Spontaneously Ordered Electronic States in Graphene

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Simons Symposium: Quantum Physics Beyond Simple Systems
Caneel Bay, 02/02/2012

New ordered states in SLG and BLG

- Weak interactions in undoped SLG (low DOS) **both a blessing and a curse:** robustness vs. functionality
- Strengthen the effects of interaction: **use weakly dispersing states, $E_{kinetic} < E_{potential}$**

(i) SLG doped to saddle point: chiral d-wave superconductivity (broken time-reversal symmetry)

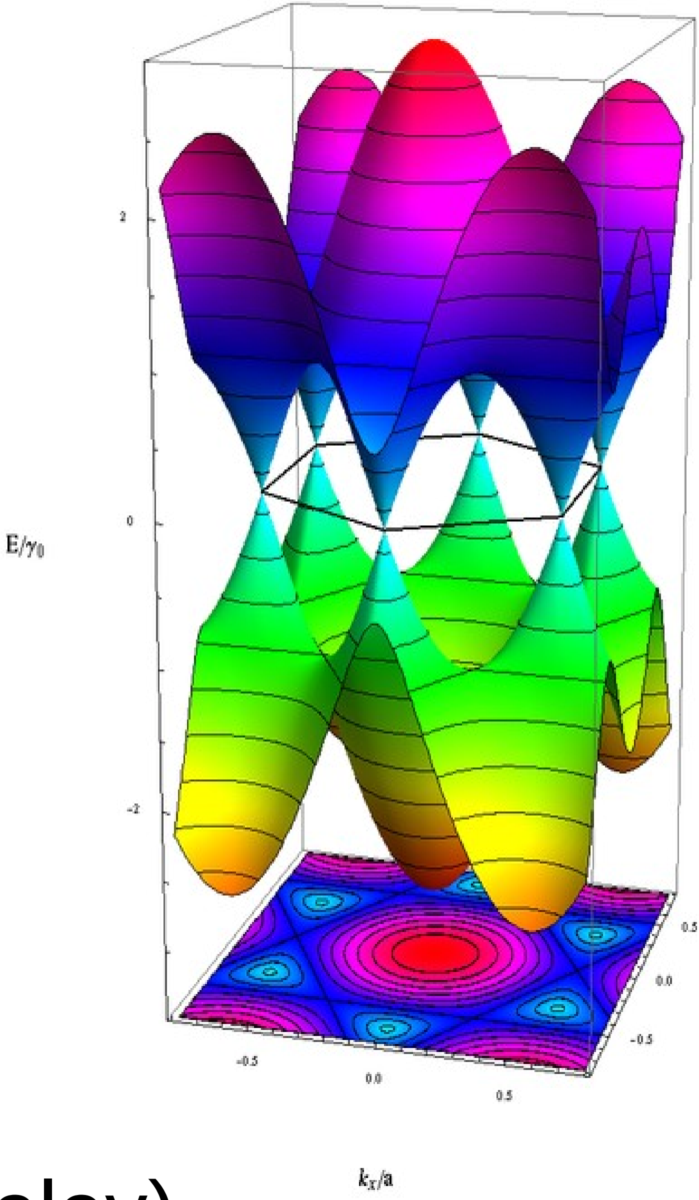
(ii) BLG at charge neutrality: excitonic insulator, spontaneous Hall effect at B=0 [charge QHE, spin QHE or valley QHE], nematic order

(iii) alter electronic states using external fields (QHE, FQHE)

- Ways to experimentally distinguish different ordered states in BLG

Electronic states in strongly doped graphene

- Quadratic dispersion near saddle points $E=+t_0,-t_0$
- Logarithmic Van Hove singularity
- Hexagonal FS @ n=3/8,5/8
- Similar to square lattice @ n=1/2
- Various competing orders: CDW, SDW, superconductivity, nematic order (Pomeranchuk instability)



High doping required ($\delta n=1/8$)
Electrostatic gating challenging
Can be achieved chemically (Berkeley)
or with liquid dielectric gating (Columbia, Geneva)

Different scenarios

- Nesting and vH singularity enhance interaction effects
- d-wave pairing, Kohn-Luttinger framework (Gonzalez 2008)
- Pomeranchuk (nematic) order, mean field (Valenzuelo, Vozmediano 2008)
- SDW order, mean field

(Li arxiv:1103.2420, Makogon et al arxiv:1104.5334)

- Legitimate mean-field states: superconductor, metal, insulator
- Need renormalization group (RG) to compare these orders on equal footing

Attraction from repulsion

- Approach developed for square lattice

Schulz 1987, Dzyaloshinskii 1987, Furukawa, Rice, Salmhofer 1998, LeHur, Rice 2009

- RG treats all potential instabilities on equal footing
- Progressively integrate out high energy states, examine flow of couplings
- Marginal with log corrections
- Three sources of log divergences: DOS, BCS, nesting

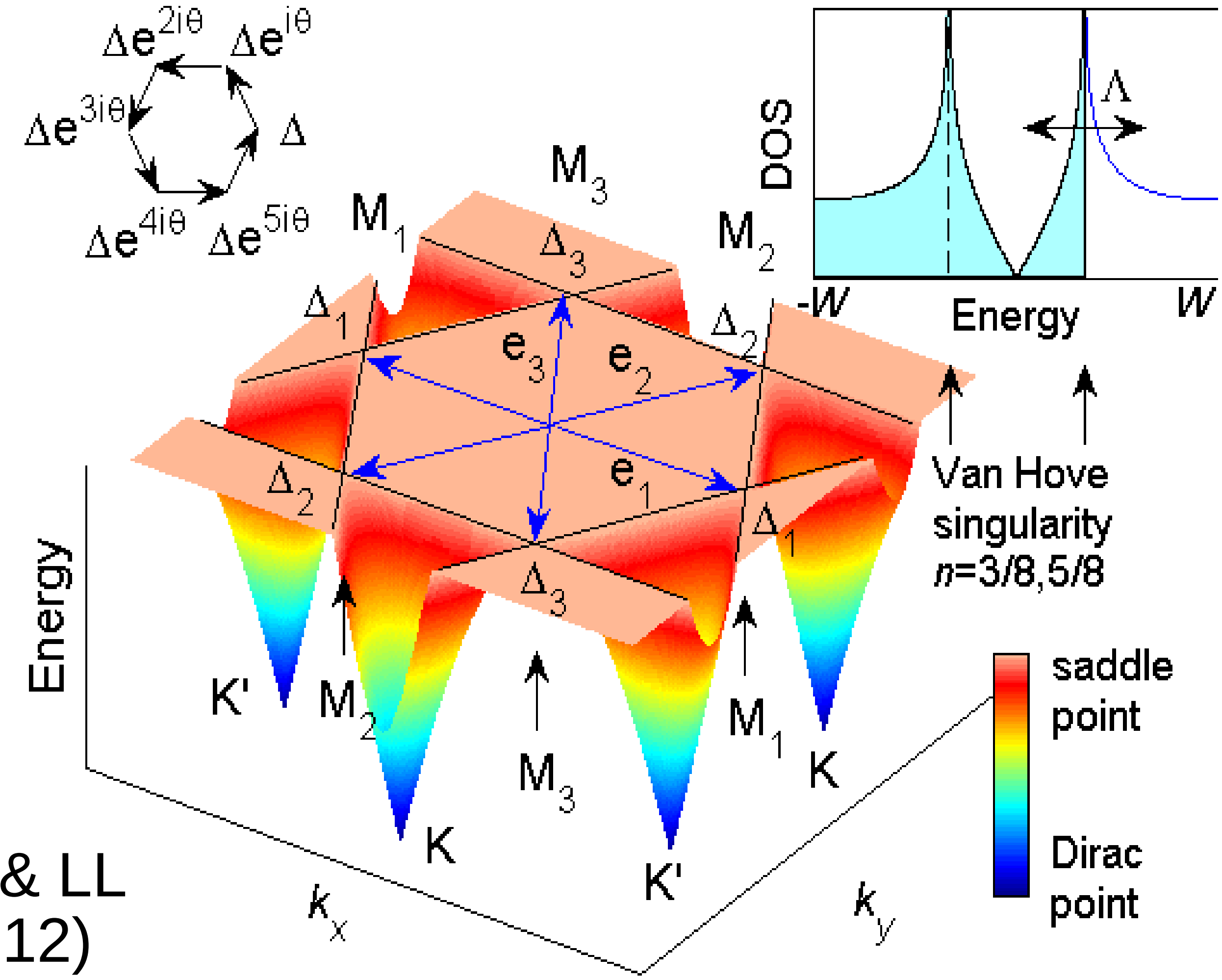
$$L=\sum_{\alpha=1}^{3}\psi_{\alpha}^{+}\left(\partial_{t}-\epsilon_{k}+\mu\right)\psi_{\alpha}-H_{two-particle}$$

- Pairing interaction induced by spin fluctuations
- New scenario for the competition of SDW and SC

Low energy description: three inequivalent patches

$$L=\sum_{\alpha=1}^{3}\psi_{\alpha}^{+}(\partial_t-\epsilon_{\boldsymbol{k}}+\mu)\psi_{\alpha}$$
$$-H_{two-particle}$$

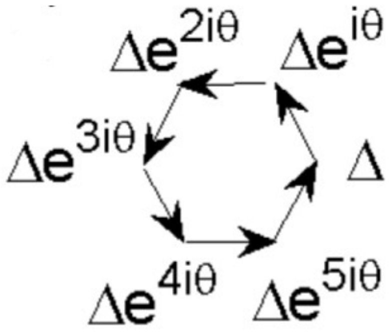
four interactions
(i) marginal at tree level
(ii) log's



Nandkishore, Chubukov & LL
Nat Phys (22 January 2012)

Chiral superconductivity from repulsive interaction

- Pairing gap winds around the Fermi surface



- Induced by (weak) repulsive interactions
- d-wave pairing wins over s-wave pairing
- d+id state: *time reversal symmery broken*
- Once a candidate for high Tc, long abandoned
- Rich phenomenology, similar to p+ip states in 3He films, SrRuO, FQHE $\nu=5/2$ (Volovik 1988, Laughlin 1998, Senthil, Marston, Fisher 1999, Fu, Kane 2008, Zhang 2009):

(i) nonzero Chern class (“charge QHE” at B=0);

(ii) spin and thermal QHE; edge charge current in B field

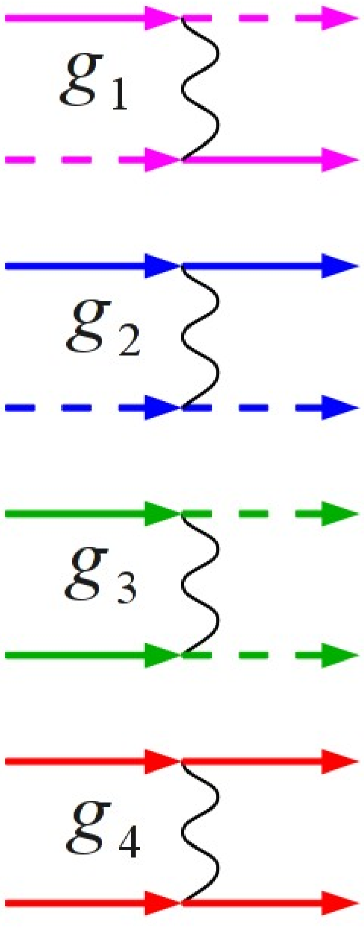
(iv) Majorana states @ vortices and boundaries

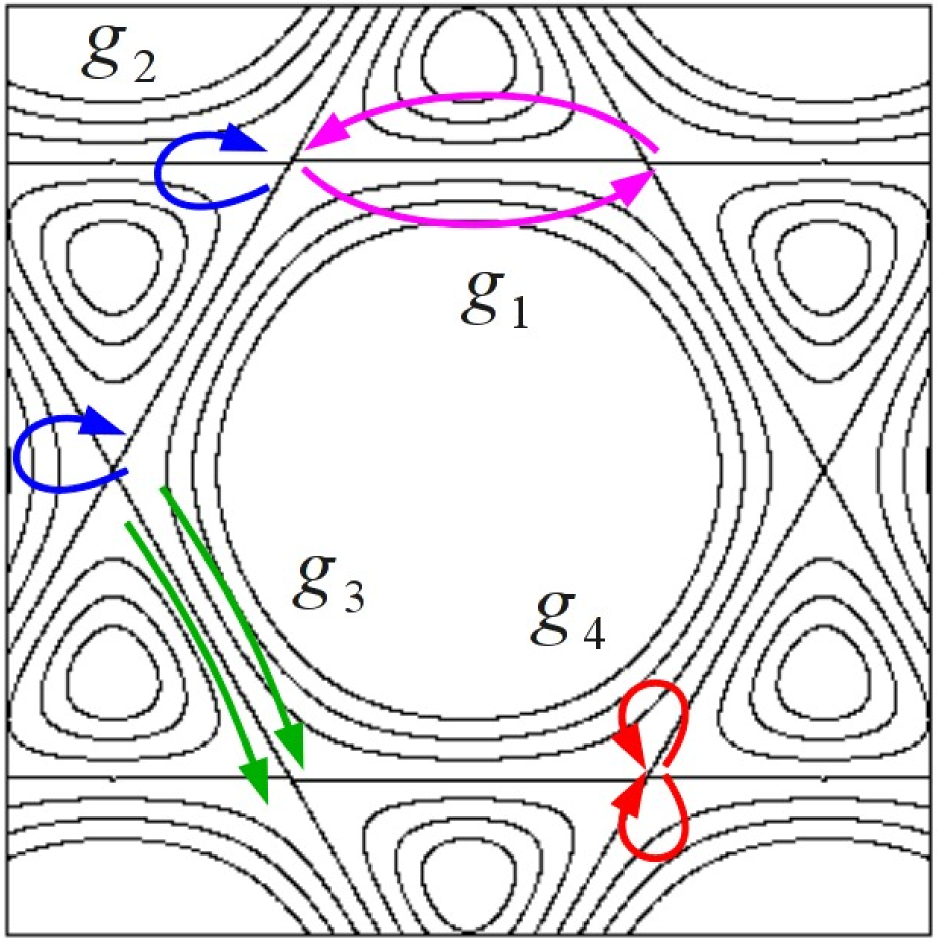
(v) Kerr effect, interesting Andreev states, etc

Two-particle inter- and intra-patch scattering processes

$$H_{two-particle}=\sum_{\alpha,\beta=1}^{3}\frac{g_1}{2}\Psi_\alpha^+\Psi_\beta^+\Psi_\alpha\Psi_\beta+\frac{g_2}{2}\Psi_\alpha^+\Psi_\beta^+\Psi_\beta\Psi_\alpha+\frac{g_3}{2}\Psi_\alpha^+\Psi_\alpha^+\Psi_\beta\Psi_\beta$$

$$+\sum_{\alpha=1}^{3}\frac{g_4}{2}\Psi_\alpha^+\Psi_\alpha^+\Psi_\alpha\Psi_\alpha$$





Diverging susceptibilities

SC pairing (spin-up, spin-down)

$$\Pi_{pp}(0)=\frac{\nu_0}{4}\ln\frac{\Lambda}{max(\mu,T)}\ln\frac{\Lambda}{T}$$

SDW susceptibility

$$\Pi_{ph}(Q_i)=\frac{\nu_0}{4}\ln\frac{\Lambda}{max(\mu,T)}\ln\frac{\Lambda}{max(\mu,T,t_3)}$$

Imperfect nesting

Lesser susceptibilities:

$$\Pi_{pp}(Q_i),\Pi_{ph}(0)=\frac{\nu_0}{4}\ln\frac{\Lambda}{max(\mu,T)}$$

RG flow of couplings for n patches

$$\frac{dg_1}{dy} = 2\mathrm{d}_1 g_1 (g_2 - g_1) \qquad \frac{dg_2}{dy} = 2\mathrm{d}_1 (g_2^2 + g_3^2) \qquad \frac{dg_4}{dy} = -(n-1) g_3^2 - g_4^2$$

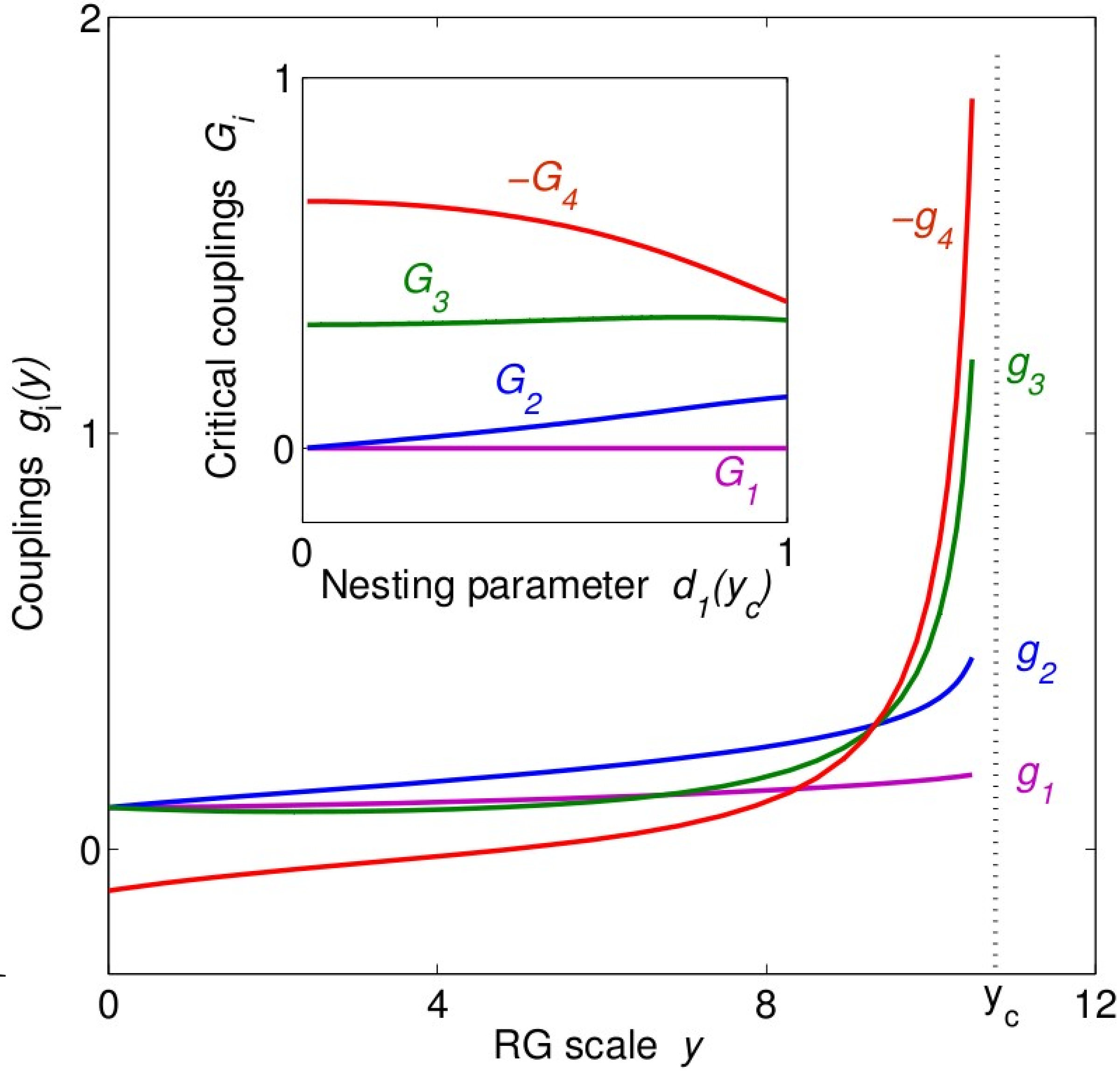
$$\frac{dg_3}{dy} = -(n-2) g_3^2 - 2\mathrm{g}_3 g_4 \\ + 2\mathrm{d}_1 g_3 (2\mathrm{g}_2 - g_1)$$

RG time $y = \ln^2 \xi = \Pi_{pp}$

Nesting parameter $d_1 = \frac{d\,\Pi_{ph}(Q)}{d\,\Pi_{pp}(0)} < 1$

Critical couplings $g_i(y) \approx \frac{G_i}{y_c - y}$

Initial values $g_i(y=0) \approx 0.1$



RG flow features

- Agrees with the square lattice (n=2)
- Unique fixed trajectory (“stable fixed point”) for repulsive bare couplings
- g1, g3, g2 cannot change sign, stay positive
- g4 decreases & reverses sign
- g3-g4 large & positive, drives SC instability

positive g3 penalizes s-wave **favors d-wave SC**

- Susceptibility χsc diverges faster than χsdw
- **SC a clear winner** (cf. square lattice)
- High Tc from weak coupling physics

$$T_c \approx \Lambda e^{-\frac{A}{\sqrt{g_0 \nu_0}}}$$

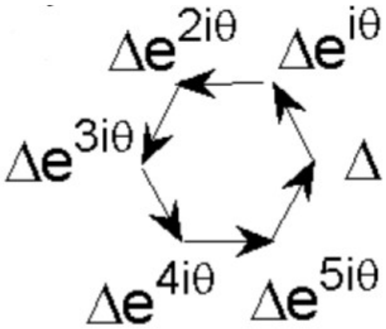
Competition of d-wave orders below Tc

- By symmetry, two degenerate d-wave states
- Ginzburg-Landau analysis of competiton

$$\Delta=\Delta_a(x^2-y^2)+\Delta_b 2xy$$

$$F(\Delta_a,\Delta_b)=\alpha(T-T_c)(|\Delta_a|^2+|\Delta_b|^2)+K_1(|\Delta_a|^2+|\Delta_b|^2)^2$$

$$+K_2\left|\Delta_a^2+\Delta_b^2\right|^2$$



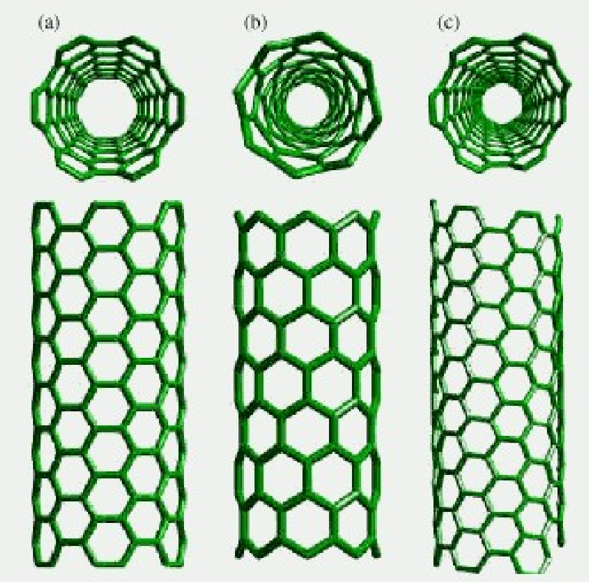
- Calculation of GL functional yields $K_2>0$
- d+id and d-id ground states $\Delta_a=\pm\Delta_b$
- Superconductivity with TRS breaking

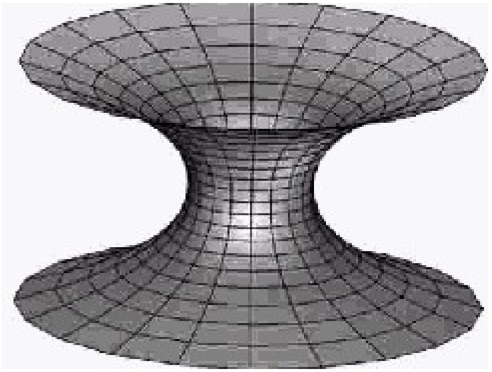
Summary: chiral SC in doped graphene

- Interaction driven instability in graphene doped at saddle points
- Weak repulsive interaction stabilizes chiral superconducting state d+id or d-id
- Enhanced Tc
- Topological superconductor with broken TRS

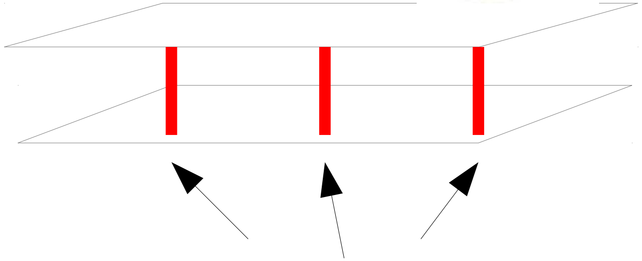
Outlook:

- Topological superconductor with broken TRS
- Zoo of interesting phenomena
- Higher-genus fullerens
- Graphene easily combined with other materials into hybrid structures and heterostructures: pathway to applications of chiral superconductivity



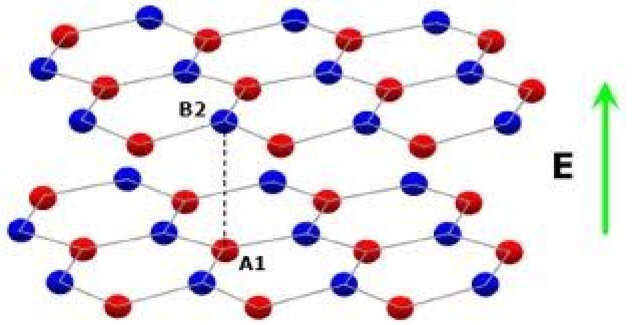


genus g=2:

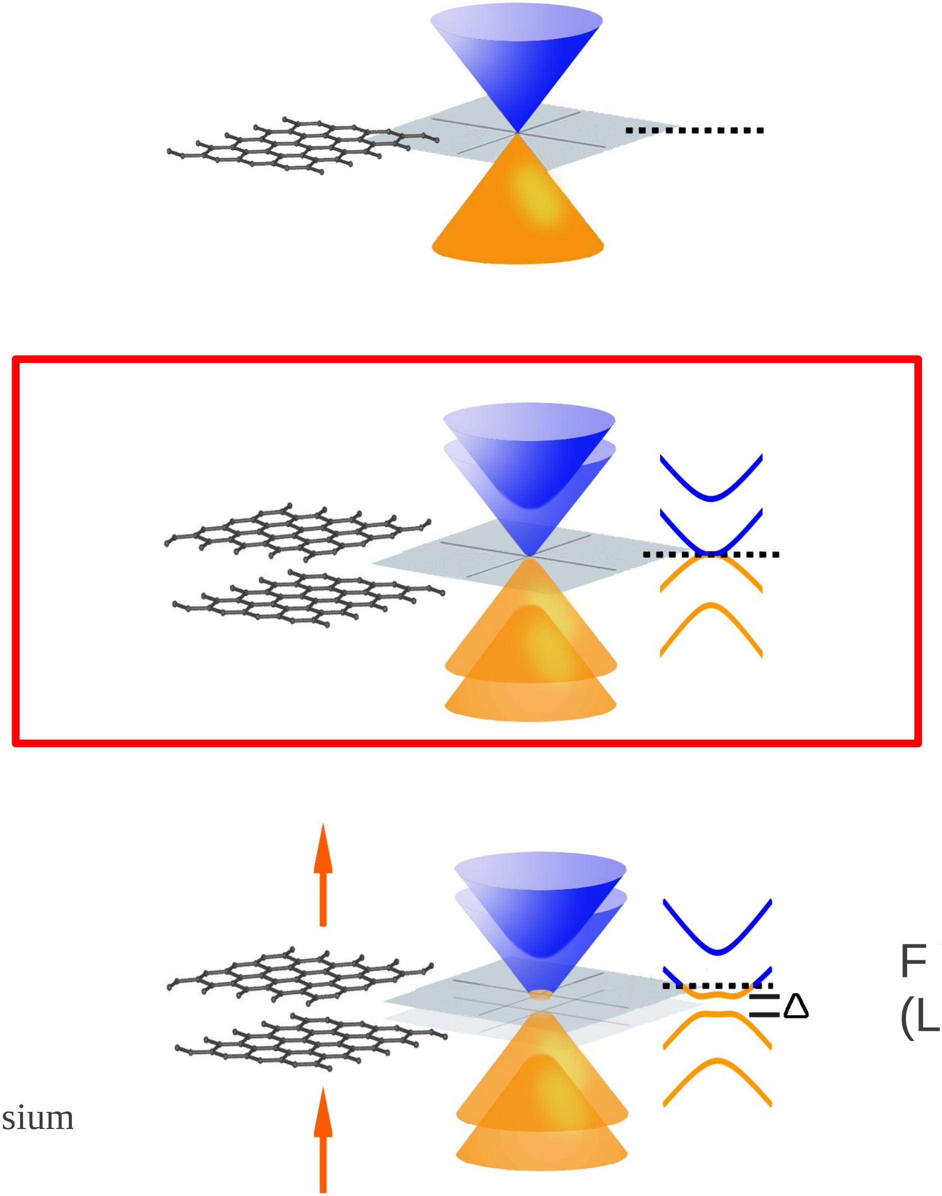


point junctions

Spontaneously ordered states in bilayer graphene

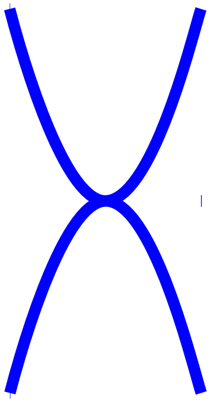


$$\hat{H} = -\frac{\hbar^2}{2m}\begin{pmatrix} 0 & (k_x - ik_y)^2 \\ (k_x + ik_y)^2 & 0 \end{pmatrix} .$$



F Wang (LBL)

Bilayer at charge neutrality (no disorder, no trigonal warping)



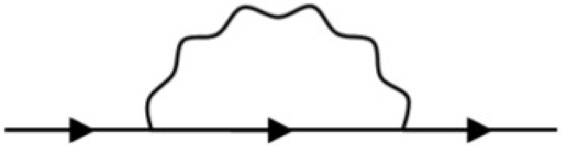
- Finite DOS at ε=0 (quadratic dispersion)
- Fermi surface reduced to a point
- Fermi liquid unstable due to interband transitions
- Log-divergent 2-particle interaction vertices, self-energy, effective mass, etc
- RG similar to g-ology in D=1

Non-Fermi liquid even at weak interaction: Greens function log^2 renormalization

$$G(\omega,k)=\frac{Z(\xi)}{i\omega-H_0(k)},\quad \xi=\ln\frac{\Lambda_0}{\sqrt{\omega^2+(k^2/2m)^2}}\qquad V_{RPA}(\omega,k)=\frac{2\pi e^2}{\kappa k-2\pi e^2 N\Pi(\omega,k)}$$

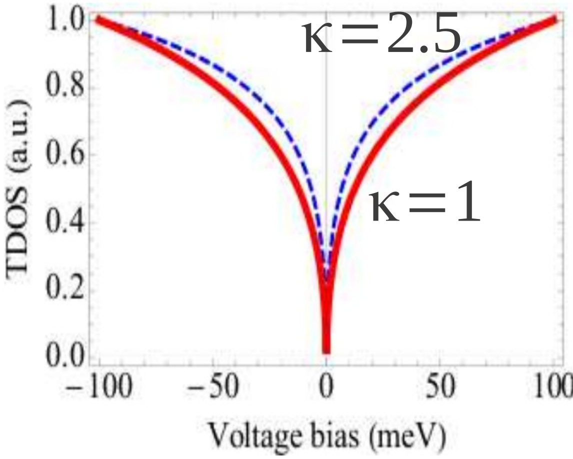
RG flow at log^2 order
(Nandkishore & LL 2010)

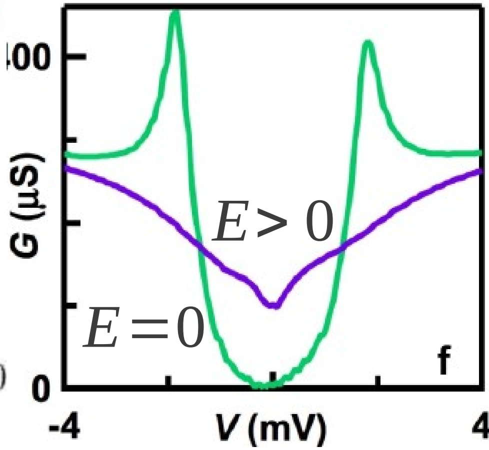
$$\Sigma\sim\xi^2(i\omega-H_0(k))$$

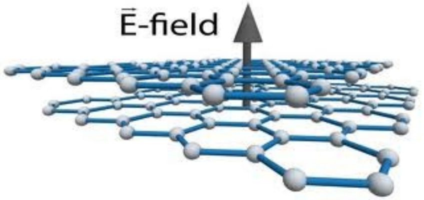


$$\frac{\partial Z}{\partial\xi}=-\xi\frac{2Z(\xi)}{N\pi^2}\qquad N=4$$

$$G(\xi)=AG_0(\omega,k)\exp(-\xi^2/N\pi^2)$$







Compare with the diffusive Coulomb Anomaly (Altshuler, Aronov, Lee 1980)

$$\frac{\partial Z}{\partial\xi}=-\frac{\xi}{4\pi^2 g}Z(\xi),\quad \omega\tau\ll 1$$

2D conductance

Effective mass and interaction not renormalized at log^2 order

$$\delta m=\frac{0.56\,\xi}{2N\pi\ln 4}m_0\approx 0.016\,\xi\,m_0$$

RG for interaction, see Falko's talk

Theory:

- Min, Borghi, Polini & MacDonald, Pseudospin magnetism in graphene. Phys. Rev. B 77, 041407 (2008).
- Nandkishore & Levitov, Dynamical screening and excitonic instability in bilayer graphene, Phys. Rev. Lett. 104, 156803 (2009)
- Nandkishore & Levitov, Flavor symmetry and competing orders in bilayer graphene. arXiv:1002.1966v1001 (2010).
- Zhang, Min, Polini, & MacDonald, Spontaneous inversion symmetry breaking in graphene bilayers. Phys. Rev. B 81, 041402 (R) (2010).
- Nandkishore & Levitov, Quantum Anomalous Hall State in Bilayer Graphene, Phys Rev B 82, 115124 (2010)
- Vafek & Yang, Many-body instability of Coulomb interacting bilayer graphene: Renormalization group approach. Phys. Rev. B 81, 041401 (2010).
- Lemonik, Aleiner, Toke & Falko, Spontaneous symmetry breaking and Lifshitz transition in bilayer graphene. Phys. Rev. B 82, 201408 (2010).
- Zhang, Jung, Fiete, Niu & MacDonald, Spontaneous quantum Hall states in chirally stacked few-layer graphene systems. Phys. Rev. Lett. 106, 156801 (2011).
- Kharitonov, Canted antiferromagnetic phase of the ν=0 quantum Hall state in bilayer graphene. preprint, arXiv:1105.5386v1101 (2011).

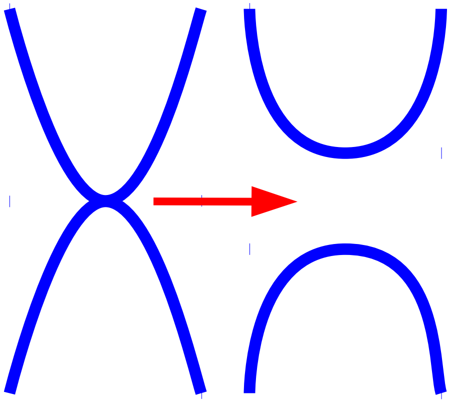
Experiment:

- Martin, Feldman, Weitz, Allen & Yacoby, Local Compressibility Measurements of Correlated States in Suspended Bilayer Graphene. Phys. Rev. Lett. 105, 256806 (2010).
- Weitz, Allen, Feldman, Martin, & Yacoby, Broken-symmetry states in doubly gated suspended bilayer graphene. Science 330, 812-816 (2010).
- Freitag, Trbovic, Weiss & Schonenberger, Spontaneously gapped ground state in suspended bilayer graphene. arXiv:1104.3816vs (2011)
- Zhao, Cadden-Zimansky, Jiang, & Kim, Symmetry Breaking in the Zero-Energy Landau Level in Bilayer Graphene. Phys. Rev. Lett. 104, 066801 (2010).
- Feldman, Martin & Yacoby, Broken-symmetry states and divergent resistance in suspended bilayer graphene. Nat. Phys. 5, 889-893 (2009).
- Bao, W. et al. Magnetoconductance oscillations and evidence for fractional quantum Hall states in suspended bilayer and trilayer graphene Phys. Rev. Lett. 105, 246601 (2010).
- Velasco, Jing, Bao, Lee, Kratz, Aji, Bockrath, Lau, Varma, Zhang, Jung & MacDonald, Transport Spectroscopy of Symmetry-Broken Insulating States in Bilayer Graphene, arXiv:1108.1609
- Mayorov, Elias, Mucha-Kruczynski, Gorbachev, Tudorovskiy, Zhukov, Morozov, Katsnelson, Falko, Geim, Novoselov, Interaction-Driven Spectrum Reconstruction in Bilayer Graphene, Science 333, 860 (2011)

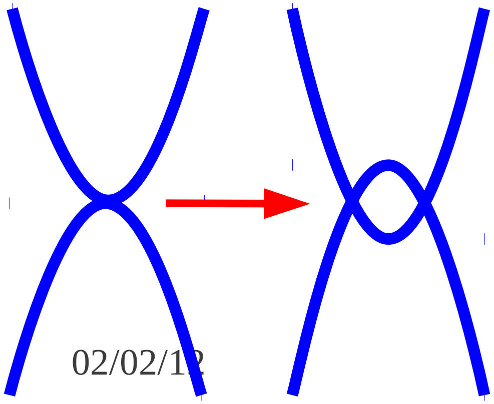
Spontaneous ordering in BLG at DP

- Particle-hole pairing instability
- BCS-like exciton condensate, no superfluifity, phase locking
- Gapped spectrum $\Delta = \pm\Delta_0$
- Another candidate state: “nematic” order, gapless spectrum, broken rotational symmetry

Min et al 2008;
Nandkishore, LL 2010
Zhang et al 2010



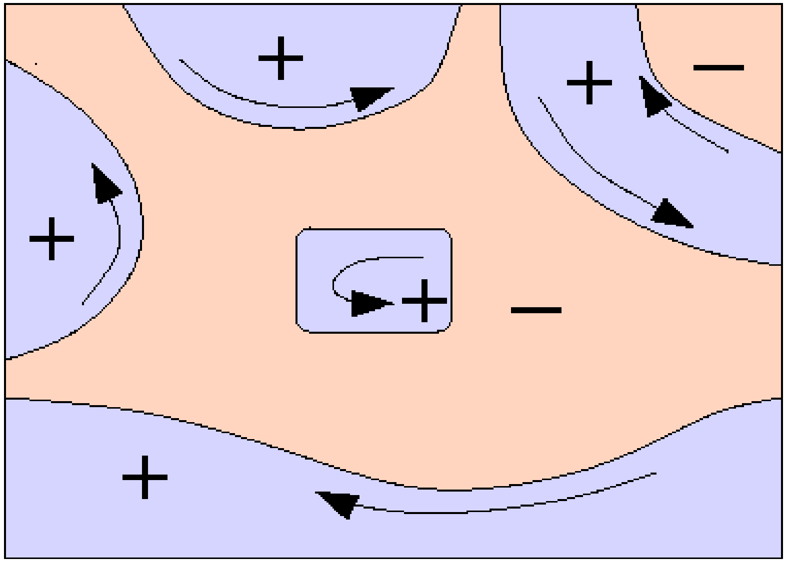
Vafek, Yang 2010; Lemonik et al 2010

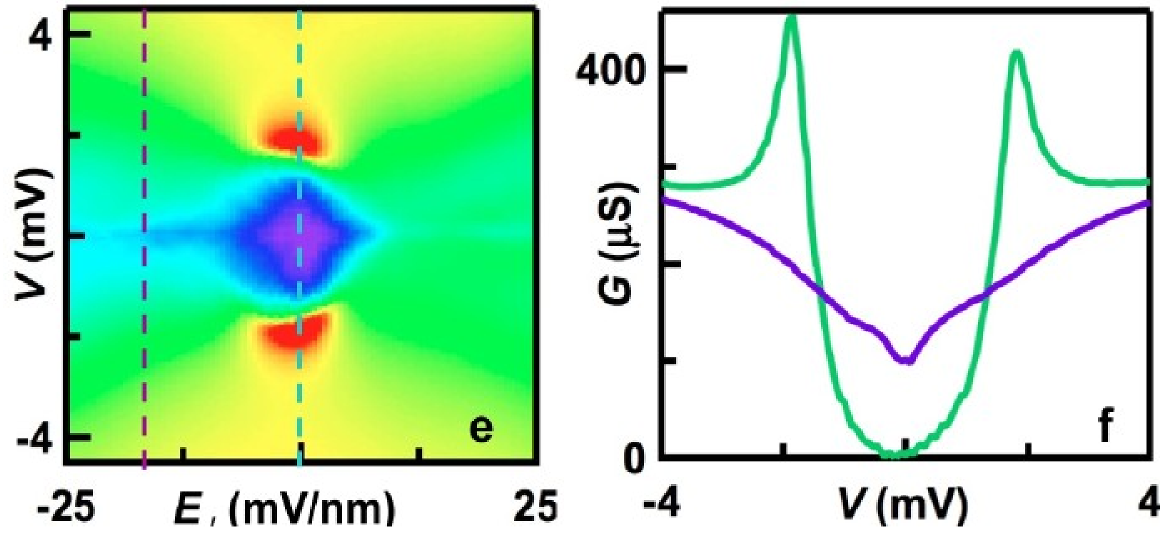


$$H_{nema} = \begin{pmatrix} 0 & \frac{p_-^2}{2m} + \Delta \\ \frac{p_+^2}{2m} + \Delta & 0 \end{pmatrix} \qquad H_{gapped} = \begin{pmatrix} \Delta & \frac{p_-^2}{2m} \\ \frac{p_+^2}{2m} & -\Delta \end{pmatrix}$$

Spontaneous gap opening in BLG

Nandkishore & LL, PRL 104, 156803 (2010), PRB 82, 115124 (2010)





Velasco et al arXiv:1108.1609

- 'Which-layer' symmetry breaking
- Domains of + and – polarization
- Charge, valley or spin polarized current along domain boundaries, QHE, VQHE, SQHE, etc
- SU(4) symmetry and the variety of possible states
- Time reversal symmetry breaking at B=E=0: Anomalous Quantum Hall state, quantized σ_{xy}
- Experiment (Yacoby, Lau and Geim groups)

Large variety of possible states

$$H_K = \begin{pmatrix} \Delta_K & p_-^2/2\mathrm{m} \\ p_+^2/2\mathrm{m} & -\Delta_K \end{pmatrix} \qquad H_{K'} = \begin{pmatrix} \Delta_{K'} & p_+^2/2\mathrm{m} \\ p_-^2/2\mathrm{m} & -\Delta_{K'} \end{pmatrix}$$

$$\Delta_{K,\sigma} = \pm\Delta_{K',\sigma} = \pm\Delta_{K,-\sigma} = \pm\Delta_{K',-\sigma} \qquad p_\pm = p_1 \pm i\, p_2$$

- Four-fold spin/valley degeneracy
- Many gapped states: valley “antiferromagnet”, ferromagnetic, ferrimagnetic, ferroelectric, etc (Min et al 2008, Nandkishore & LL 2010, Zhang et al 2010)
- Degeneracy on a mean field level: instability threshold **the same for all states**: short-range interaction, screened long-range interaction models
- SU(4) symmetry?

Opposite chirality of two valleys conceals SU(4) symmetry, made manifest by performing unitary transformation

$$H_0 = \frac{p_+^2}{2m}\tilde{\tau}_- + \frac{p_-^2}{2m}\tilde{\tau}_+,$$

Approximate SU(4) symmetry (weakly broken by trigonal warping and capacitor energy)

$$H = \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} H_0 \psi_{\mathbf{p}} + \frac{1}{2}\sum_{\mathbf{q}} V_+(q)\rho_{\mathbf{q}}\rho_{-\mathbf{q}} + V_-\lambda_{\mathbf{q}}\lambda_{-\mathbf{q}},$$

Strategy: Diagonalize SU(4) invariant Hamiltonian and incorporate anisotropies perturbatively

Mean field description of gapped states

$$H = \frac{p_+^2\tau_- + p_-^2\tau_+}{2m} + \Delta\tau_3 Q$$

Classification into manifolds (4,0), (3,1), (2,2) and distinction between symmetry protected and accidental degeneracies

$$\sigma_{xy} = \left(M_> - M_<\right)\frac{e^2}{h},$$

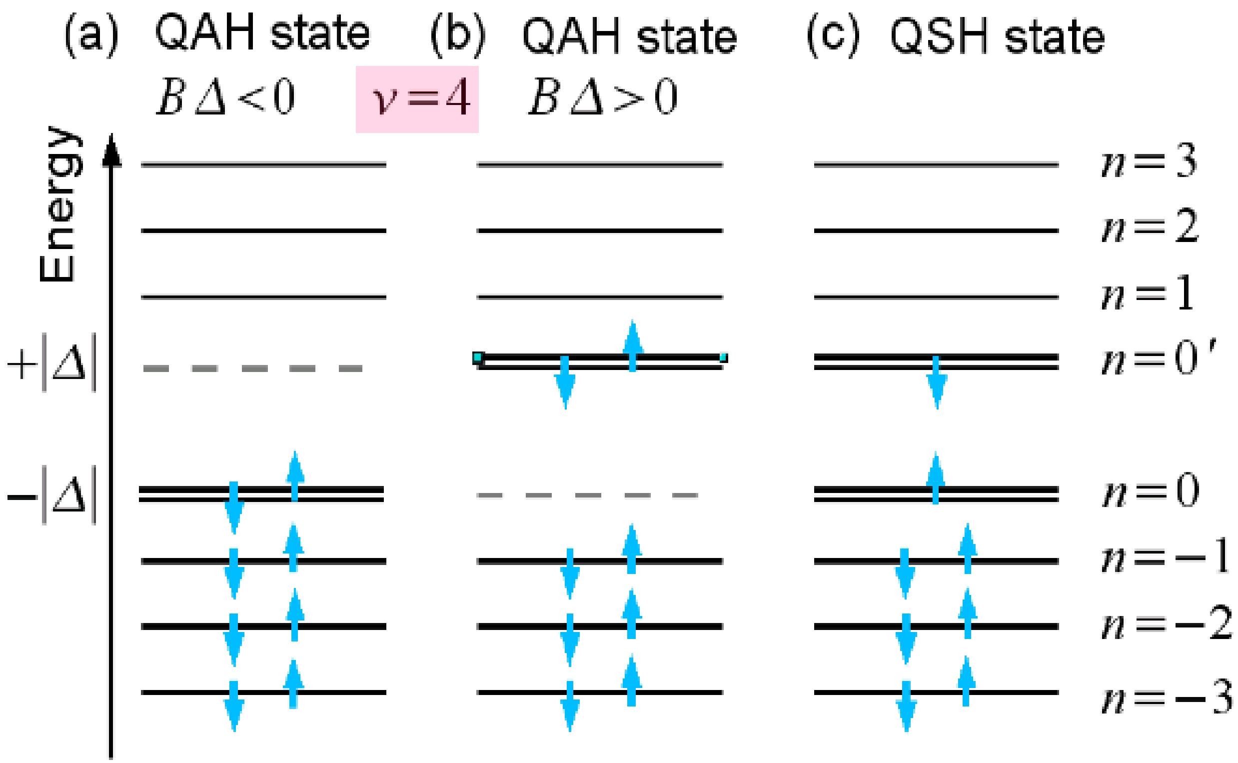
(4,0) and (3,1) states feature QHE, “anomalous QHE”, B=0

Nandkishore, LL 2010   Vafek, Yang 2010

Near-degeneracy and selection:

Quantum fluctuations favor (4,0) state; thermal fluctuations favor (2,2) state

The QAH state stabilized by a B field

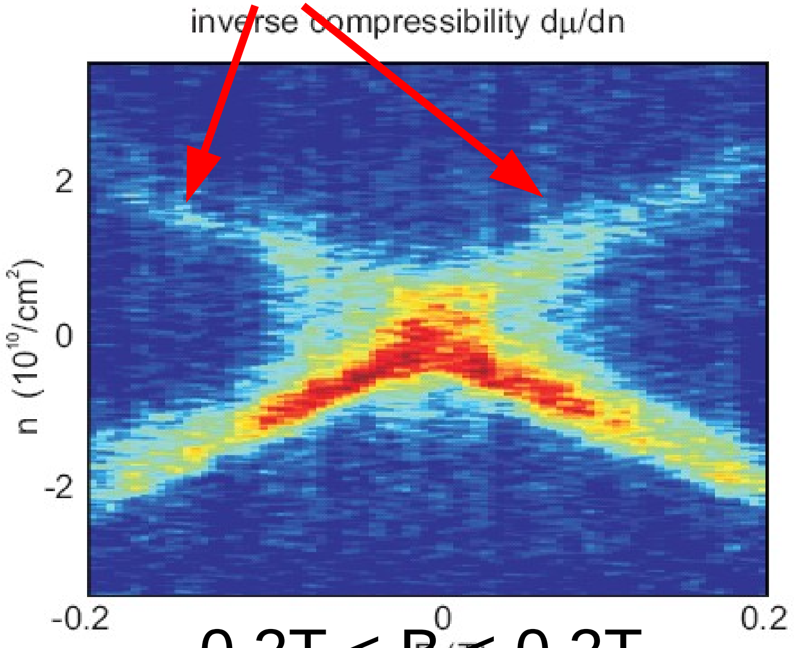


Theory: the QAH state favored at small nonzero B and ν=+4,-4

Experiment: Incompressible ν=+4,-4 states observed at **very low B**

Consistent with the QAH state

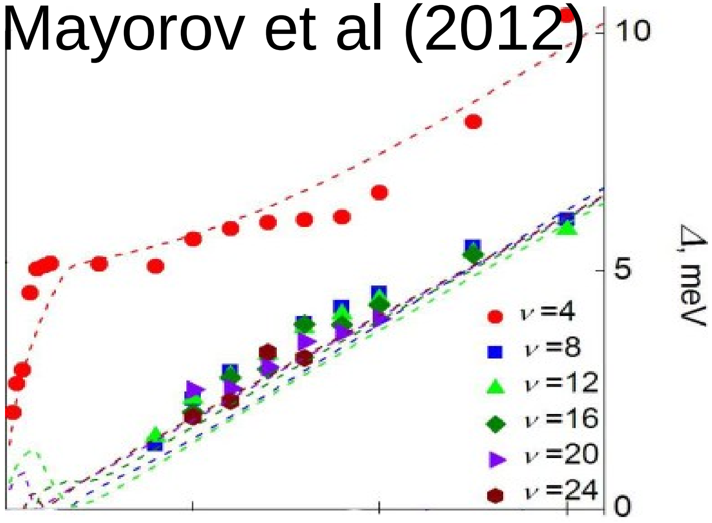
$$n=\pm 4\times eB/h$$



-0.2T < B < 0.2T

Martin et al (2010)

Mayorov et al (2012)



Transport experiments compatible with the QAH state (but indecisive)

- Incompressible regions at low B, $\nu=4$ (if field induced), $\nu=+4$ and $\nu=-4$ (if intrinsic); no such feature at higher filling factor (unlike nematic or other states)
- Incompressible (bulk gap)+finite two-probe conductivity; distinguishes QAH state from (2,2) state but not from nematic state or trigonal warping
- Phase transition at zero ν, finite B to (2,2) QHFM state (likewise)
- Phase transition at finite E to trivial insulator (Ising universality class)

The QAH state not yet observed

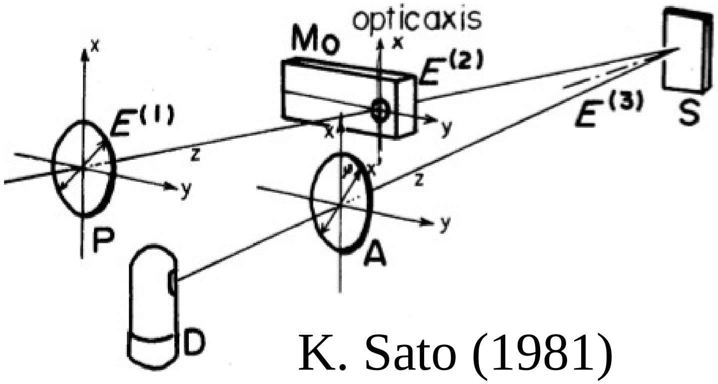
EXPERIMENTAL SIGNATURES

1) Direct test: measurement of QHE at B=0; requires four-probe measurement on suspended BLG at low T

2) TRS breaking via violation of Onsager symmetry B,-B in a four-probe measurement

3) Optically detect TRS breaking: contactless measurement of σ_{xy} by polar Kerr effect (not Faraday effect)

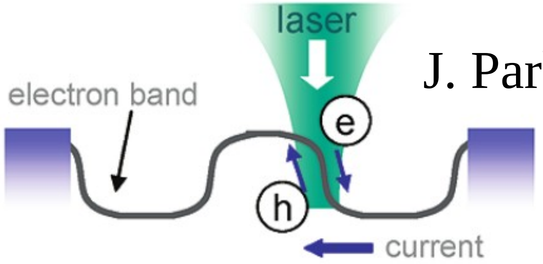
Nandkishore & LL, PRL 107, 097402 (2011)



K. Sato (1981)

4) Scanning photocurrent imaging: domains with different chirality, p-n droplets, edge states

Song & LL, arXiv:1112.5654 (2011)



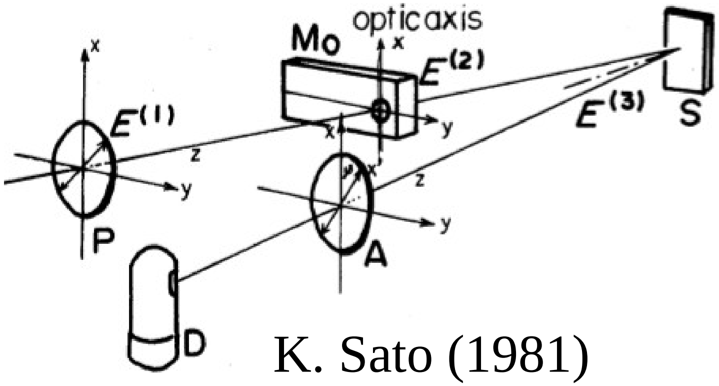
J. Park et al (2009)

5) Tunneling probes and local capacitance probes: local gap, filling factor, compressibility

Kerr effect: optical detection of TRS breaking, contactless measurement of σ_{xy}

Large polar Kerr effect in TRS-broken states: interband transitions sensitve to low-energy physics at Dirac point

Nandkishore & LL
PRL 107, 097402
(2011)



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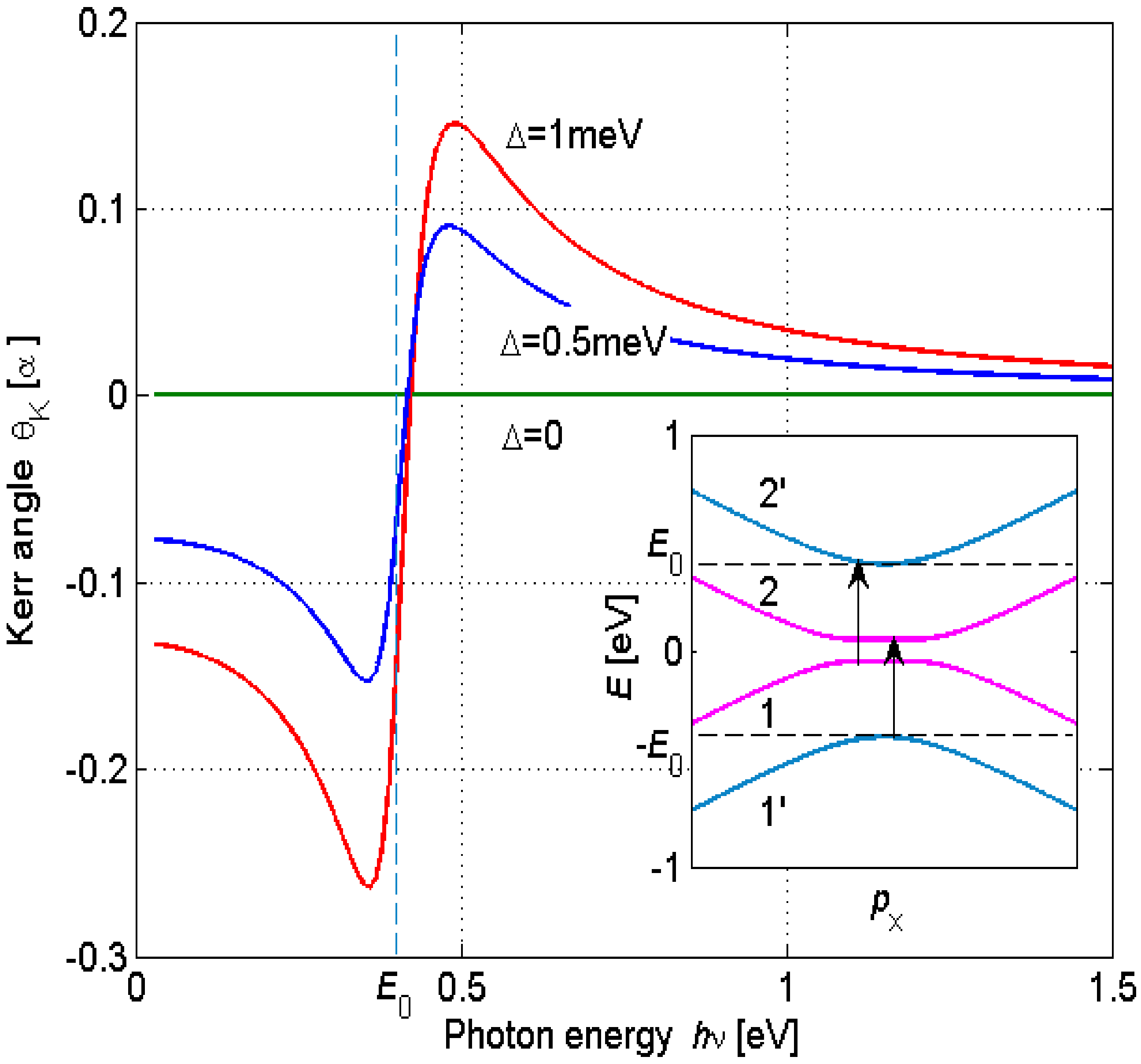
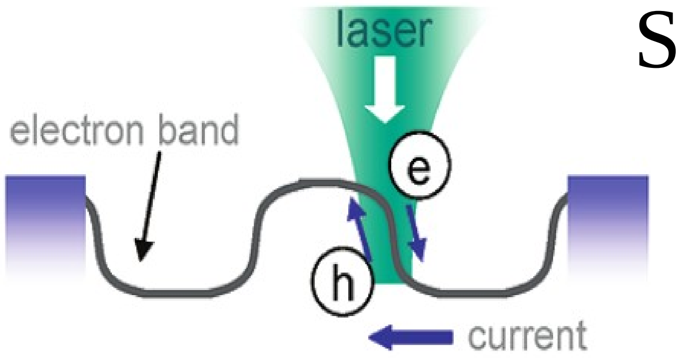


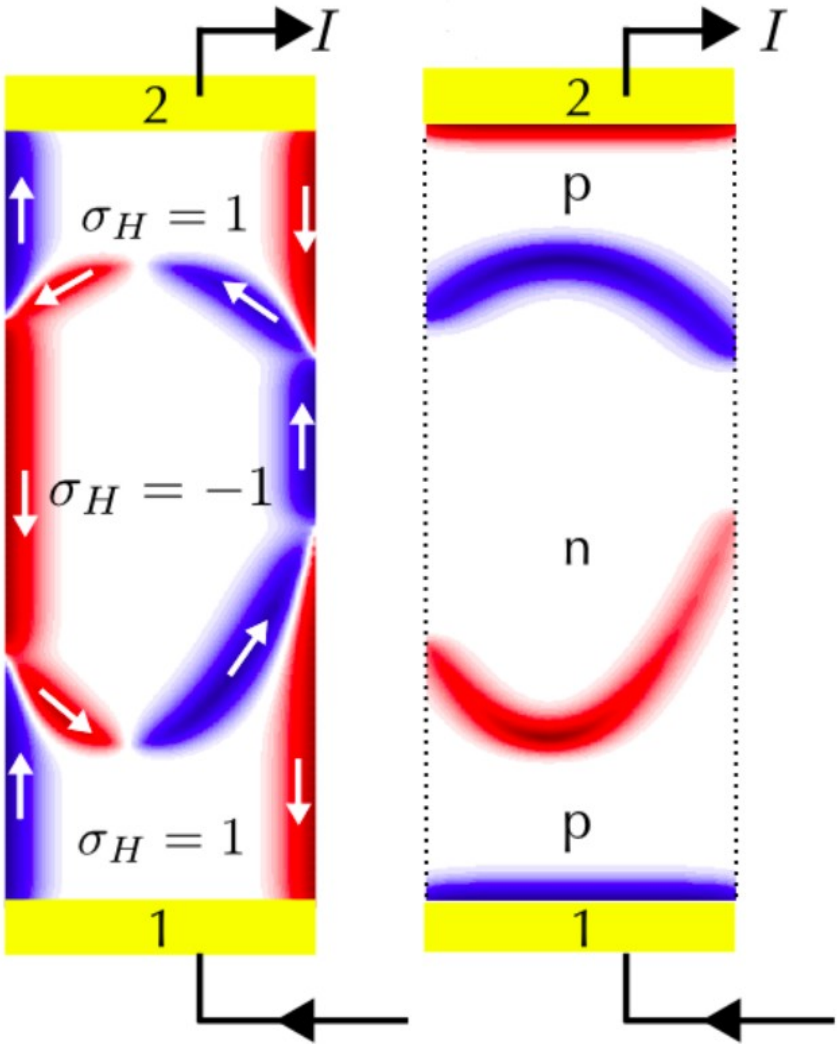
Scanning photocurrent (PC) imaging



Song & LL, arXiv:1112.5654 (2011)

$$j_{local} = (a\nabla n + b\hat{z}\times\nabla n)\, J_{laser}$$

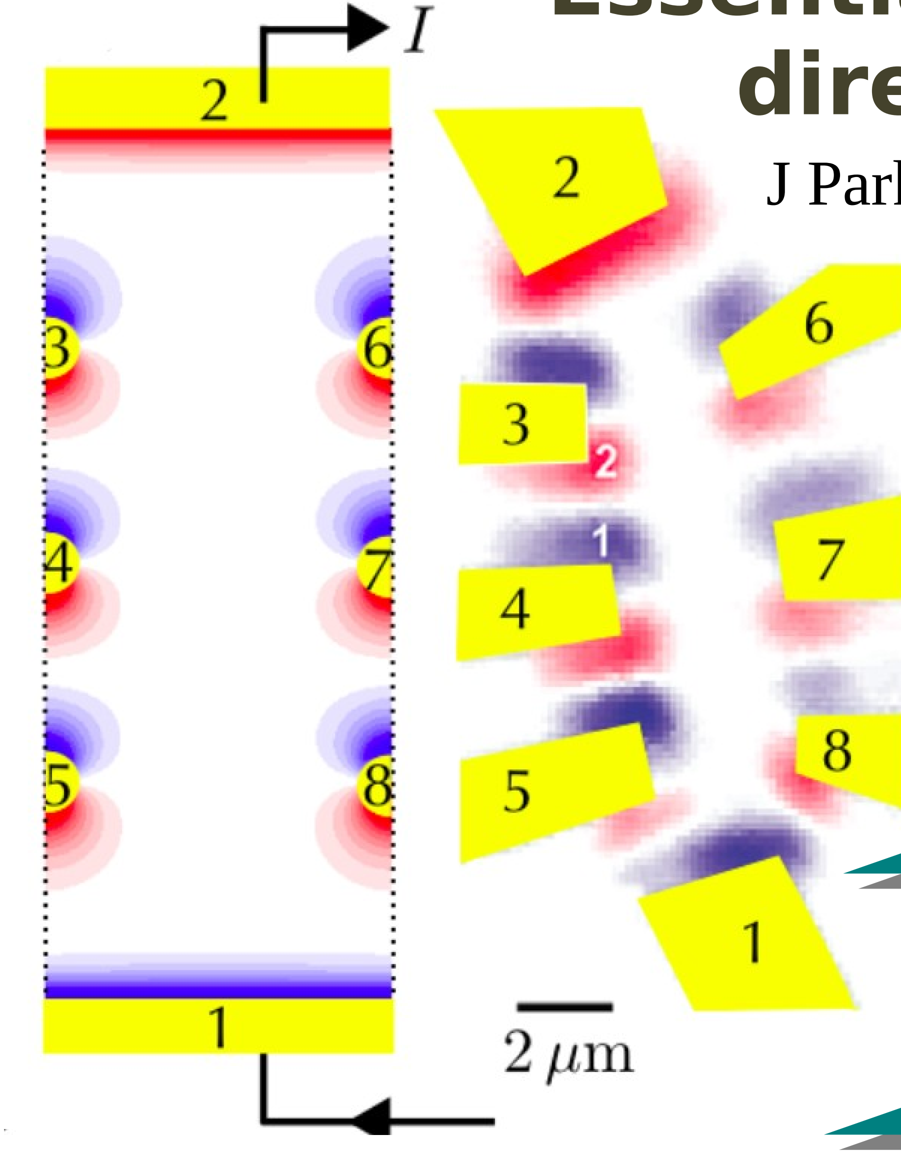
- Unpolarized light generates PC at interfaces, inhomogeneities, edges
- PC can image domains of opposite chirality, p-n bondaries, etc
- How are local properties manifested in system-wide PC?



System-wide (global) PC in gapless materials: imaging local properties

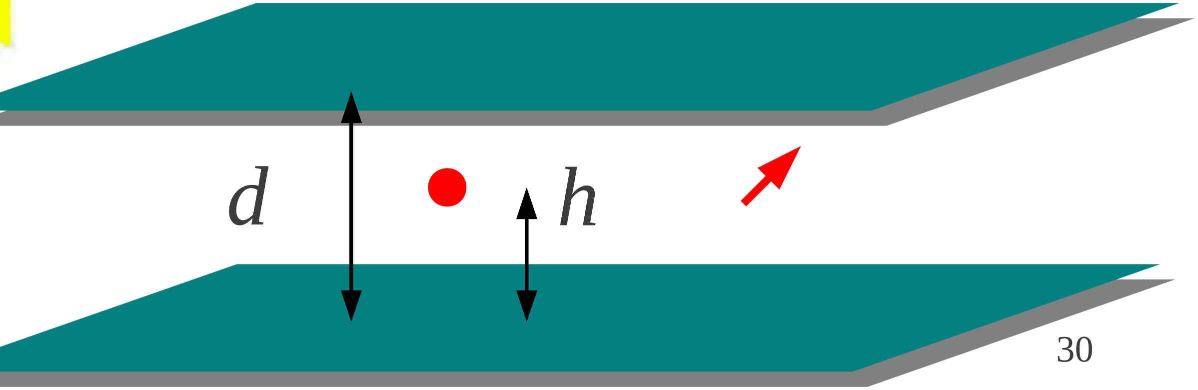
Essential nonlocality and directional effect

J Park et al (2009), J Song & LL (2011)

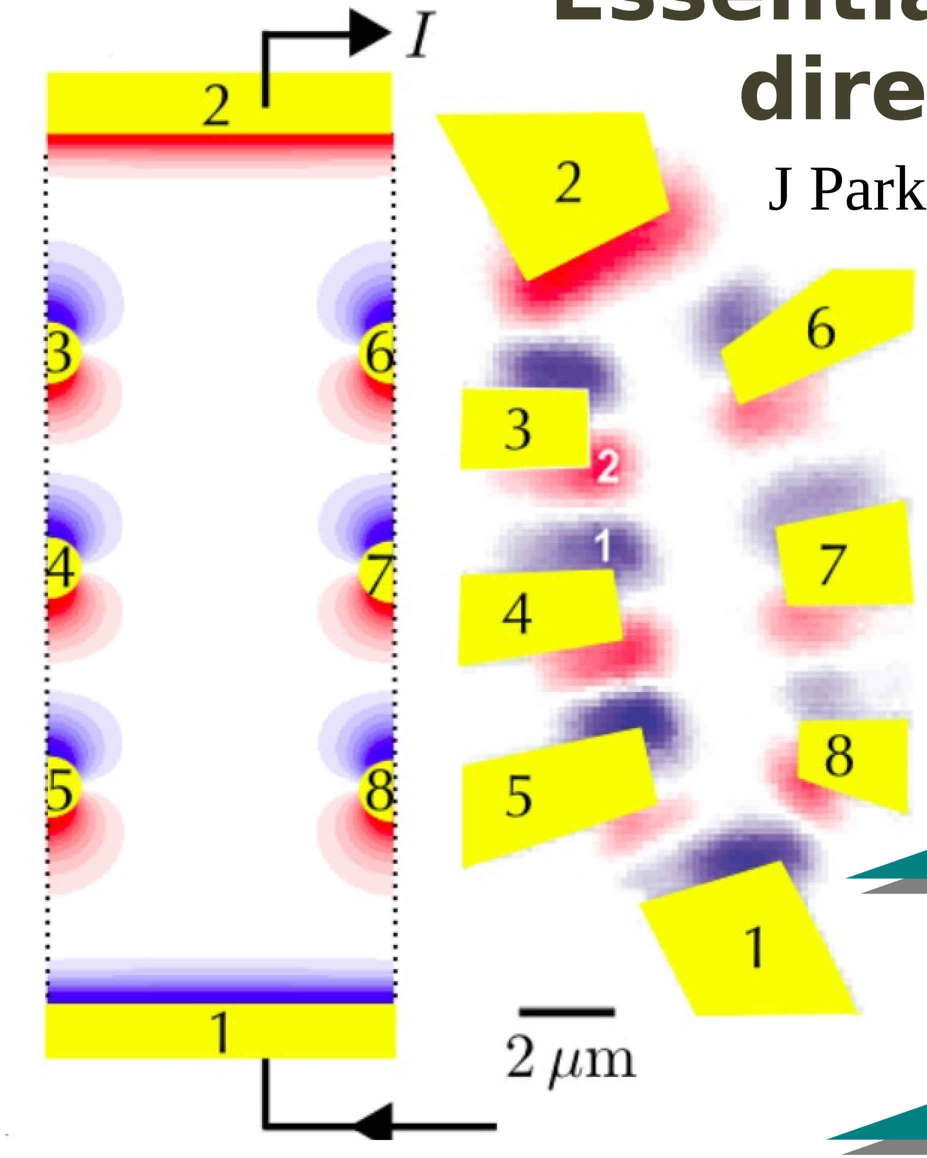


Electrostatic analogy

h dependence?



System-wide (global) PC in gapless materials: imaging local properties



Essential nonlocality and directional effect

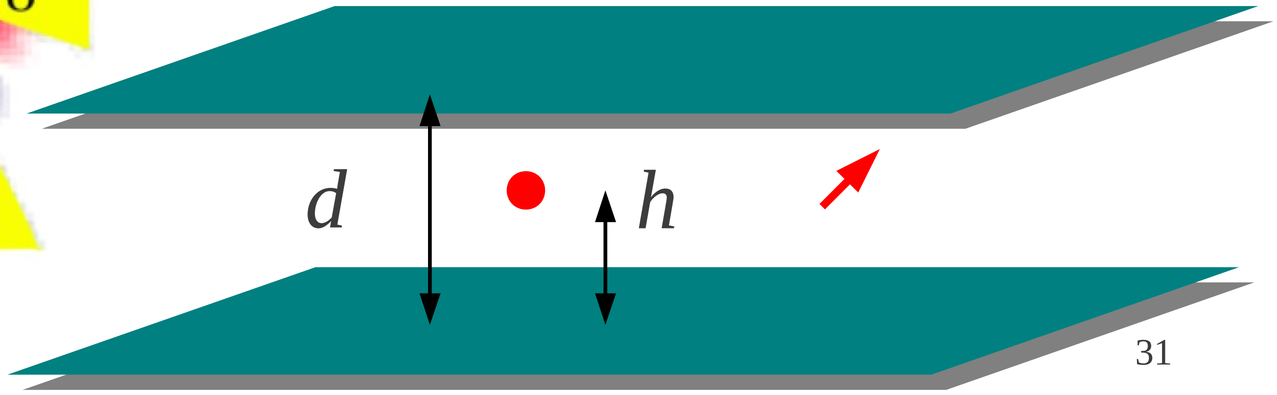
J Park et al (2009), J Song & LL (2011)

Electrostatic analogy

$$q' = -\frac{h}{d} q \qquad q'_{dipole} = -p_z / d$$

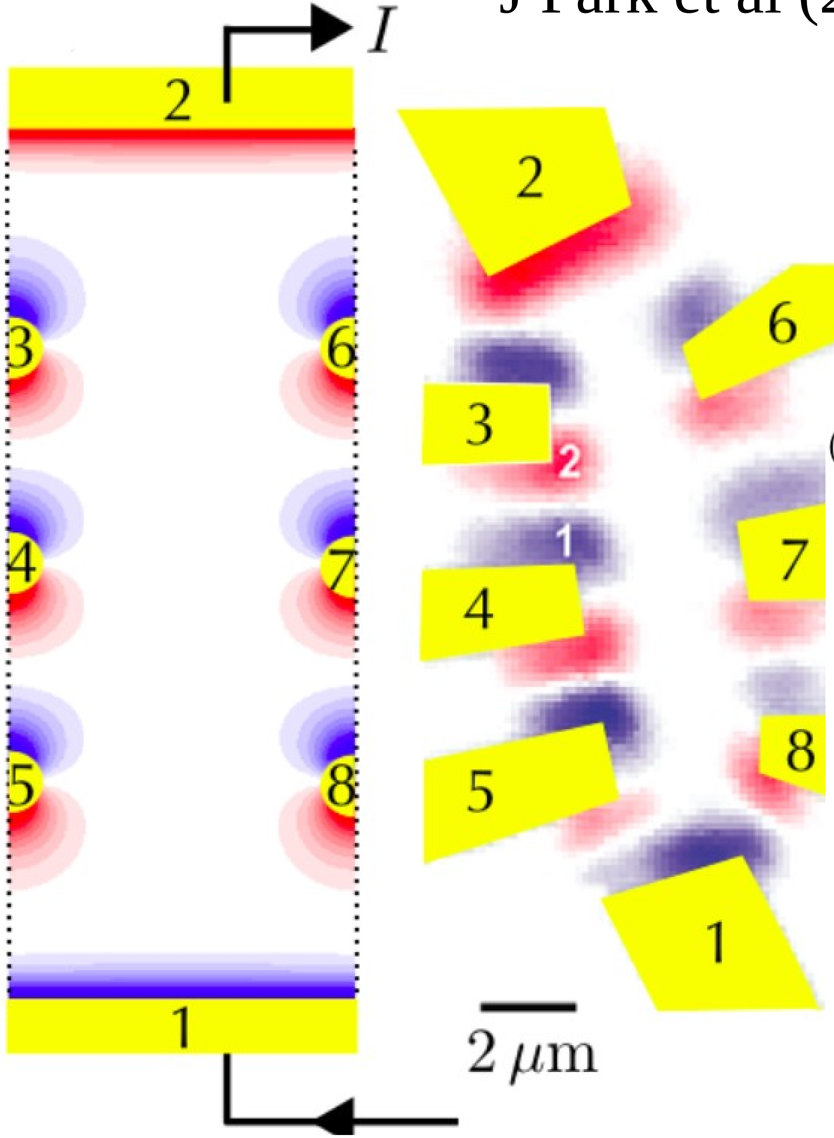
$$q'' = -\frac{d-h}{d} q \qquad q''_{dipole} = p_z / d$$

***h*-independent!**

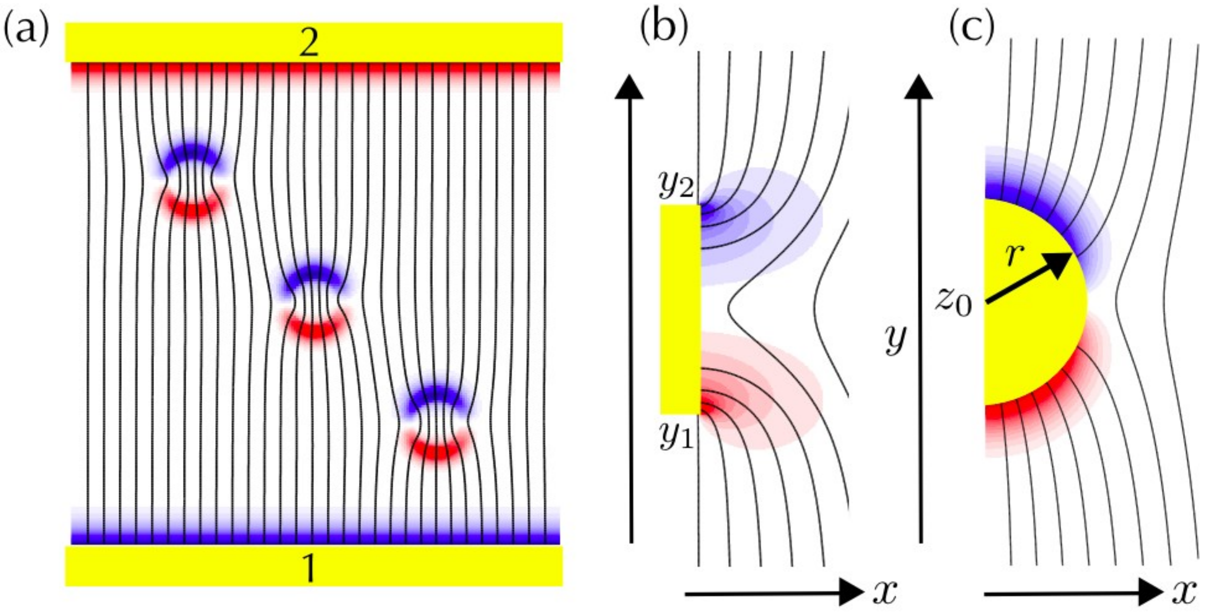


Nonlocality and directional effect “Shockley-Ramo theory”

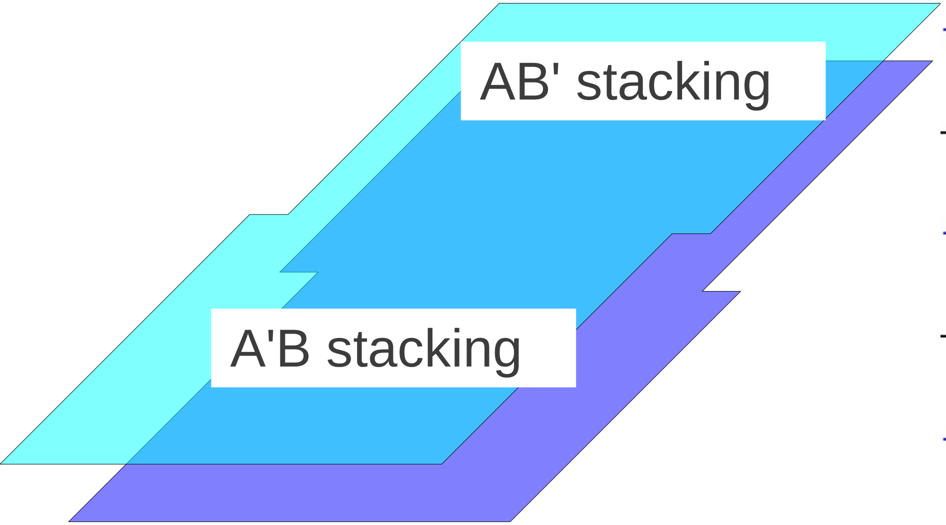
J Park et al (2009), J Song & LL (2011)

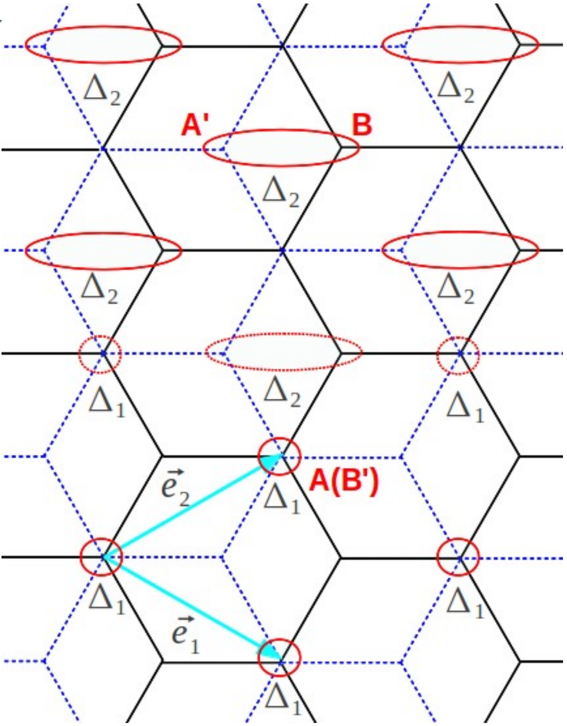


Angle-dependent global response, no position dependence

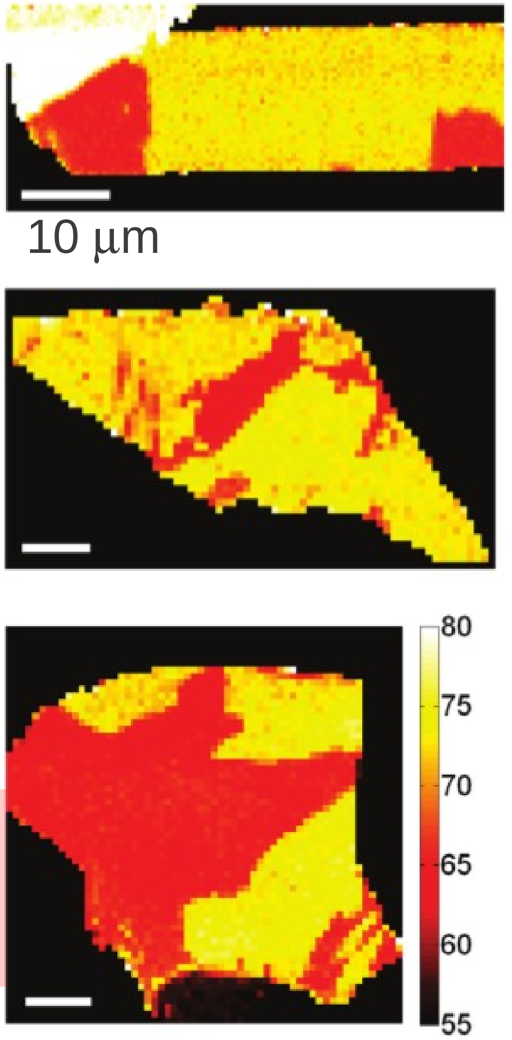


Tunneling heterojunctions in BLG: domains with different stacking order





ABA & ABC stacking in trilayer, Lui et al (2010)



Features:

- Tunneling transport (depends on orientation)
- New tunneling probe of ordered states
- Energy-dependent conductance, suppressed near DP (can mimic/obscure gapped state)

BLG summary

- Rich pattern of phases, SU(4) classification
- Possibility of realizing QAH state at low T
- Inducing QAH state with B field
- **Experimental verdict**: QAH order plausible, but more work needed
- Additional experimental probes: optical Kerr effect, photocurrent imaging, tunneling

Collaboration

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