

# On the control of buoyancy-driven devices in stratified, uncertain flowfields

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## Abstract

The feasibility of a buoyancy-based control approach for regulating the horizontal position of atmospheric balloons in stratified and turbulent environmental flows is investigated. Two different control rules are considered, and numerical investigations are performed for a challenging case related to tropical cyclones. The low energy requirement of the buoyancy-based control approach allows continuous, in-situ observations that extend over much longer periods, and at a significantly reduced cost, than what is achievable with current observation platforms, while maintaining a high degree of maneuverability.

## 1 Introduction

The usefulness of in-situ, high quality, and persistent observations in the atmosphere and the ocean is undisputed. Validation of theoretical and numerical models, calibration and verification of remote sensing instruments, and data assimilation algorithms rely heavily on accurate measurements of, e.g., flow velocity, pressure, temperature, and humidity as a function of location, as well as on the ability of the measurement system to target specific regions of interest in a flowfield.

Balloons, dropsondes, drones, floaters, airplanes, and ships are routinely employed to sample the atmosphere and the oceans (Houze et al., 2006; Reasor et al., 2014; Carval et al., 2015); measurements can be taken while moving with the flow as quasi-Lagrangian tracers, or while fighting it with actively controlled devices. Trade-offs between control authority — the ability of steering a device to a target destination — and endurance — the time duration of the data-acquisition mission — are often required. Airplanes and drones, which guarantee the maximum control authority, are often limited in duration by their energy requirements: their mission can last from tens of minutes to a maximum of a day (Pieri and Diaz, 2015). In contrast, super-pressure balloons can stay afloat for a month or more at a time (Doerenbecher et al., 2016), and the Argo floats can operate in the ocean environment for years, but provide more limited maneuverability.

A closer analysis reveals a unique opportunity to achieve both good control authority and extended device operational life: in highly stratified environmental flows, the different directions of the mean velocity field at different depths or altitudes can be leveraged directly to steer buoyancy-driven measurement devices with very little control energy. Forecasts of the flow velocity field can be used in a Model Predictive Control (MPC) fashion to plan in advance a schedule of density — and altitude — changes leveraging the flow stratification in order to keep the devices as close as possible to target balloons' trajectories and distributions.

At the same time, the forecast trajectory is only as good as the forecast velocity field. Uncertainty in the knowledge of the flowfield results in a divergence between the planned and actual trajectories. In this work, we address the design of a feedback control strategy

to counteract the effect of the flowfield unsteadiness due to turbulence, which is treated in this work as uncertainty or noise in the underlying velocity field.

Specifically, the control of a device in a stratified flowfield, the mean of which is depicted by blue arrows in Figure 1a, is analyzed. Its horizontal motion, representing a linearization of the motion around a forecast trajectory, is described by a random process with mean  $gz$  and spectral density  $c^2$ , where  $g$  is the gradient of the horizontal velocity with respect to the vertical direction  $z$ .

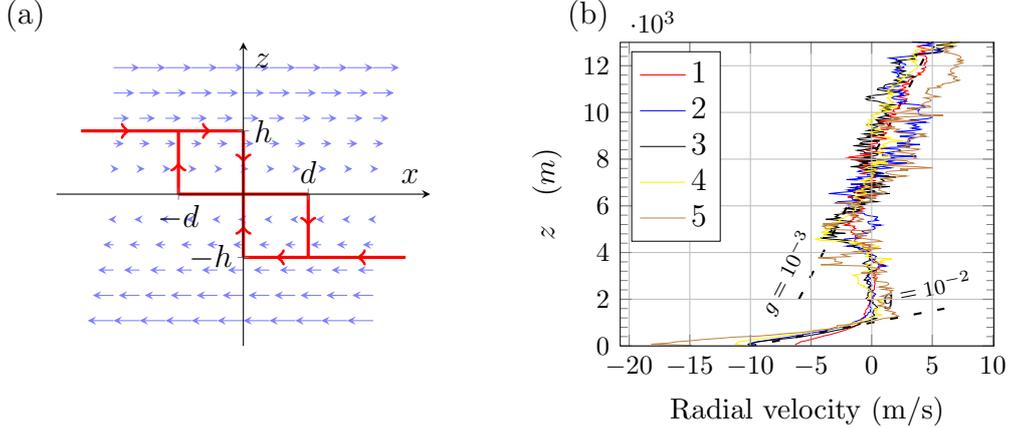


Figure 1: Left: flow field model (blue arrows) and three-step control rule (red lines). Right: radial velocity composite from dropsonde measurements between 1996 and 2012 within 200 km of the hurricane center (Wang et al., 2015), binned into 50m altitude intervals and sorted according to hurricane category (1 – 5). Dashed lines indicate a rough estimate of the mean velocity gradient  $g$ .

Assume the average velocity of the balloon is the same as the mean velocity of the flow. The stochastic differential equations (Pope, 2000, appendix J) governing the horizontal and vertical position  $(X, Z)$  read

$$\dot{X} = gZ + \xi \quad (1a)$$

$$\dot{Z} = u(X, Z) \quad (1b)$$

where capital letters denote random variables,  $u(X, Z)$  is the control rule, and  $\xi$  represents stochastic forcing, which models the uncertainty of the flowfield. For a device starting at the origin, and in the absence of control ( $u = 0$ ), the resulting horizontal position is Brownian motion with mean  $\mu_X(t) = 0$  and variance  $\sigma_X^2(t) = c^2t$ , while the vertical position remains unchanged.

The goal of the feedback control strategy  $u(X, Z)$  is to maintain the variance of the horizontal position to a constant target value  $\bar{\sigma}_X^2$  using minimal control cost  $w = \mathbf{E}[|u|]$ , where  $\mathbf{E}[\cdot]$  is the expected value operator and  $|\cdot|$  is the  $L_1$  norm. Two feedback control rules are introduced and tested: (i) a *linear* control rule  $u(X, Z) = k_1 X + k_2 Z$ , and (ii) a *three-step* control rule with hysteresis, consisting essentially of step-changes of altitude,  $\pm h$ , in the vertical position when the device reaches horizontal positions of  $\mp d$  (when  $g > 0$ ) from the target trajectory (see the red line in Figure 1a). The use of the  $L_1$  norm assures mathematical tractability of the three-step control rule, which would have infinite energy if the more commonly used  $L_2$  norm were employed.

The optimal feedback control relationship can then be computed by minimizing the objective functional

$$J = \bar{\sigma}_X^2 + l^2 w = \mathbf{E}[X^2] + l^2 \mathbf{E}[|u|] \quad (2)$$

over the values  $h, d$  and  $k_1, k_2$ , where  $l^2$  is a Lagrangian multiplier. For an imposed  $\bar{\sigma}_X^2$ , minimizing (2) is equivalent to minimizing the control cost  $w$ .

This paper is organized as follows. In §2, the reference length, time, and velocity are defined, and the functional form of the control cost  $w$  is obtained via dimensional analysis. In §3 and §4, the optimal solution for the linear and three-step feedback control rules are computed. Noting the dimensional analysis provided in §2, these solutions may easily be adapted for any values of  $g, c^2$ , and  $\bar{\sigma}_X$ . In §5 the optimal value for the parameters  $h, d, k_1, k_2$ , and the cost  $w$  are computed for both control rules, for values of  $g, c^2$ , and  $\bar{\sigma}_X$  that are typical for the control of atmospheric balloons in a hurricane. The feasibility of this buoyancy-based control approach is discussed briefly in §6. The simplicity of the three-step control rule in particular opens the door for evaluating the application of this effective and energy-efficient approach to the observation and monitoring of many other stratified flowfields in both the atmosphere and the ocean.

## 2 Dimensional analysis

The control problem is governed by three parameters: the velocity gradient  $g$ , the spectral density  $c^2$ , and the target horizontal standard deviation  $\bar{\sigma}_X$ . A single dimensionless parameter can be defined as

$$R = \frac{\bar{\sigma}_X^2 g}{c^2}. \quad (3)$$

while length, time, and velocity can be scaled via  $L = \sqrt{c^2/g}$ ,  $T = g^{-1}$  and  $U = L/T = \sqrt{c^2 g}$ . Accordingly, the control cost can be written  $w = \mathbf{E}[|u|] = U \mathcal{F}(R)$ , where  $\mathcal{F}(R)$  is an unknown dimensionless function.

Additionally, rescaling the vertical coordinate and control velocity by the time scale  $g^{-1}$  reduces the parameters governing the problem to the standard deviation  $\bar{\sigma}_X$  and the spectral density  $c^2$ . Dimensional analysis (Barenblatt, 1996) can then be used to express the control cost as:

$$w = k_w \frac{c^4}{g \sigma_X^3} = k_w U R^{-\frac{3}{2}} \quad (4a)$$

## 3 Linear control rule

We first consider a linear feedback control rule in the form  $u = -k_1 x - k_2 z$ . The governing equations (1) can be recast in matrix form as

$$\dot{\mathbf{X}} = A\mathbf{X} + \boldsymbol{\xi} \quad \text{with} \quad \mathbf{X} = \begin{bmatrix} X \\ Z \end{bmatrix}, \quad A = \begin{bmatrix} 0 & g \\ k_1 & k_2 \end{bmatrix}, \quad \boldsymbol{\xi} = \begin{bmatrix} \xi \\ 0 \end{bmatrix}, \quad (5)$$

where  $A$  is the closed-loop system matrix. Let  $(\cdot)^T$  denote the transpose. Under the white noise hypothesis, the variance  $\sigma_X^2$  of the horizontal position  $x$ , and the control cost  $w$ , can be computed as  $\sigma_X^2 = \frac{c^2(k_2^2 + gk_1)}{2gk_1k_2}$  and  $w = \frac{ck_1}{\sqrt{\pi k_2}}$ , as detailed in (Meneghello et al., 2016, under preparation). The control gains  $k_1, k_2$  and the optimal control cost  $w$  can be computed by minimization of the objective functional (2). Results are summarized in Figure 2.

## 4 Three-step control rule

We now consider the three-step, nonlinear feedback control rule indicated by red lines in Figure 1a, corresponding to step changes in altitude at  $x = 0, \pm d$ . This is a valid approximation when the control velocity  $u$  is much larger than the horizontal velocity scale  $\sqrt{c^2 g}$ . In this limit, the governing equations (1) reduce simply to

$$\dot{X} = -g\bar{Z} + \xi, \quad (6)$$

where  $\bar{Z} = \int^t u d\tau$  can take the discrete values  $\{-h, 0, h\}$ .

The steady-state solution of (6) can be obtained analytically in terms of the probability density function (PDF) of  $X$ , as detailed in (Meneghello et al., 2016, under preparation). Defining the length scale  $\lambda = \frac{c^2}{gh}$ , the variance can be computed as

$$\sigma_X^2 = \frac{d^3 + 2\lambda d^2 + 3\lambda^2 d + 3\lambda^3}{6(d + \lambda)} \quad (7)$$

The control cost can be computed as the total transition probability between the states  $\bar{Z} = 0$  and  $\bar{Z} = h$ , multiplied by the cost of the single control activation  $h$ :

$$w = \frac{2ghc^2}{d(d + \lambda)} \quad (8)$$

The control parameters  $h$  and  $d$ , and the corresponding control cost  $w$ , can then be computed by minimization of the objective functional (2).

It is also of interest to compute the minimum variance attainable for given values of  $d$  and  $h$ , and from there obtain the limiting values of  $d$  and  $h$  for a specified  $\bar{\sigma}_X$ :

$$\lim_{h \rightarrow \infty} \sigma_X^2 = \frac{d^2}{6} \implies d < \sqrt{6} \bar{\sigma}_X \quad \lim_{d \rightarrow 0} \sigma_X^2 = \frac{c^4}{2g^2 h^2} \implies h > \frac{1}{\sqrt{2}} \frac{c^2}{\bar{\sigma}_X g} \quad (9)$$

In both limits the control cost tends to infinity, in the first case because  $h \rightarrow \infty$ , and in the second because the frequency of the steps increases without bound. Reasonable (finite) values of  $h$  and  $d$ , away from these limiting values, are thus important in application. Results are summarized in Figure 2.

## 5 Application to the control of atmospheric balloons within a hurricane

Realistic values for the parameters of both control rules in the case of atmospheric balloons within a hurricane are computed in this section. A velocity gradient  $g = 10^{-3} \text{ s}^{-1}$  (see Figure 1b) and a spectral density  $c^2 = 1500 \text{ m}^2 \text{ s}^{-1}$  (Zhang and Montgomery, 2012) are used. It is important to remark that these are order-of-magnitude estimates; the actual values can be measured directly by the instrumented balloons, and are expected to show significant variations even within a single hurricane.

We additionally impose a target standard deviation  $\bar{\sigma}_X = 3 \text{ km}$ , resulting in a dimensionless parameter  $R = 6$ . Control parameters and control cost can be obtained for both control rules. For the *linear* control rule,

$$w = 4.32 \times 10^{-2} \text{ m/s}, \quad k_1 = 3.125 \times 10^{-5} \text{ s}^{-1}, \quad k_2 = 2.500 \times 10^{-4} \text{ s}^{-1}. \quad (10)$$

For the *three-step* control rule

$$w = 4.54 \times 10^{-2} \text{ m/s}, \quad d = 4886.4 \text{ m}, \quad h = 558.3 \text{ m}, \quad f = 8.11 \times 10^{-5} \text{ s}^{-1}, \quad (11)$$

where  $f$  corresponds to a period of 3 hours 25 minutes. Simulations results are shown in Figure 2d,e.

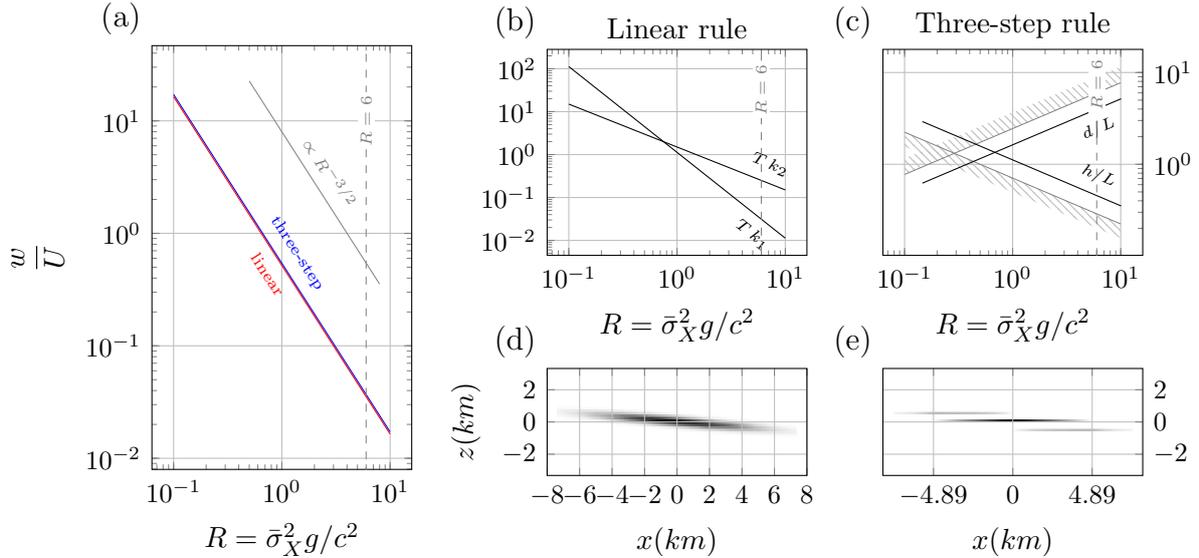


Figure 2: Comparison of control rules: (a) control cost  $w = \mathbf{E}[|u|]$  as a function of the dimensionless parameter  $R$  for the two different control rules; the two control costs are almost indistinguishable, differing by less than 5%. Control parameters (b,c) and probability density of the device position for the hurricane case described in §5 ( $R = 6$ ) (d,e) for the linear and the three-step feedback control rules; gray patterns in (c) define the limit of each curve for which a given value of  $R$  is attainable, given by (9).

## 6 Conclusions

Two control rules for buoyancy controlled devices have been introduced, and expressions for the control parameters and cost have been obtained. It is remarkable how, despite the completely different control approaches used, the cost associated with maintaining a target standard deviation of the balloon’s horizontal position is almost identical for the two control strategies. In application, the three-step control rule appears to be much more practical and efficient to implement, as the actuators can be shut off completely for the majority of the time; this also facilitates the taking of measurements for extended periods of time while the balloon is freely drifting as a Lagrangian tracer of the large-scale fluid motions.

Realistic values of the control parameters have been estimated for a hurricane. Remarkably, even in this challenging environment, the buoyancy-control approach introduced here provides the ability to track a target trajectory within the highly turbulent velocity field of a hurricane using time-averaged vertical velocities of only a few centimeters per second. Estimates for energy requirements (not included here) for a 2 kg balloon are of the order of 20 Wh (that is, the total battery charge in three iPhones) over a week of operation, even accounting reasonably for operational inefficiencies, thus allowing much longer observation missions than possible using current operational methods.

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