Development and Implementation of SLAM Algorithms

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Autonomous Mobile Robots

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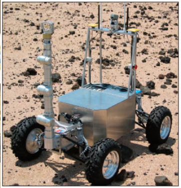

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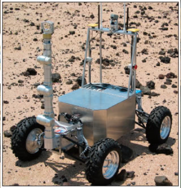


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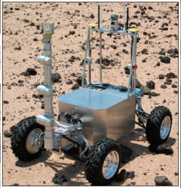

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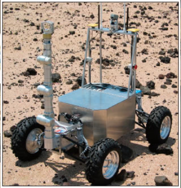

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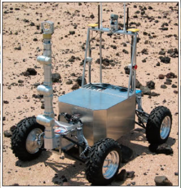

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The SLAM Problem

Assumptions

- No *a priori* knowledge about the environment (i.e. map)
- No independent position information (i.e. GPS)
- Static environment

Given

- Observations of the environment
- Control signals

Goal

- Estimate the map of the environment (e.g. locations of the features)
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Map Representation

- Topological maps
- Grid maps
- Feature-based maps

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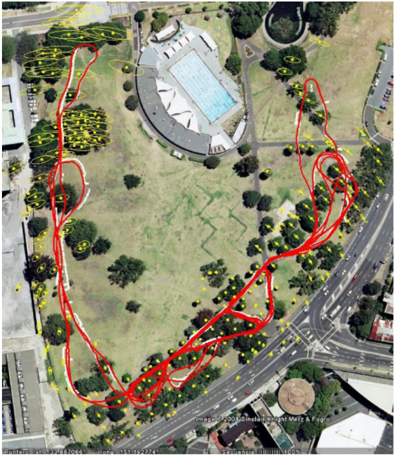

Probabilistic Methods

Probabilistic methods outperform deterministic algorithms

- Describe uncertainty in data, models and estimates
- Bayesian estimation
  - Robot pose $\mathbf{s}_t$
  - Robot pose is assumed to be a Markov process with initial distribution $p(\mathbf{s}_0)$
  - Feature's location θ_i
  - Map $\boldsymbol{\theta} = \{\theta_1, \ldots, \theta_N\}$
  - Observation $\mathbf{z}_t$, and control input $\mathbf{u}_t$
  - $\mathbf{x}_t = [\mathbf{s}_t \quad \boldsymbol{\theta}]^T$
  - $\mathbf{x}_{1:t} \triangleq \{\mathbf{x}_1, \ldots, \mathbf{x}_t\}$
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State-Space Equations

- Robot motion equation:

$$\mathbf{s}_t = f(\mathbf{s}_{t-1}, \mathbf{u}_t, \mathbf{v}_t)$$

- Observation equation:

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- $f(\cdot,\cdot,\cdot)$ and $g(\cdot,\cdot,\cdot)$ are non-linear functions
- $\mathbf{v}_t$ and $\mathbf{w}_t$ are zero-mean white Gaussian noises with covariances matrices $\mathbf{Q}_t$ and $\mathbf{R}_t$

Bayes Filter

How to estimate the posterior distribution recursively in time? Bayes filter!

1. Prediction
2. Update

$$p(\mathbf{x}_t|\mathbf{z}_{1:t-1},\mathbf{u}_{1:t}) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t)\, p(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1},\mathbf{u}_{1:t-1})\, d\mathbf{x}_{t-1} \quad (1)$$

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Motion model

Observation model

There is a similar recursive formula for the smoothing density

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Bayes Filter Cont'd.

Example

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So we have to use approximation ...

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- Unscented Kalman Filter (UKF)
- Gaussian-Sum Filter
- Extended Information Filter (EIF)
- Particle Filter (A.K.A. Sequential Monte Carlo Methods)

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Perfect Monte Carlo Sampling

Q. How to compute expected values such as $\mathbb{E}_{p(\mathbf{x})}[h(\mathbf{x})] = \int h(\mathbf{x})p(\mathbf{x})d\mathbf{x}$ for any integrable function $h(\cdot)$?

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Perfect Monte Carlo (A.K.A. Monte Carlo Integration)

1. Generate N i.i.d. samples $\{\mathbf{x}^{[i]}\}_{i=1}^{N}$ according to $p(\mathbf{x})$
2. Estimate the PDF as $P_N(\mathbf{x}) \triangleq \frac{1}{N}\sum_{i=1}^{N} \delta(\mathbf{x} - \mathbf{x}^{[i]})$
3. Estimate $\mathbb{E}_{p(\mathbf{x})}[h(\mathbf{x})] \approx \int h(\mathbf{x})P_N(\mathbf{x})d\mathbf{x} = \frac{1}{N}\sum_{i=1}^{N} h(\mathbf{x}^{[i]})$

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- Error decreases with $O(N^{-1/2})$ regardless of the dimension of $\mathbf{x}$

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But ...

It is usually impossible to sample directly from the filtering or smoothing distribution (high-dimensional, non-standard, only known up to a constant)

Importance Sampling (IS)

Idea

Generate samples from another distribution called the importance function like $\pi(\mathbf{x})$, and weight these samples according to $w^*(\mathbf{x}^{[i]}) = \frac{p(\mathbf{x}^{[i]})}{\pi(\mathbf{x}^{[i]})}$:

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Importance Sampling (IS)

Idea

Generate samples from another distribution called the importance function like $\pi(\mathbf{x})$, and weight these samples according to $w^*(\mathbf{x}^{[i]}) = \frac{p(\mathbf{x}^{[i]})}{\pi(\mathbf{x}^{[i]})}$:

$$\mathbb{E}_{p(\mathbf{x})}[h(\mathbf{x})] = \int h(\mathbf{x})\frac{p(\mathbf{x})}{\pi(\mathbf{x})}\pi(\mathbf{x})d\mathbf{x} = \frac{1}{N}\sum_{i=1}^{N} w^*(\mathbf{x}^{[i]})h(\mathbf{x}^{[i]})$$

But . . .

How to do this recursively in time?

Sequential Importance Sampling (SIS)

Sampling from scratch from the importance function $\pi(\mathbf{x}_{0:t}|\mathbf{z}_{1:t}, \mathbf{u}_{1:t})$ implies growing computational complexity for each step over time

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Sequential Importance Sampling

At time t, generate $\mathbf{x}_t^{[i]}$ according to $\pi(\mathbf{x}_t|\mathbf{x}_{0:t-1}^{[i]}, \mathbf{z}_{1:t}, \mathbf{u}_{1:t})$ (proposal distribution), and merge it with the previous samples $\mathbf{x}_{0:t-1}^{[i]}$ drawn from $\pi(\mathbf{x}_{0:t-1}|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$:

$$\mathbf{x}_{0:t}^{[i]} = \{\mathbf{x}_{0:t-1}^{[i]}, \mathbf{x}_t^{[i]}\} \sim \pi(\mathbf{x}_{0:t}|\mathbf{z}_{1:t}, \mathbf{u}_{1:t})$$

$$w(\mathbf{x}_{0:t}^{[i]}) = w(\mathbf{x}_{0:t-1}^{[i]}) \frac{p(\mathbf{z}_t|\mathbf{x}_t^{[i]})p(\mathbf{x}_t^{[i]}|\mathbf{x}_{t-1}^{[i]}, \mathbf{u}_t)}{\pi(\mathbf{x}_t^{[i]}|\mathbf{x}_{0:t-1}^{[i]}, \mathbf{z}_{1:t}, \mathbf{u}_{1:t})}$$

Degeneracy and Resampling

Degeneracy Problem

After a few steps, all but one of the particles (samples) would have very insignificant normalized weight

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Eliminate particles with low normalized weights and multiply those with high normalized weights in a probabilistic manner

- Resampling will cause sample impoverishment
- Effective sample size (ESS) is a measure of the degeneracy of SIS that can be used in order to avoid unnecessary resampling steps

$$\hat{N}_{\text{eff}} = \frac{1}{\sum_{i=1}^{N} \tilde{w}(\mathbf{x}_{0:t}^{[i]})^2}$$

Perform resampling only if N_{eff} is lower than a fixed threshold N_{T}

Proposal Distribution

- Selecting an appropriate proposal distribution $\pi(\mathbf{x}_t|\mathbf{x}_{0:t-1}, \mathbf{z}_{1:t}, \mathbf{u}_{1:t})$ plays an important role in the success of particle filter
- The simplest and most common choice is the motion model (transition density) $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)$
- $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \boxed{\mathbf{z}_t}, \mathbf{u}_t)$ is known as the **optimal proposal distribution** and limits the degeneracy of the particle filter by minimizing the conditional variance of unnormalized weights
- Importance weights for the optimal proposal distribution can be obtained as:

$$w(\mathbf{x}_{0:t}^{[i]}) = w(\mathbf{x}_{0:t-1}^{[i]})p(\mathbf{z}_t|\mathbf{x}_{t-1}^{[i]})$$

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In SLAM, neither $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{z}_t, \mathbf{u}_t)$ nor $p(\mathbf{z}_t|\mathbf{x}_{t-1}^{[i]})$ can be computed in closed form and we have to use approximation

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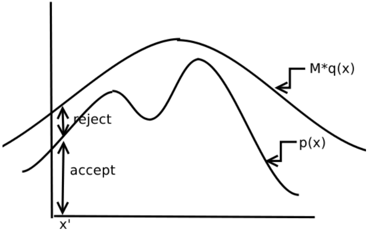

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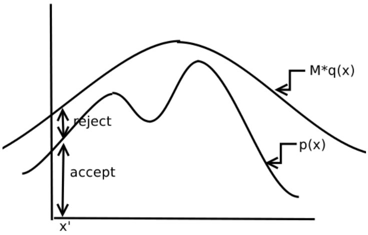

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Rejection Sampling

1. Generate $u^{[i]} \sim \mathcal{U}[0,1]$ and $\mathbf{s}_t^{[i,j]} \sim p(\mathbf{s}_t|\mathbf{s}_{t-1}^{[i]}, \mathbf{u}_t)$

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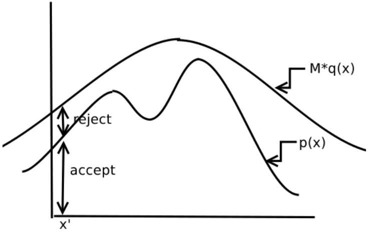

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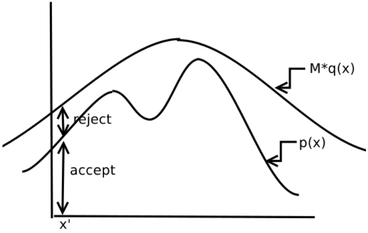

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LRS Cont'd.

Now we have to compute the importance weights for the set of accepted samples

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MC Integration

LRS Cont'd.

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MC Integration $\Rightarrow$ Large number of local particles M

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LIS-1

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Local weights are used in LIS-2 to filter those samples with low $p(\mathbf{z}_t|\mathbf{s}_t^{[i,j]})$

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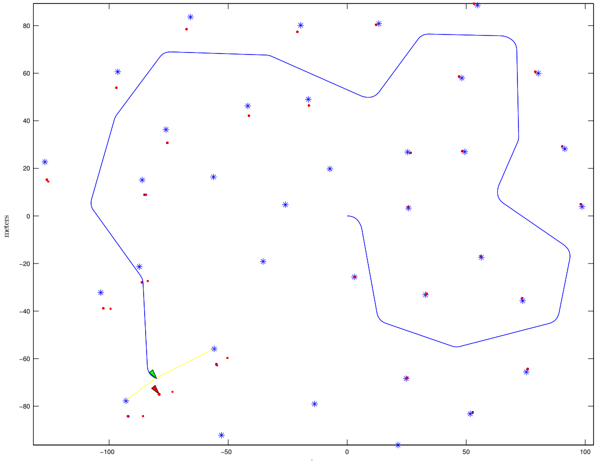
Simulation

- 50 Monte Carlo runs with different seeds
- 200m $\times$ 200m simulated environment



Number of Resamplings

Table: Average number of resampling steps over 50 MC simulations

N	FastSLAM 2.0	LRS ($M = 50$)	LIS-2 ($M = 3$)
20	221.01	180.84	188.85
30	229.79	186	193.13
40	235.71	186.55	195.33
50	237.80	188.96	196.32
60	239.55	189.62	195.99
70	240.10	189.40	197.85
80	243.61	190.39	197.96
90	244.16	190.94	198.73
100	245.70	192.69	199.50

Runtime

Table: Average Runtime over 50 MC simulations

N	FastSLAM 2.0 (sec)	LRS $(M = 50)$ (sec)	LIS-2 $(M = 3)$ (sec)
20	36	318	38
30	52	480	56
40	68	635	75
50	84	794	93
60	103	953	112
70	117	1106	130
80	134	1265	148
90	149	1413	165
100	167	1588	183

Mean Square Error

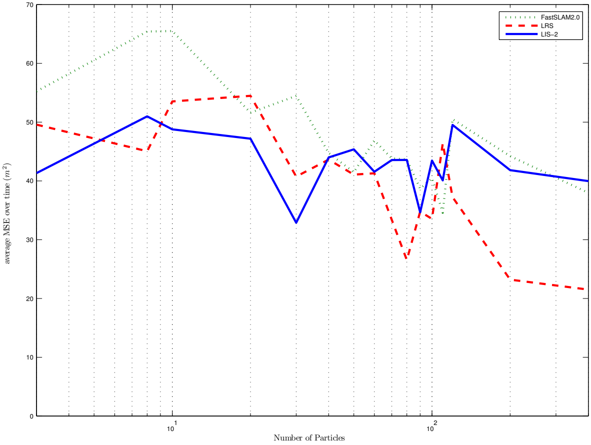


Figure: Average MSE of estimated robot pose over time. The parameter M is set to 50 for LRS and 3 for LIS-2.

Mean Square Error Cont'd.

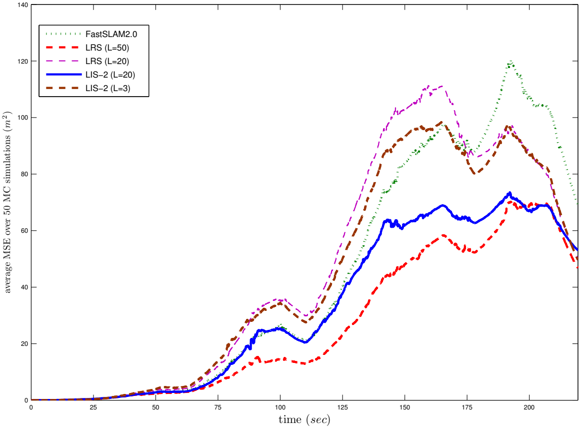


Figure: Average MSE of estimated robot position over 50 Monte Carlo simulations.

Victoria Park Dataset

- Victoria Park dataset by E. Nebot and J. Guivant

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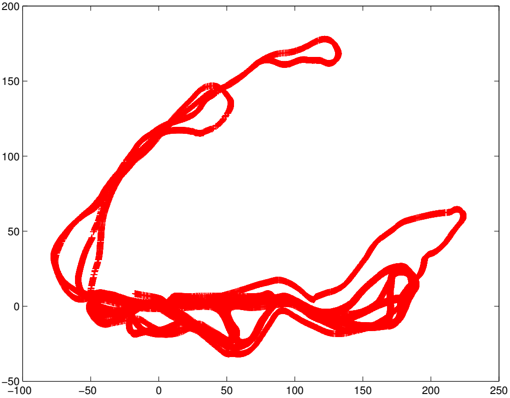

Figure: Estimated Robot Path

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Papers

- IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS) 2011: Monte-Carlo Approximation of The Optimal Proposal Distribution in RBPF-SLAM (submitted)
- . . .

Thank You . . .

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- SLAM is a very high-dimensional problem
- Estimating the $p(\mathbf{s}_{0:t}, \boldsymbol{\theta}|\mathbf{z}_{1:t}, \mathbf{u}_{1:t})$ using a particle filter can be very inefficient
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- We can estimate $p(\mathbf{s}_{0:t} | \mathbf{z}_{1:t}, \mathbf{u}_{1:t})$ using a particle filter
- $p(\theta_k | \mathbf{s}_{0:t}^{[i]}, \mathbf{z}_{1:t}, \mathbf{u}_{1:t})$ can be estimated using an EKF for each robot path particle $\mathbf{s}_{0:t}^{[i]}$
- A map (estimated locations of the features) is attached to each robot path particle

Rao-Blackwellized Particle Filter in SLAM

- SLAM is a very high-dimensional problem
- Estimating the $p(\mathbf{s}_{0:t}, \boldsymbol{\theta} | \mathbf{z}_{1:t}, \mathbf{u}_{1:t})$ using a particle filter can be very inefficient
- We can factor the smoothing distribution into two parts as

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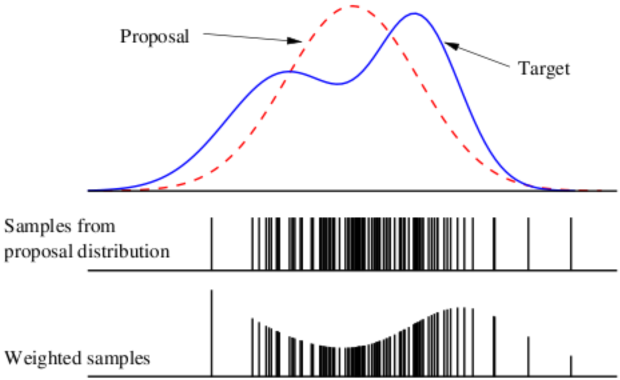
Importance Sampling



Importance Sampling (IS)

Idea

Generate samples from another distribution called the importance function like $\pi(\mathbf{x})$, and weight these samples according to $w^*(\mathbf{x}^{[i]}) = \frac{p(\mathbf{x}^{[i]})}{\pi(\mathbf{x}^{[i]})}$:

$$\mathbb{E}_{p(\mathbf{x})}[h(\mathbf{x})] = \int h(\mathbf{x})\frac{p(\mathbf{x})}{\pi(\mathbf{x})}\pi(\mathbf{x})d\mathbf{x} = \frac{1}{N}\sum_{i=1}^{N} w^*(\mathbf{x}^{[i]})h(\mathbf{x}^{[i]})$$

In practice we compute importance weights $w(\mathbf{x})$ proportional to $\frac{p(\mathbf{x})}{\pi(\mathbf{x})}$ and normalize them to estimate the expected value as:

$$\mathbb{E}_{p(\mathbf{x})}[h(\mathbf{x})] \approx \sum_{i=1}^{N} \frac{w(\mathbf{x}^{[i]})}{\sum_{j=1}^{N} w(\mathbf{x}^{[j]})} h(\mathbf{x}^{[i]})$$

But ...

How to do this recursively in time?

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SIR