dv} + \frac{dr_L}{dv} \\ &> \frac{d\alpha_S}{dv}(r_S - r_L) + \frac{1}{2}\frac{d(r_L + r_S)}{dv} \geq \frac{d\alpha_S}{dv}(r_S - r_L). \end{aligned} \tag{58}$$

The last inequality holds because we know from Proposition 4 that $d(r_L + r_S)/dv \geq 0$ as X is sufficiently large and $0 \leq G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f - s + v]$. That means

$$\begin{aligned}\frac{dW}{dv} &\geq \frac{d\alpha_S}{dv}(r_S - r_L) + (v - y)G'(y)\frac{dy}{dv} + G(y) \\ &= G'(y)\frac{dy}{dv}(r_S - r_L + v - y) + G(y) = G(y) > 0. \end{aligned} \tag{59}$$

Proof of Proposition 6

We proved that $\alpha_S(f - r_S)$ and $\alpha_L(f - s)$ are both decreasing in s in the constrained equilibrium in Proposition 3. The two banks' profits Π_S and Π_L are dominated by $X\alpha_S(f - r_S)$ and $X\alpha_L(f - r_L)$, respectively, if X is sufficiently large. So both banks' profits are decreasing in s, and so is the total profit.

To establish the comparative statics with respect to v, we start with the unconstrained equilibrium and then consider the constrained one.

The unconstrained equilibrium

In Proposition 4, we proved that $\alpha_L(f - r_L)$ is decreasing in v, and $\alpha_S(f - r_S)$ is increasing. When X is sufficiently large, this means large bank's profit on interest rate spread is decreasing, and the small bank's is increasing in v. When X is sufficiently large, each bank's total profit is dominantly determined by profit on interest rate spread. So large bank's total profit is decreasing and small bank's total profit is increasing in v .

Total profit of both banks is dominantly determined by $X[\alpha_L(f - r_L) + \alpha_S(f - r_S)] = X[f - \alpha_L r_L - \alpha_S r_S]$ when X is large. Therefore, the impact of v on total bank profit is the opposite of its impact on weighted average deposit interest rate. The rest follows from Propostion 4.

The constrained equilibrium

Same as the unconstrained case, large bank's profit on interest rate spread and total profit are decreasing, and small bank's profit on interest rate spread and total profit are increasing in v. The proof is omitted.

When X is sufficiently large, we know from Proposition 4 that the weighted average interest rate is decreasing in v, so the total bank profit is increasing in v.

Finally, we show that $\Pi_S < \Pi_L$. In the unconstrained equilibrium, taking the difference of the two FOCs, we have $(r_S - r_L)G'(r_S - r_L + v) = 1 - 2G(r_S - r_L + v)$. So X does not affect the spread $r_S - r_L$, and consequently α_S and α_L. Since we know $\alpha_L > \alpha_S$ in the unconstrained equilibrium, there exists a constant c such that $\alpha_L - \alpha_S > c$ for any X. Denote:

$$M_L = \alpha_L \left[\frac{A}{2}(1 - q_L^{*2}) - (1+f)(1 - q_L^*)\right] + \left[\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)\right] \alpha_L(f - r_L) \quad (60)$$

$$M_S = \alpha_S \left[\frac{A}{2}(1 - q_S^{*2}) - (1+f)(1 - q_S^*)\right] + \left[\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)\right] \alpha_S(f - r_S) \quad (61)$$

Then, take the difference of the two banks' profits:

$$\Pi_L - \Pi_S = X[\alpha_L(f - r_L) - \alpha_S(f - r_S)] + M_L - M_S \quad (62)$$

Further, let

$$B(X) \equiv \frac{\frac{1}{A}\alpha_L\alpha_S(f - r_L)(f - r_S)G'(r_S - r_L + v)}{X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)} > 0. \quad (63)$$

Then from the FOCs of the two banks, we have $\alpha_L(f - r_S) - \alpha_S(f - r_S) = \frac{(\alpha_L - \alpha_S)(1 + B(X))}{G'(r_S - r_L + v)} > \frac{c}{G'(r_S - r_L + v)}$. As $0 < M_L < \frac{A}{2} + f$, $0 < M_S < \frac{A}{2} + f$, (that is, both are bounded), when $X > (\frac{A}{2} + f)\frac{G'(r_S - r_L + v)}{c}$, we have $\Pi_L - \Pi_S > 0$.

In the constrained equilibrium, let $l(x) = (f - x)G'(x - s + v) - G(x - s + v)$, then we have $\frac{dl(x)}{dx} = (f - x)G''(x - s + v) - 2G'(x - s + v) < 0$ under the condition that $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$. We know from the small bank's FOC that $l(r_S) = B(X)$, where $B(X) \equiv \frac{\frac{1}{A}\alpha_L\alpha_S(f - s)(f - r_S)G'(r_S - r_L + v)}{X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)} > 0$. That means, when $B(X)$ decreases, r_S increases.

We know that $r_S > s$ for any given X. So when $X = X_1$, there exists a constant c such that $r_S - s > c$. There must exist an X_2, such that for any $X > X_2$, $B(X) < B(X_1)$, and hence $r_S - s > c$. Take the difference of the banks' total profits:

$$\begin{aligned}\Pi_L - \Pi_S &= X[\alpha_L(f-s) - \alpha_S(f-r_S)] + M_L - M_S \\ &> X[(\alpha_L - \alpha_S)(f-r_S) + \alpha_L c] - (\frac{A}{2} + f) \qquad (64) \\ &> X\frac{c}{2} - (\frac{A}{2} + f) \qquad (65)\end{aligned}$$

So when $X > \max(X_2, (A+2f)/c)$, we have $\Pi_L - \Pi_S > 0$.

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