Online Load Balancing with Learned Weights

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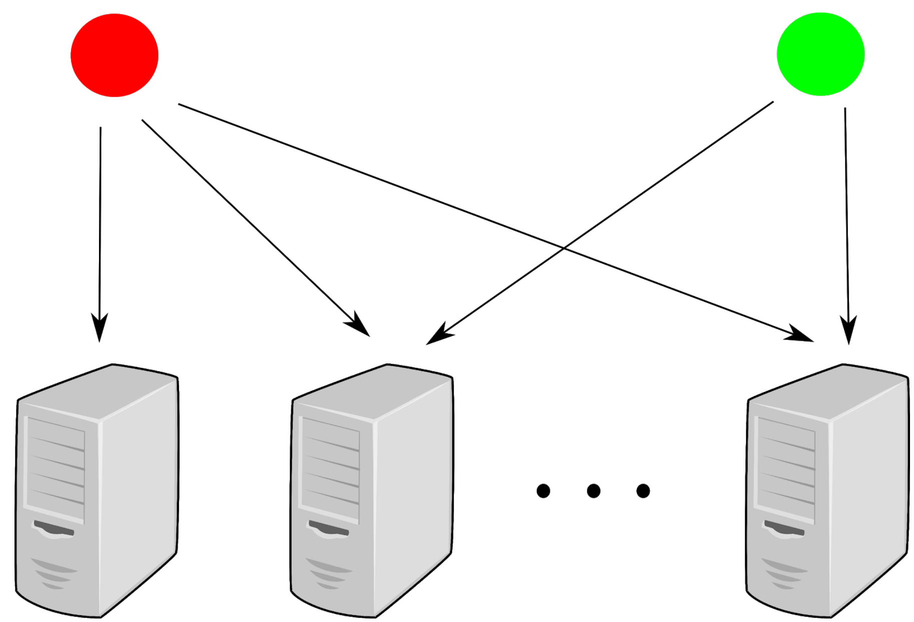
rAI

Data Center Scheduling

- Client Server Scheduling
  - Processed in m machines in the **restricted assignment** setting (more generally **unrelated machines**)
  - Jobs arrive over time in the **online-list** model
  - Assign jobs to the machines to minimize **makespan**

Load Balancing under Restricted Assignment

- m machines
- n jobs
  - Online list: a job must be immediately assigned before the next job arrives
  - N(j): feasible machines for job j
  - p(j): size of job j (complexity essentially the same if ***unit sized***)
- Minimize the maximum load
  - Optimal load is T



Online Competitive Analysis Model

- c-competitive $$\frac{ALG(I)}{OPT(I)} \leq c$$
- Worst case relative performance on each input I

- Problem well understood:
    - A $\Omega(\log m)$ lower bound on any online algorithm
    - Greedy is a $O(\log m)$ competitive algorithm [Azar, Naor, and Rom 1995]

Beyond Worst Case

- Reasonable assumption:
  - Access to job traces
- Desire a **model** to assist in assigning future jobs based on the past.
  - Predict the future based on the past.
  - What should be predicted?
  - How can it be predicted?

Learning and Online Algorithms

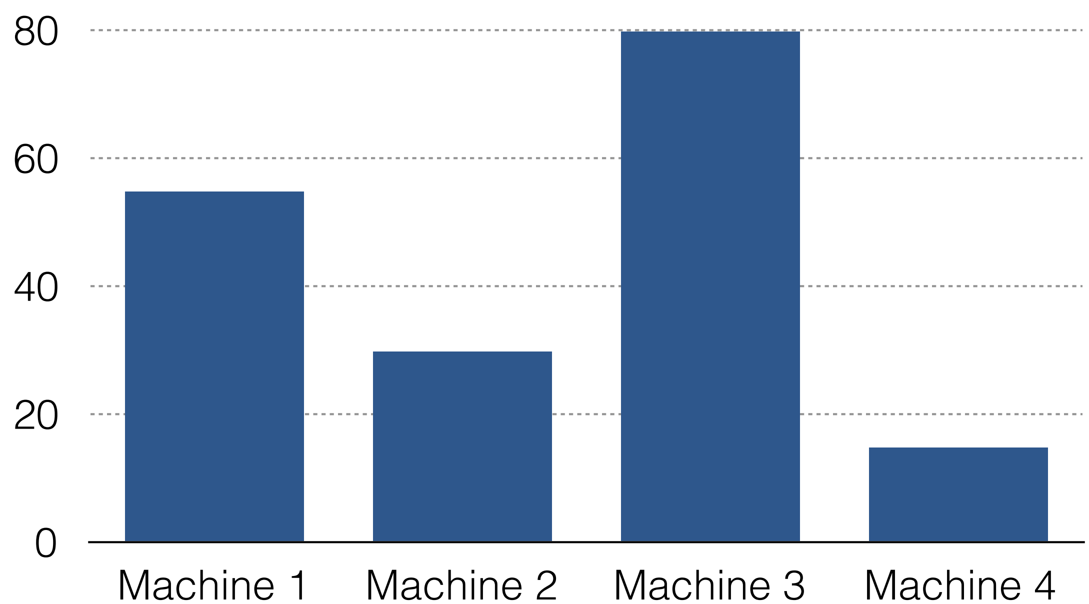
- Combining learning and optimization
  - Caching [Lykouris and Vassilvitskii 2018]
  - Ski Rental [Purohit et al 2018]
  - Non-clairvoyant scheduling [Purohit et al 2018]

Building a Model

- Guiding principals
  - **Computable** based on prior job traces
  - Predictions should be **reasonably sized**
  - Should be **robust** to error or inconsequential changes to the input
- Focus on **quantity** to predict
  - Independent of learning algorithm used to construct the prediction
  - Focus on the worst case with access to the prediction
- Goal: beat log(m) when error is small
  - Competitive ratio should depend on the error

What to Predict?

- **Load** of the machines in the optimal solution?
  - Perhaps we can identify the contentious machines?

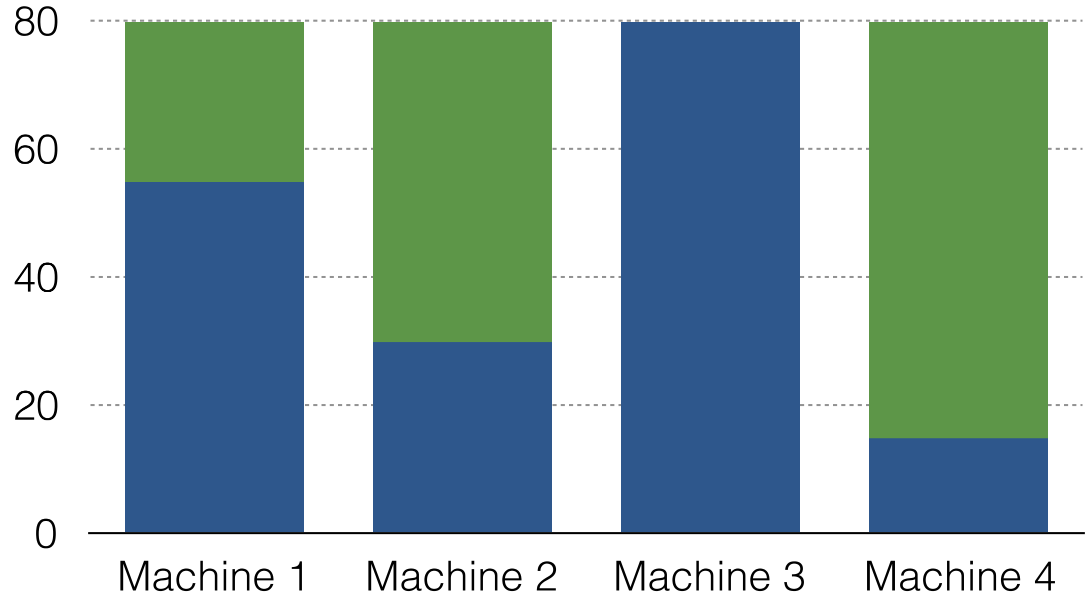


makespan 80

optimal solution

What to Predict?

- **Load** of the machines in the optimal solution?
  - Perhaps we can identify the contentious machines? **No**



optimal solution

new instance padded with dummy jobs

loads the **same**

What to Predict?

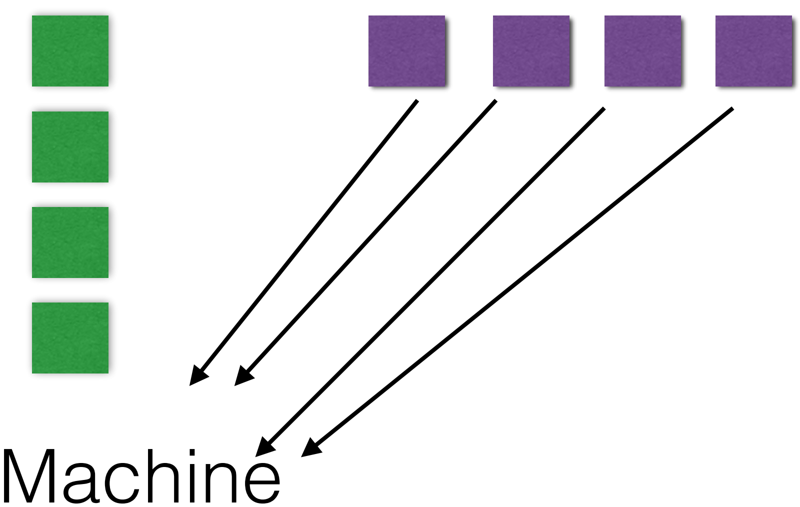
- **Number** of jobs that can be assigned to a machine?
  - Perhaps machines that can be assigned more jobs are more contentious?

What to Predict?

- **Number** of jobs that can be assigned to a machine
    - Consider the following gadget to any instance

New jobs can be assigned to old machines, skewing 'degrees' adversarially

New jobs say have a private machine.



What to Predict?

- Distribution on job types

- Is this the best predictive model?
  - 2^m job types possible
    - Need to predict a lot of information in some cases
  - Perhaps not the right model if information is sparse

What to Predict?

- Predict **dual variables**
- Known to be useful for **matching** in the **random order model** [Devanur and Hayes, Vee et al.]
  - Read a portion of the input
  - Compute the duals
  - Prove a primal assignment can be (approximately) constructed from the duals online
  - Use duals to make assignments on remaining input

What to Predict?

- Predict **dual variables** for makespan scheduling
  - Can derive primal based on dual
  - Sensitive to small error (e.g. changing a variable by a factor of $1/n^{1/2}$ has the potential to drastically change the schedule)

What to Predict?

- Idea: Capture **contentiousness** of a machine
  - Seems like the most important quantity besides types of jobs

Machine Weights

- Predict a weight for each machine
  - **Single number** (compact)
  - Lower weight means more restrictive machine
  - Higher weight less restrictive
- Framework:
  - Predict machine **weights**
  - Using to construct **fractional** assignments
  - **Round** to an **integral** solution online

Results on Predictions

- Existence of weights
  - **Theorem 1**: Let T be optimal max load. For any $\varepsilon > 0$, there **exists** machine weights and a rule to convert the weights to fractional assignments such that the resulting fractional max load is at most $(1+\varepsilon)T$.
  - **Theorem 2:** Given predictions of the machine weights with **maximum relative error** $\eta > 1$, there exists an online algorithm yielding fractional assignments for which the fractional max load is bounded by $O(T\min\{\log(\eta), \log(m)\})$.

Results on Rounding

- **Theorem 3**: There exists an **online** algorithm that takes as **input fractional assignments** and **outputs integer assignments** for which the maximum load is bounded by $O((\log\log(m))^3 T')$, where T' is maximum fractional load of the input. The algorithm is randomized and succeeds with probability at least $1 - 1/m^c$.

- **Corollary**: There exists an $O(\min\{(\log\log(m))^3 \log(\eta), \log m\})$ competitive algorithm for restricted assignment in the online algorithms with learning setting

- **Theorem 4**: Any randomized online rounding algorithm has worst case load at least $\Omega(T' \log\log m)$

Existence of Good Weights

- Each machine i has a weight w_i

- Job j is assigned to machine i **fractionally** as follows:

$$x_{i,j} = \frac{w_i}{\sum_{i' \in N(j)} w_{i'}}$$

Existence of Good Weights

- There exists weights that satisfy the following for all machines i

$$\sum_j x_{i,j} \leq (1+\epsilon)T$$

- Existence builds from [Agrawal, Zadimoghaddam, Mirrokni 2018]
  - Used for approximate maximum matching

Finding the Weights

- Algorithm sketch for computing weights **given an instance**
  - Initialize all weights to be the same
  - **While** there is an overloaded machine
    - **For** each machine i
      - Current load of machine i: $L_i = \sum_j x_{i,j} = \sum_j \frac{w_i}{\sum_{i' \in N(j)} w_{i'}}$
      - **If** $L_i \geq (1+\epsilon)T$
        - Divide w_i by $(1+\epsilon)$

Accounting for Error in the Predicted Weight

- Say we are given a prediction $\hat{w}$
- Let the error be the maximum $\eta = \max_i \frac{\hat{w}_i}{w_i}$
- If a machine is overloaded, run an iteration of the weight computation algorithm online
  - Converges in $\log \eta$ steps
  - If the load is greater than a $\log m$ factor off then revert to another online algorithm (i.e. greedy)
- Get a fractional makespan at most $O(T \min\{\log \eta, \log m\})$

Setup for Rounding Algorithm

- Jobs arrive online
- When j arrives it reveals all $x_{i,j}$ over all machines i
- Assign each job immediately when it arrives
- Compare maximum load to the **maximum factional load** seen so far

Rounding Algorithm

- Possible approaches
  - Prior LP rounding techniques
    - Techniques are too sophisticated to be used online i.e.[Lenstra, Shmoys, Tardos 1990] needs a basic solution, BFS on support graph,…
  - Deterministic rounding
    - We show a $\Omega(\log m)$ lower bound
  - Vanilla randomized rounding
    - Easy to construct instances where a machine is over loaded by $\Omega(\log m)$

Rounding Algorithm

- Use **randomized rounding** with **deterministic assignments**
- Assign jobs to machines using the distribution defined by the fractional assignment
- If a job picks a machine with load more than $Tc \log \log m$
  - c is some constant
  - The job fails
  - Let **F** be the set of failed jobs
- Assign failed jobs using **greedy** (i.e. assign to the the least loaded feasible machine)

Analysis of the Rounding Algorithm

- Assume jobs (machines) have at most $\log m$ machines (jobs) in the support of their fractional assignment.
  - Most interesting case
- Only care about failed jobs (others have small makespan)
- Consider conceptually creating a graph G
  - Nodes are failed jobs
  - Two jobs are connected if they share the same machine

Greedy on Failed Jobs

- Prove components have polylogarithmic size, say $O(\log m)$ with high probability
- Greedy is an $O(\log m')$ approximation for an instance with m' machines
  - Each component is a separate instance with number machines $m' = \texttt{polylog m}$
- Greedy gives a $O(\log m') = O(\log \log m)$ approximation to the fractional load

Future Work

- How to combine learning with optimization
  - Can predictions be used to discover improved *algorithms*?
- Theoretical model characterizing good predictions?
- Does there a exist generic algorithm for using data?

Thank you!

Questions?