For interested mathematicians, there is a longer, more detailed research statement on my website.

I study **arithmetic geometry** and **arithmetic statistics**. The fundamental problem of **arithmetic geometry** is as follows: given a collection $f_1, \dots, f_n$ of polynomials in k variables, can one understand the set of all tuples $(x_1, \dots, x_k)$ of *integers* such that $f_1(x_1, \dots, x_k) = f_2(x_1, \dots, x_k) = \cdots = f_n(x_1, \dots, x_k) = 0$, i.e. all *integral solutions* to the system $f_1 = \cdots = f_n = 0$? For example, one can set $n = 1$, set $k = 2$, look at the one polynomial $F(x, y) = y^2 - x^3 + x$ in two variables, and notice it has the solution $F(1, 0) = 0^2 - 1^3 + 1 = 0$, among others. In general, "understanding" this set of solutions can be interpreted either qualitatively (is it empty, finite, or infinite?) or quantitatively (exactly how big is it?). However, proving that a particular system of equations has even a single integral solution can be quite difficult[1]. This is where **arithmetic statistics** comes in. Instead of looking at a single system of equations, one considers a family of equations and tries to understand *statistical* properties of the integral solutions in this family (see e.g. **(2)** below). My Ph.D research follows three themes.

(1) Integral points on higher-dimensional varieties; see [AM23].

Let $f_1, \dots, f_n$ be a system of polynomials, as discussed above. The '*geometry*' in '*arithmetic geometry*' refers to the fact that one usually studies the *integral solutions* to $f_1 = \cdots = f_n = 0$ by first looking at the shape/space X formed by plotting all the real (or even complex) solutions to $f_1 = \cdots = f_n = 0$. This will often be a nice, smooth space whose geometry informs the arithmetic (e.g. set of integral solutions) of the system $f_1, \dots, f_n$. In my first project in grad school, I looked at systems where the associated X is a high-dimensional space and, together with Jackson Morrow, was able to prove, in effect, that knowing a mild geometric condition on X is enough to prove that a suitable transformation of your system only has finitely many integral solutions.[2] *I propose to make this result more explicit; for the experts, I would like to do this by defining and studying an "étale Nevalinna constant" akin to the work of Ru and Ru–Vojta* [Ru17, RV20, RV21].

(2) Statistics on the family $F_{a,b}(x, y) = y^2 - (x^3 + ax + b)$; see [Ach23].

These are the so-called '*elliptic curves*'. In this context, one usually studies rational solutions (i.e. x, y can be fractions). The polynomials $F_{a,b}$ are special because two solutions (x_1, y_1), (x_2, y_2) (possibly the same point twice) can be "combined" to produce a third (x_3, y_3). Often, repeating this procedure can lead one to constructing infinitely many solutions to this equation. One naturally asks, "Given a, b, how many solutions does one need to start with in order to generate all of them, up to finitely many exceptions?" This number is called the *rank* of $F_{a,b}$, and there has been much literature (e.g. [dJ02, BS15, HLHN14, BKL+15]) studying the *distribution* of the ranks of these equations. In particular, we are interested in computing the *average rank* of these equations. In [Ach23], I studied this average rank question in a modified setting, in which these sorts of questions had not previously seen much progress.[3] *I propose to continue this work both by improving the average rank bound I attained and by studying theoretic limitations of the "parameterize-and-count" strategy typically employed in this area.*[4]

(3) Geometric invariants of the family $F_{a,b}$; see [ABJ+24, Ach24].

Remarkably, the *collection of all* $F_{a,b}$*'s* itself forms a geometric object, the 'moduli space of elliptic curves' $Y(1)$. A *rational point* on $Y(1)$ is, equivalently, a choice of some $F_{a,b}$.[5] Moduli spaces (spaces which parameterize other geometric objects of interest) are often studied in mathematics. In [ABJ+24, Ach24], we study a particular geometric invariant (the 'Brauer group') of spaces like $Y(1)$, building on earlier work of Antieau–Meier [AM20]. Interests in this invariant stems from its ability to obstruct points on these spaces. *I propose to both expand the techniques of* [Ach24] *so they apply to more general classes of moduli spaces and to use these Brauer groups to prove that* $Y(1)$ *has no integral points.*[6] This is well-known via other means, but I hope a Brauer obstruction-theoretic proof may better generalize to studying integral points on some currently less well understood moduli spaces.

[1] This is related to *Hilbert's 10th Problem* (proved by Matiyasevich) which states that there is *no* general algorithm for deciding if even a single polynomial in many variables has any integer solutions. See [Coo04, Section 6.3] for more of the history of this problem.

[2] For the experts, the statement is essentially that if X is a smooth, projective variety with infinite (étale) fundamental group, then there are infinitely many *irreducible* divisors $D \subset X$ such that $X - D$ can only have finitely many integral points. Current techniques usually only prove such finiteness results when D has many components.

[3] For the experts, I bounded the average rank of elliptic curves over global function fields of characteristic $p = 2$ (in fact, any $p > 0$).

[4] For the experts, I would like to find an explicit value of n for which n-Selmer elements (thought of as genus 1, degree n curves in $\mathbb{P}^{n-1}$) provably cannot be parameterized.

[5] Up to *isomorphism*; if there is a nice matching between solutions of $F_{a,b}$ and $F_{c,d}$, they correspond to the same rational point on $Y(1)$.

[6] For the experts, I hope to show that the integral étale-Brauer obstruction set for the moduli *stack* of elliptic curves is empty.

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