Near Neighbor Problem Made Fair

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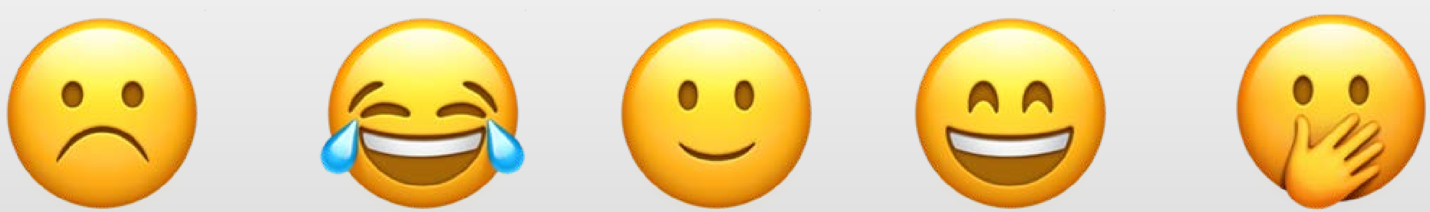
Nearest Neighbor Problems

- **Nearest Neighbor:** Given a set of objects, find the closest one to the query object.



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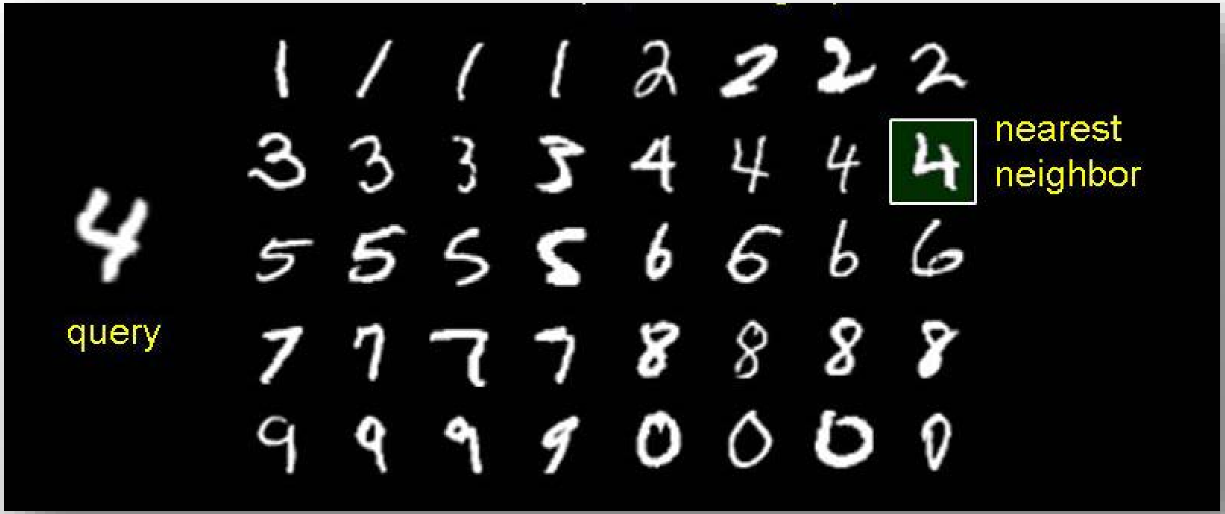
Nearest Neighbor Problems

- **Nearest Neighbor:** Given a set of objects, find the closest one to the query object.

- **Near Neighbor:** given a set of objects, find one that is close enough to the query object.

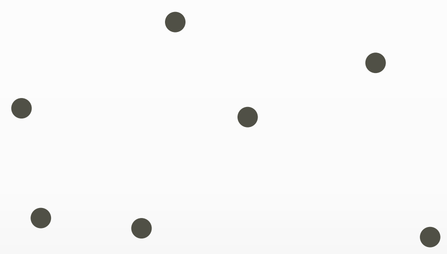
There are many applications of NN

Searching for the closest object



Near Neighbor

Dataset of n points P in a metric space, e.g. $\mathbb{R}^d$, and a parameter r



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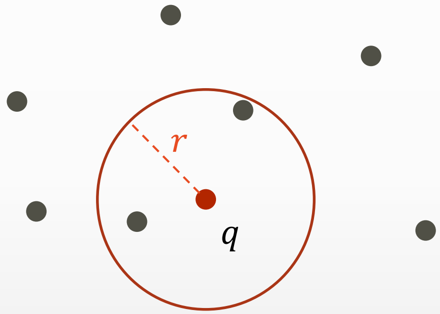
A query point q comes online



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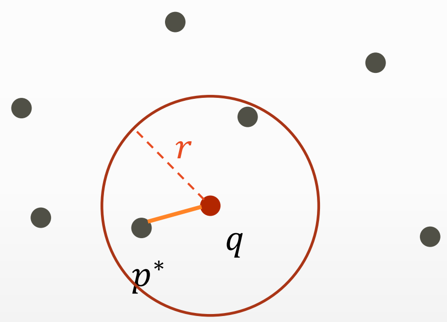

Goal:

- Find a point p^* in the r-neighborhood

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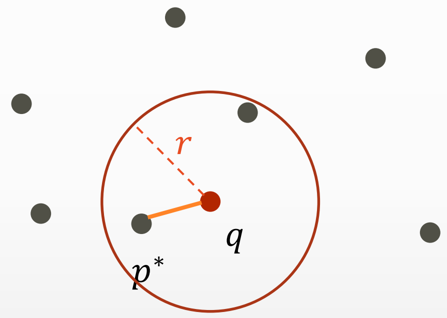

Goal:

- Find a point p^* in the r-neighborhood
- Do it in sub-linear time and small space

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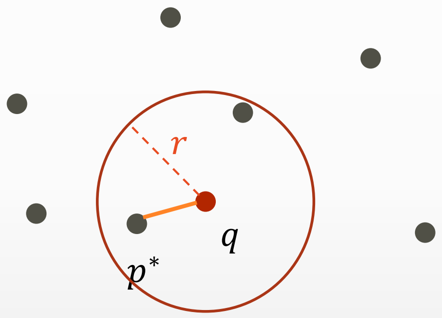

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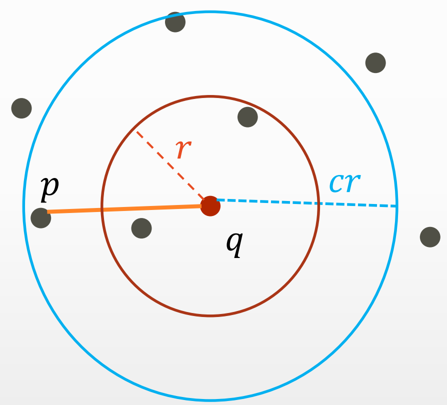
All existing algorithms for this problem

- Either space or query time depending exponentially on d
- Or assume certain properties about the data, e.g., bounded intrinsic dimension

Approximate Near Neighbor

Dataset of n points P in a metric space, e.g. $\mathbb{R}^d$, and a parameter r

A query point q comes online



Goal:

- Find a point p^* in the r-neighborhood
- Do it in sub-linear time and small space
- Approximate Near Neighbor
  - Report a point in distance cr for $c > 1$

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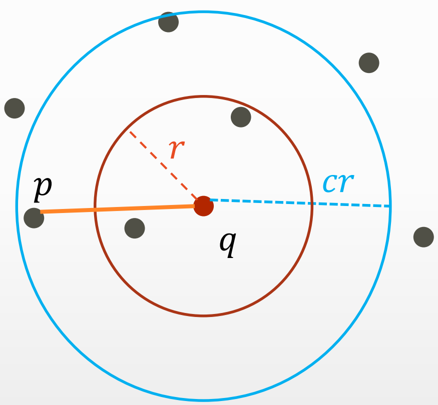

Goal:

- Find a point p^* in the r-neighborhood
- Do it in sub-linear time and small space
- Approximate Near Neighbor
  - Report a point in distance cr for $c > 1$
  - For Hamming (and Manhattan) query time is $n^{O(1/c)}$ [IM98]
  - and for Euclidean it is $n^{O(\frac{1}{c^2})}$ [AI08]

Fair Near Neighbor

Sample a neighbor of the query uniformly at random

- ❑ Individual fairness: every neighbor has the same chance of being reported.
- ❑ Remove the bias inherent in the NN data structure

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Applications:

- ❑Removing noise, k-NN classification
- ❑Anonymizing the data
- ❑Counting the neighborhood size

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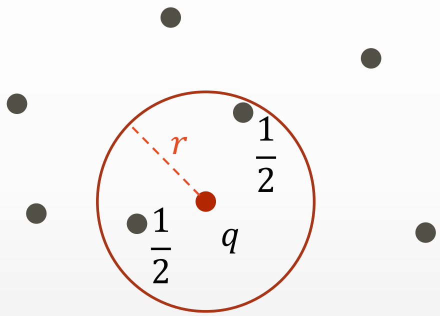

Goal:

- Return each point p in the neighborhood of q with uniform probability
- Do it in sub-linear time and small space

Approximate Fair Near Neighbor

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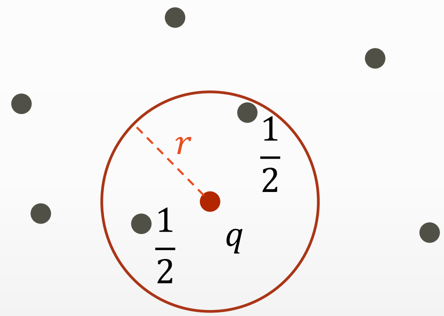


Goal of Approximate Fair NN

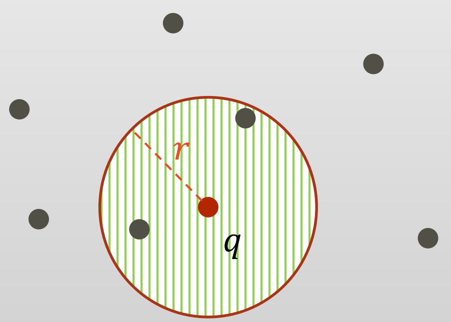
– Any point p in $N(q,r)$ is reported with "almost uniform" probability, i.e., $\lambda_q(p)$ where

$$\frac{1}{(1+\epsilon)|N(q,r)|} \leq \lambda_q(p) \leq \frac{(1+\epsilon)}{|N(q,r)|}$$

Results on $(1+\epsilon)$-Approximate Fair NN

Domain	Guarantee	Space	Query
Exact Neighborhood $N(q,r)$	w.h.p	$O(S_{ANN})$	$\tilde{O}(T_{ANN} \cdot \frac{\lvert N(q,cr)\rvert}{\lvert N(q,r)\rvert})$

- S_{ANN} and T_{ANN} are the space and query time of standard ANN



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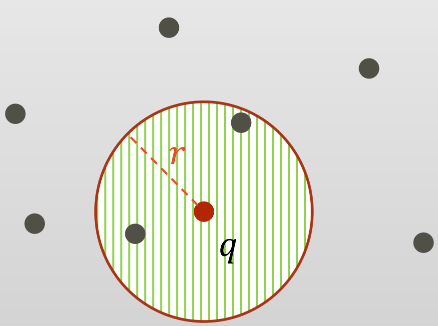

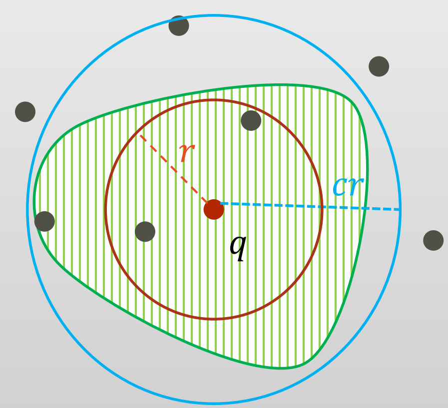

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- **Experiments**

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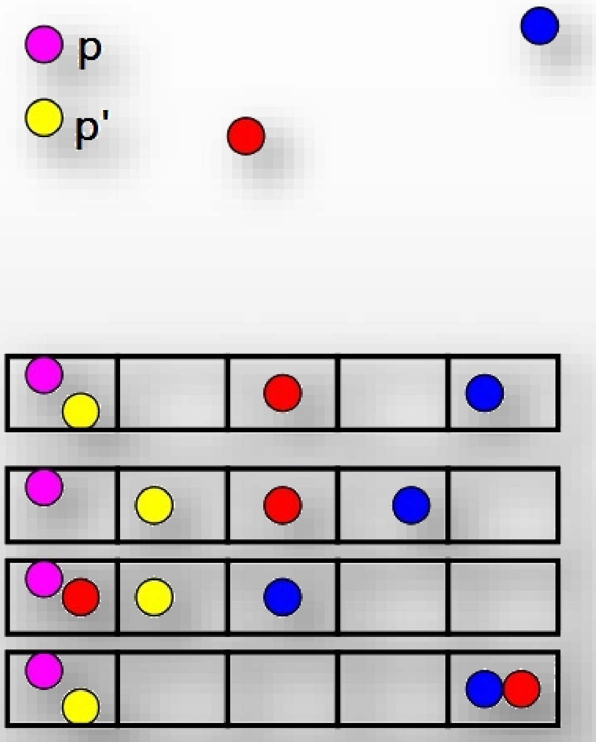
- **Experiments**
- Recent paper [Aumuller, Pagh, Silvestry'19] defining the same notion

Locality Sensitive Hashing (LSH)

One of the main approaches to solve the Nearest Neighbor problems

Locality Sensitive Hashing (LSH)

Hashing scheme s.t. close points have higher probability of collision than far points



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Hash functions: $g_1 , \ldots , g_L$

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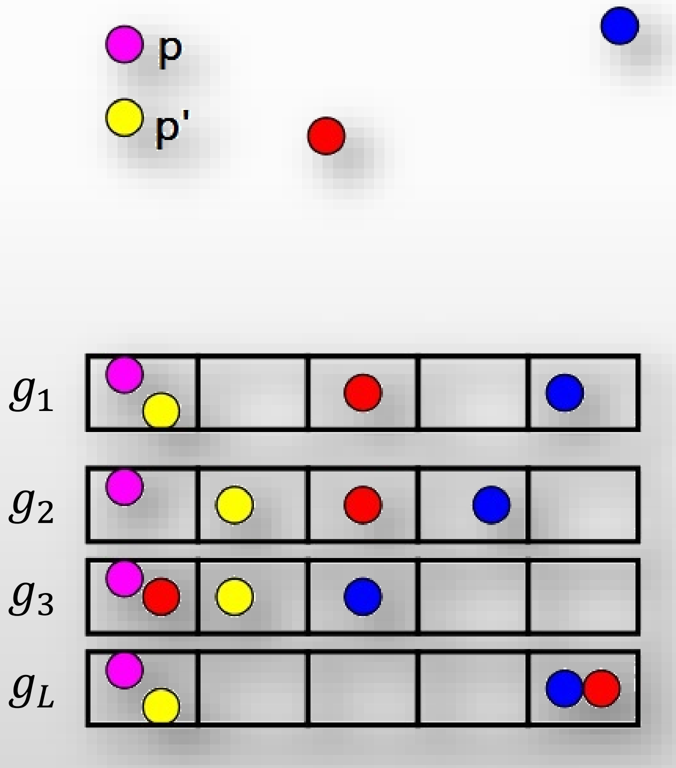

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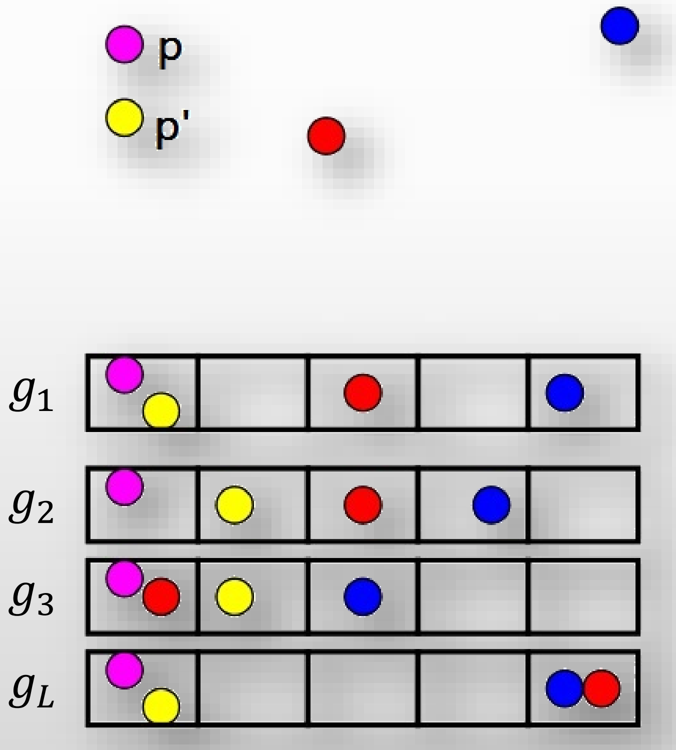
If $||p - p'|| \leq r$, they collide w.p. $\geq P_{high}$

If $||p - p'|| \geq cr$, they collide w.p. $\leq P_{low}$

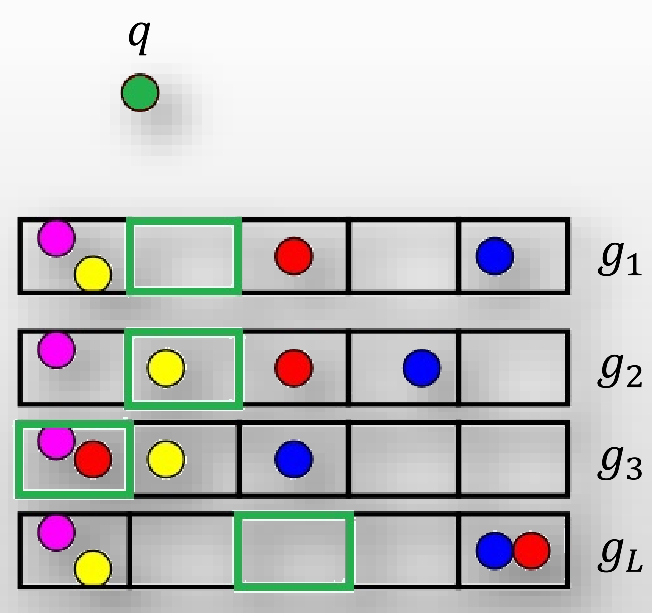
For $P_{high} \geq P_{low}$



Locality Sensitive Hashing (LSH)

Retrieval: [Indyk, Motwani'98]

- The union of the query buckets is roughly the neighborhood of q
- $\bigcup_i B_i(g_{i(q)})$ is roughly the neighborhood



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- The union of the query buckets is roughly the neighborhood of q
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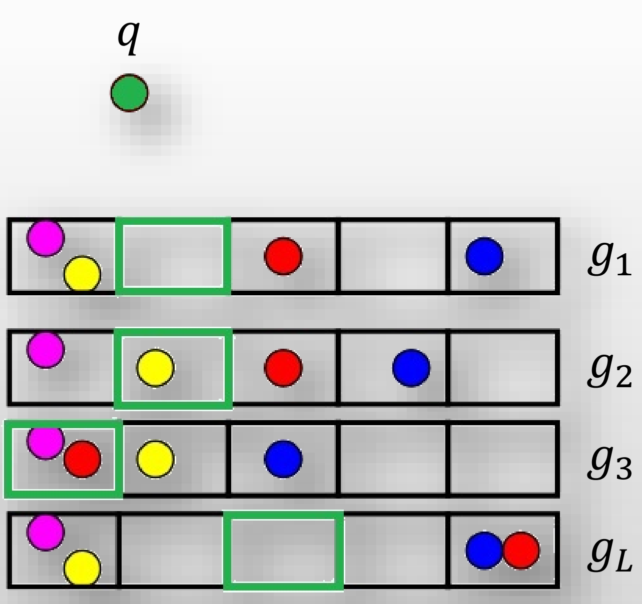
- How to report a uniformly random neighbor from union of these buckets?



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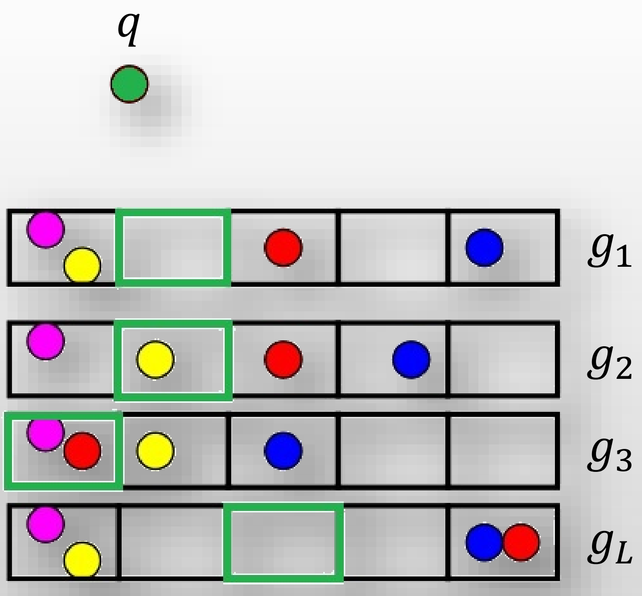
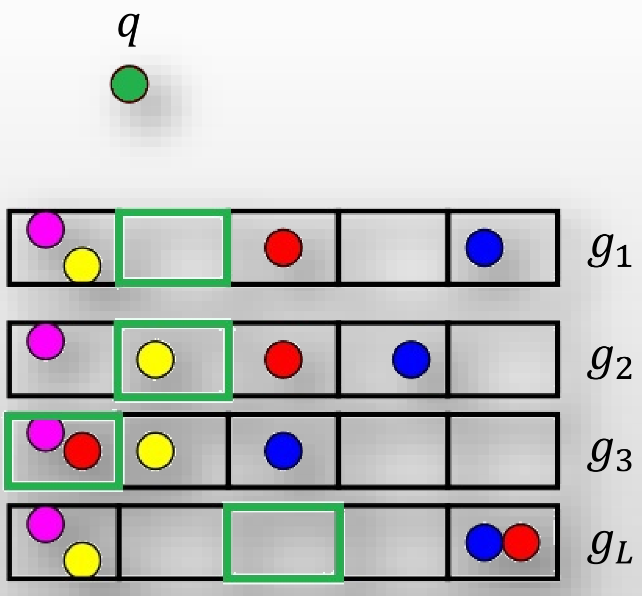


Approaches

Approach 1: Uniform/Uniform

How to output a random neighbor from $\bigcup_i B_i(g_{i(q)})$:

1. Choose a uniformly random bucket
2. Choose a uniformly random point in the bucket



Approach 2: Weighted/Uniform

How to output a random neighbor from
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1. Choose a random bucket proportional to its size
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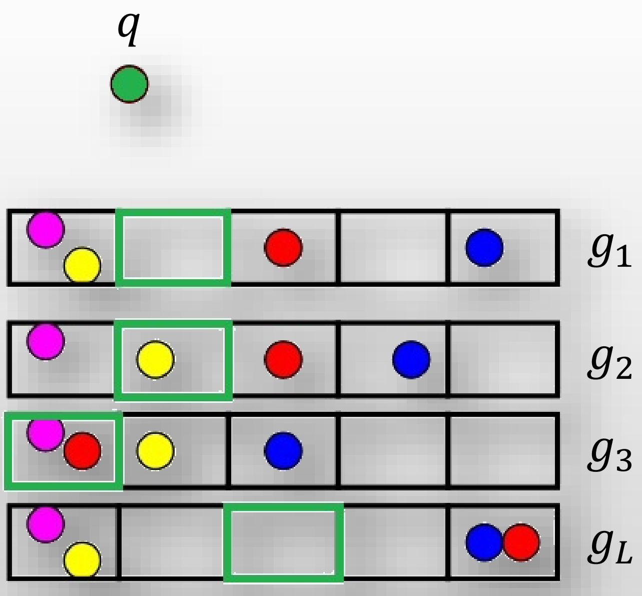

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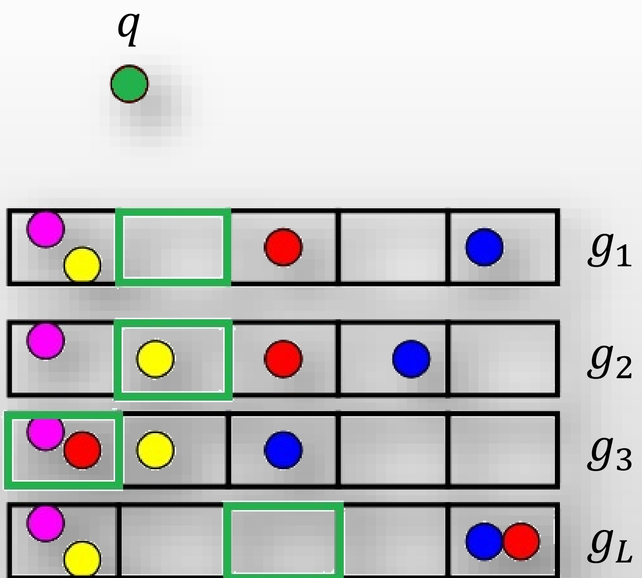
Number of buckets that p appears in



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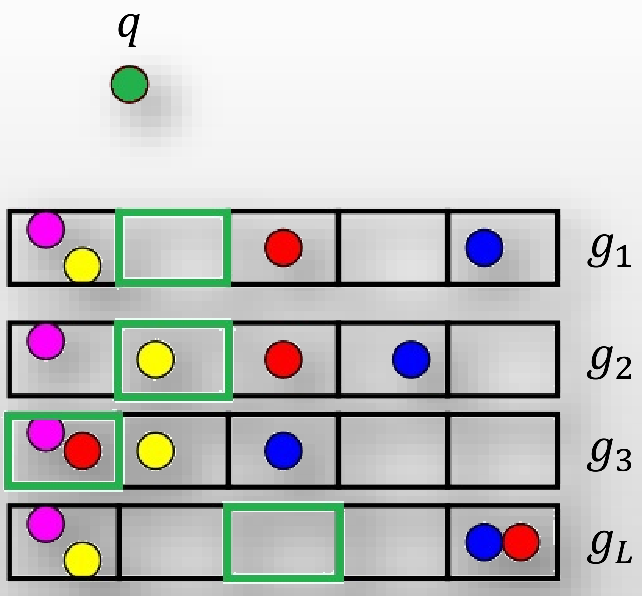
3. Keep p with probability $\frac{1}{d_p}$, o.w. repeat



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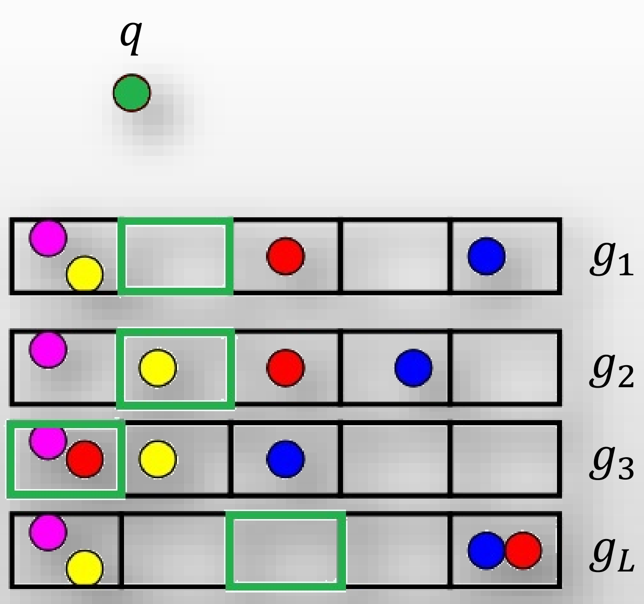

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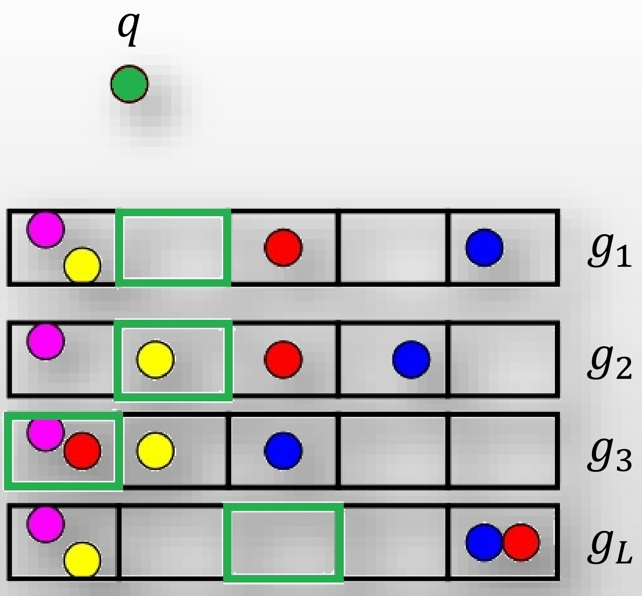

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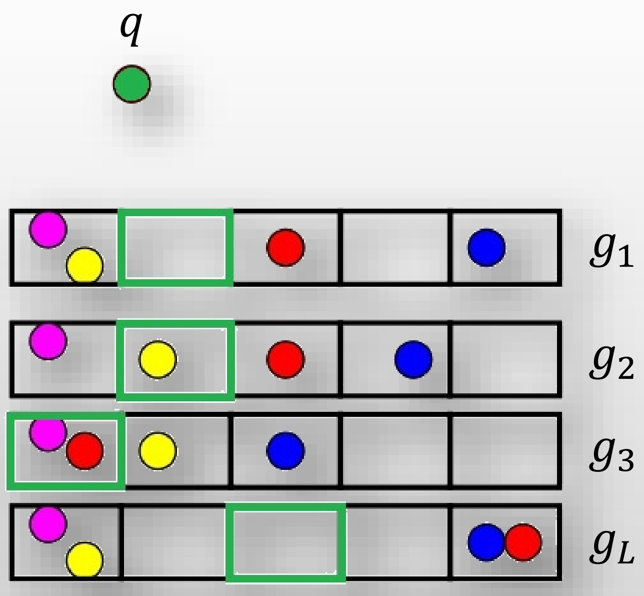
   - Need to spend $O(L)$ to find the degree
   - Might need $O(d_{max}) = O(L)$ samples
   - Total time is $O(L^2)$



Approximate the degree d_p

Sample $O(\frac{L}{d_p \cdot \epsilon^2})$ buckets out of L buckets to $(1+\epsilon)$-approximate the degree.

- Still if the degree is low this takes $O(L)$ samples.

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Case 1: Small degree d_p:

- More samples are required to estimate
- Reject with lower probability -> Fewer queries of this type

Case 2: Large degree d_p:

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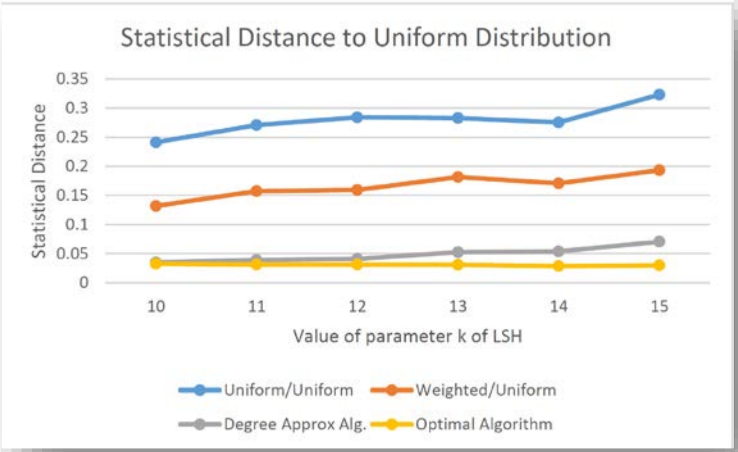
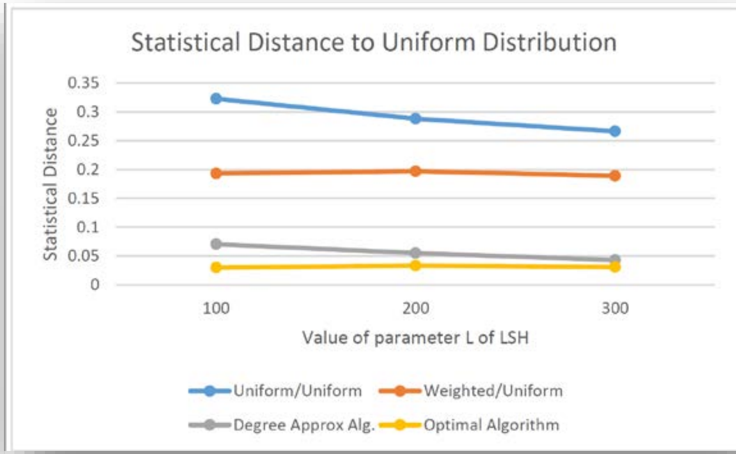
- This decreases $O(L^2)$ runtime to $\tilde{O}(L)$
- Large dependency on ϵ of the form $O(\frac{1}{\epsilon^2})$
- Via a different sampling approach we show how to reduce the dependency to logarithmic $O(\log \frac{1}{\epsilon})$.

Experiments





Setup

- Take MNIST as the data set
- Ask a query several times and compute the empirical distribution of the neighbors.
- Compute the statistical distance of the empirical distribution to the uniform distribution

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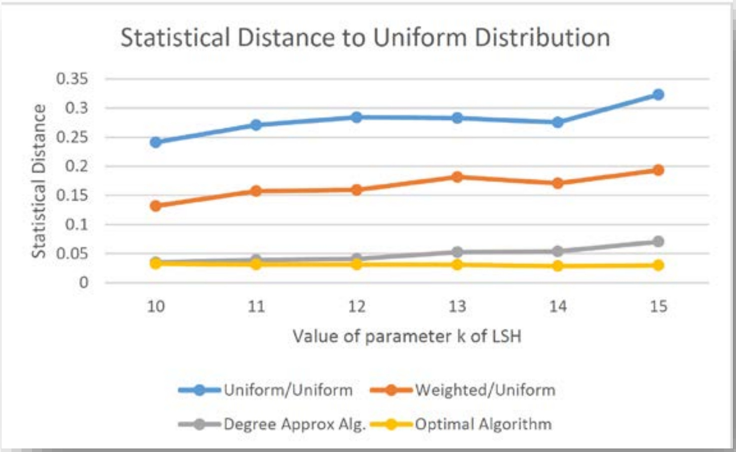

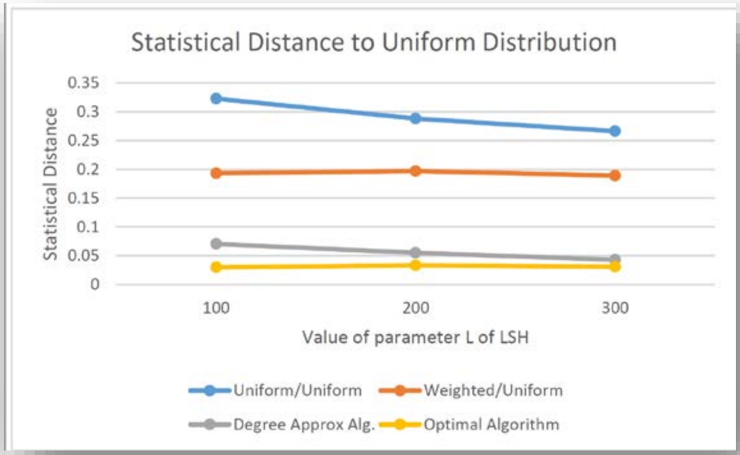

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Comparison

- Our algorithm performs 2.5 times worse than the optimal algorithm, but the other two perform 7 and 10 times worse than the optimal.
- Four times faster than the optimal but 15 times slower than the other two

Conclusion

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Exact Neighborhood $N(q,r)$	w.h.p	$O(S_{ANN})$	$\tilde{O}(T_{ANN} \cdot \frac{\lvert N(q,cr)\rvert}{\lvert N(q,r)\rvert})$
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- ✓ The dependence on the parameter n matches the standard Nearest Neighbor.
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Thanks
Questions?

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