

## Homework 6

*Multivariable Analysis, Spring 2014*

*Instructor: Mert Gurbuzbalaban*

- Assigned: Apr 18, Due: May 5 (in class)
  - Please do not submit homework by email. Submit a hard copy to me in class. For HW submissions after this, please reach out to our grader Insuk directly and send him a reminder([insuk@cims.nyu.edu](mailto:insuk@cims.nyu.edu)). Late HWs will be assessed a late penalty of 10% per day.
  - Please staple your homeworks.
  - Ground rules for homework: you can get help from any source (friends, relatives, books, the web) but you must acknowledge the source in your submission.
1. (a) (Problem 7.2.6 from the book) A function  $f$  is an integrating factor for a one-form  $\omega$  if  $f(x) \neq 0$  for every  $x \in D$  and  $f\omega$  is exact. Show that if  $w$  has an integrating factor then  $\omega \wedge d\omega = 0$ .  
(b) Discuss how integrating factors are useful for solving differential equations (ODEs) of the form:

$$\frac{dy}{dx} + P(x)y = Q(x).$$

2. Problem 7.4.8 from the book.
3. Problem 7.5.12 from the book.
4. Problem 8.1.7 from the book.
5. Problem 8.2.6 from the book. Note: Stereographic projections are useful to visualize Möbius transformations and have applications in complex analysis.
6. **(Hard)** Let  $\mathcal{O}_{d,2}$  denote the set of real  $d \times 2$  matrices with orthogonal columns. Show that this is a manifold. Compute its dimension and its tangent space. Hint: This manifold is known as the Stiefel manifold and its characterizations are available through many sources online.
7. Problem 8.3.6 from the book.