Homework 1

Multivariable Analysis, Spring 2014

- Assigned: Feb 3, Due: Feb 17 (anytime)
- Please do not submit homework by email. Submit a hard copy to me in class or leave it under my office door later, but not in my mailbox.
- Ground rules for homework: you can get help from any source (friends, relatives, books, the web) but you must acknowledge the source in your submission.

1. Section 1.5 of the book: Problems 1, 6, 9.
2. Section 2.11 of the book: Problems 2, 4.
3. For $p \geq 1$, we define the p-norm as

$$\|\mathbf{x}\|_p := \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}.$$

Note that for $p = 1$ we get the taxicab norm (or Manhattan norm)[1], for $p = 2$ we get the Euclidean norm.

(a) Prove that as $p \to \infty$, the p-norm approaches the infinity norm (or maximum norm), i.e. for $x \in E^n$, we have

$$\|x\|_\infty = \lim_{p \to \infty} \|x\|_p.$$

(b) For $0 \leq p < 1$, this is not a norm as we do not have the triangle inequality anymore. For $p = 0$, by a limit argument as p approaches zero, we get

$$\|x\|_0 = \#(i|x_i \neq 0)$$

which is the number of non-zero entries of the vector x. Prove that $\|\cdot\|_0$ is not a norm.

Even though $\|\cdot\|_0$ is not an actual norm, by the abuse of terminology, it is called the l_0 "norm" (quotation marks indicate that it is not a proper norm). It has uses in scientific computing, information theory, statistics, signal processing and compressed sensing.

[1] This name alludes to the grid layout of most streets on the island of Manhattan, which causes the shortest path a car could take between two intersections in the borough to have length equal to the intersections' distance in taxicab geometry.