Homework 5

Multivariable Analysis, Spring 2014

Instructor: Mert Gurbuzbalaban

- Assigned: Apr 18, Due: Apr 28 (in class)
- Please do not submit homework by email. Submit a hard copy to me in class. For HW submissions after this, please reach out to our grader Insuk directly and send him a reminder(insuk@cims.nyu.edu). Late HWs will be assessed a late penalty of 10% per day.
- Please staple your homeworks.
- Insuk has a one-time special office hour for claiming all the grades of assignments/exams, the exact date is:

  April 24th, Thursday, 1:00-2:30pm, WWH 1004

  If there is any error/question about your grades, please wait until then for your objections/questions.
- Ground rules for homework: you can get help from any source (friends, relatives, books, the web) but you must acknowledge the source in your submission.

Note that one more question at the end is added to this homework by April 21st.

1. Section 4.7 of the book: Problems 1, 2.
2. Section 6.2 of the book: Problem 5.
3. Section 6.3 of the book: Problem 3.
4. Section 6.4 of the book: Problem 1,2.
5. Section 7.1 of the book: Problem 3,4.
6. Section 7.2 of the book: Problem 2,3.
7. Let

$$\mathcal{P}_1^7 = \{p(z) : p(z) = z^7 + \sum_{i=1}^{7} a_i z^{7-i}, a_i \in \mathbb{R} \text{ for all } i = 1, 2, \ldots, 7\}$$

denote the set of monic polynomials of degree 7 (polynomials of degree 7 with a leading coefficient equal to 1).

Assume that we have an affine constraint on the coefficients $a_i \in \mathbb{R}$ of the form $B_0 + \sum_{i=1}^{7} B_i a_i = 0$ where $B_i \in \mathbb{R}$ and not all B_i are zero, i.e., $\sum_{i=0}^{7} |B_i| > 0$. Consider the subset of monic polynomials satisfying this constraint:

$$\mathcal{C} = \{p(z) \in \mathcal{P}_1^7 : p(z) = z^7 + \sum_{i=1}^{7} a_i z^{7-i}, B_0 + \sum_{i=1}^{7} B_i a_i = 0\}$$

$$\inf_{p \in \mathcal{C}} \max\{|z| : p(z) = 0\}$$

(a) **(Easy)** Show that the infimum above is attained. Hint: Use compactness and continuity.

(b) **(Hard)** Show that there exists an optimizer of the form $(z-\gamma)^k(z+\gamma)^{7-k}$ for some k and γ. Hint: Starting with an optimizer, use implicit function theorem to show that you can perturb the roots in certain directions if you need to, while staying optimal.