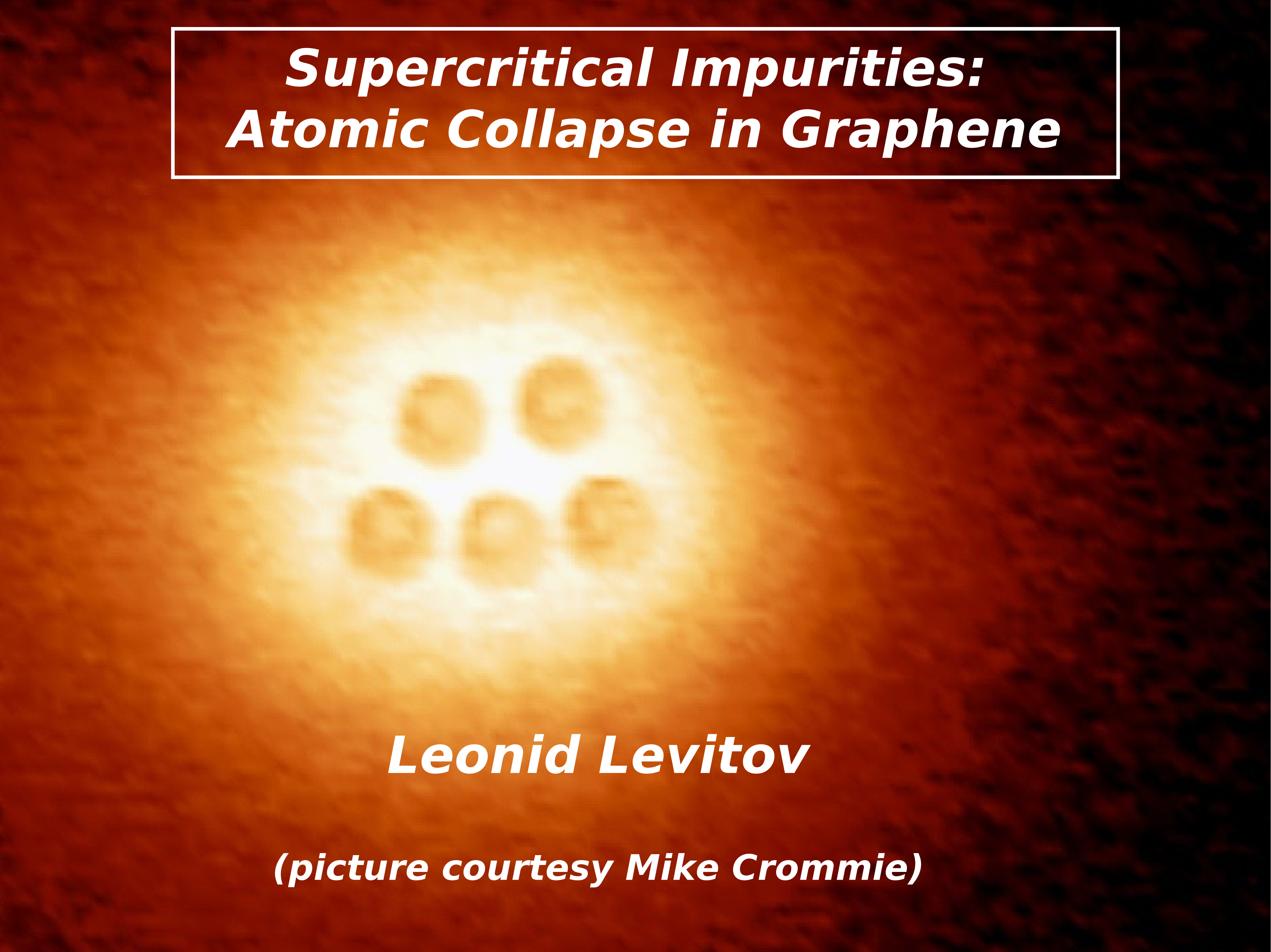


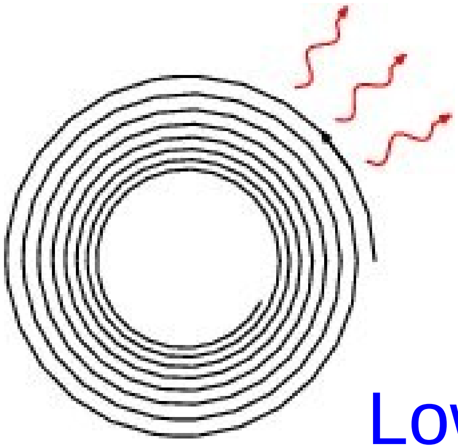
Supercritical Impurities: Atomic Collapse in Graphene

Leonid Levitov

(picture courtesy Mike Crommie)

Stability of a planetary atom

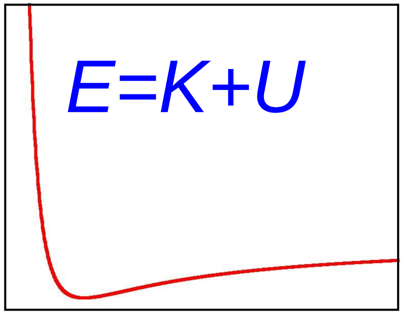
Classical physics: unstable (energy unbounded)



Quantum theory (Bohr, 1913): stable orbits, Rydberg's formula

$$E_n = -\frac{me^4Z^2}{2\hbar^2 n^2}$$

Lower bound: $E_1 = -\frac{me^4Z^2}{2\hbar^2}$



Heisenberg (1926): uncertainty principle

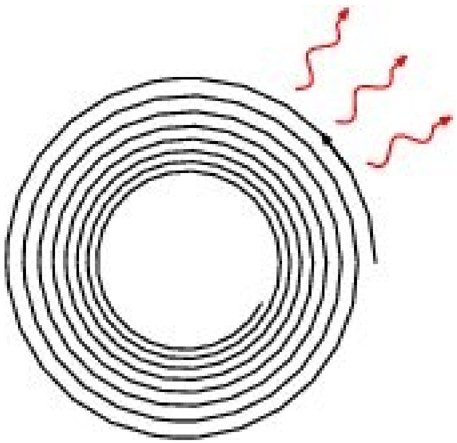
$$\Delta p \Delta x \geq \hbar/2$$

$$K_{\mathrm{nr}} = \frac{p^2}{2m} \sim \frac{\hbar^2}{2mr^2} \gg U = -\frac{Ze^2}{r}$$

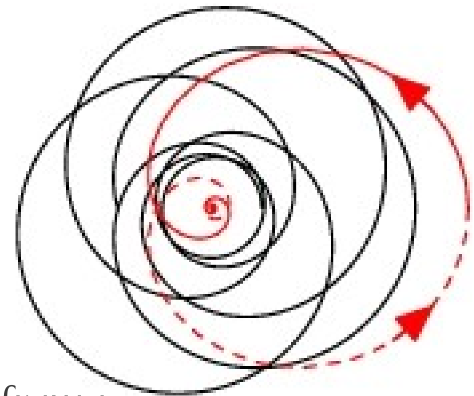
Planetary atom stabilized by QM (zero-point motion)

Stability of a Relativistic Atom

Classical physics: unstable
(energy unbounded)



Relativity: collapsing orbits



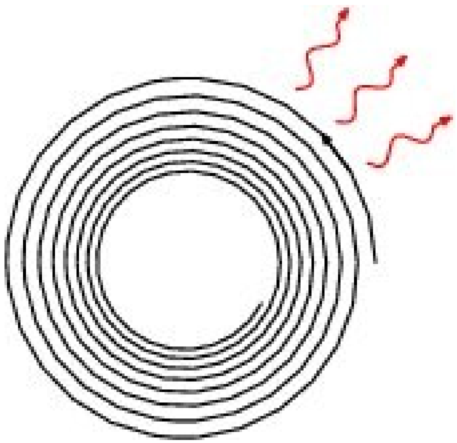
v < c

$$|Mc| > |Ze^2|$$

$$|Mc| < |Ze^2|$$

Stability of a Relativistic Atom

Classical physics: unstable (energy unbounded)

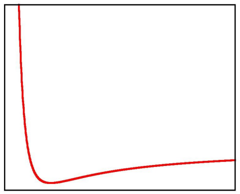


QM orbitals stabilized by zero point motion

$$K_{\rm nr} = \frac{p^2}{2m} \sim \frac{\hbar^2}{2mr^2}$$

E=K+U

$$U = -\frac{Ze^2}{r}$$



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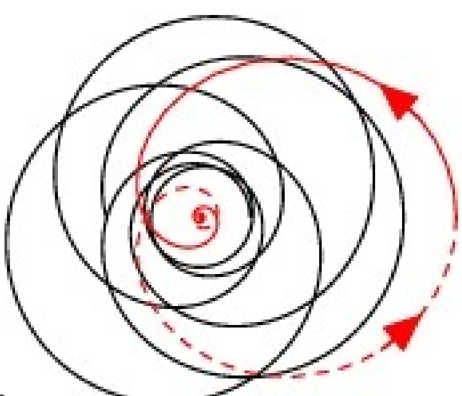

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Stability of a Relativistic Atom

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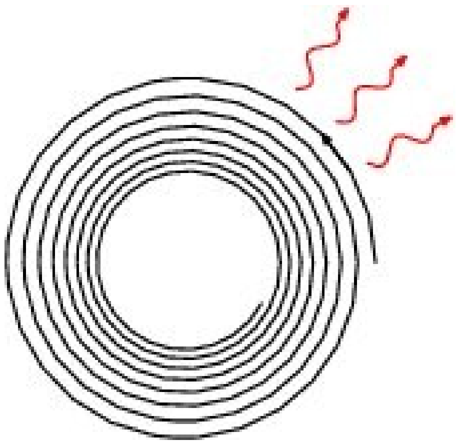
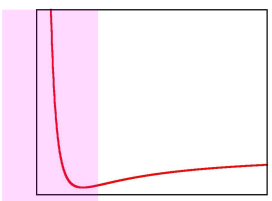


QM orbitals stabilized by zero point motion

$$K_{\rm nr} = \frac{p^2}{2m} \sim \frac{\hbar^2}{2mr^2}$$

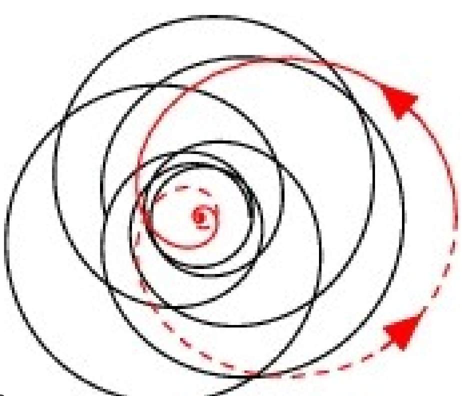
E=*K*+*U*

$$U = -\frac{Ze^2}{r}$$



Relativity: collapsing orbits

v < c

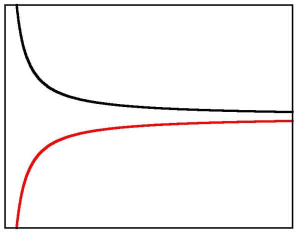


$$|Mc| > |Ze^2|$$

$$|Mc| < |Ze^2|$$

Relativity + QM:

$$K = cp \sim \frac{\hbar c}{r} \overset{?}{>} U$$



Collapse?

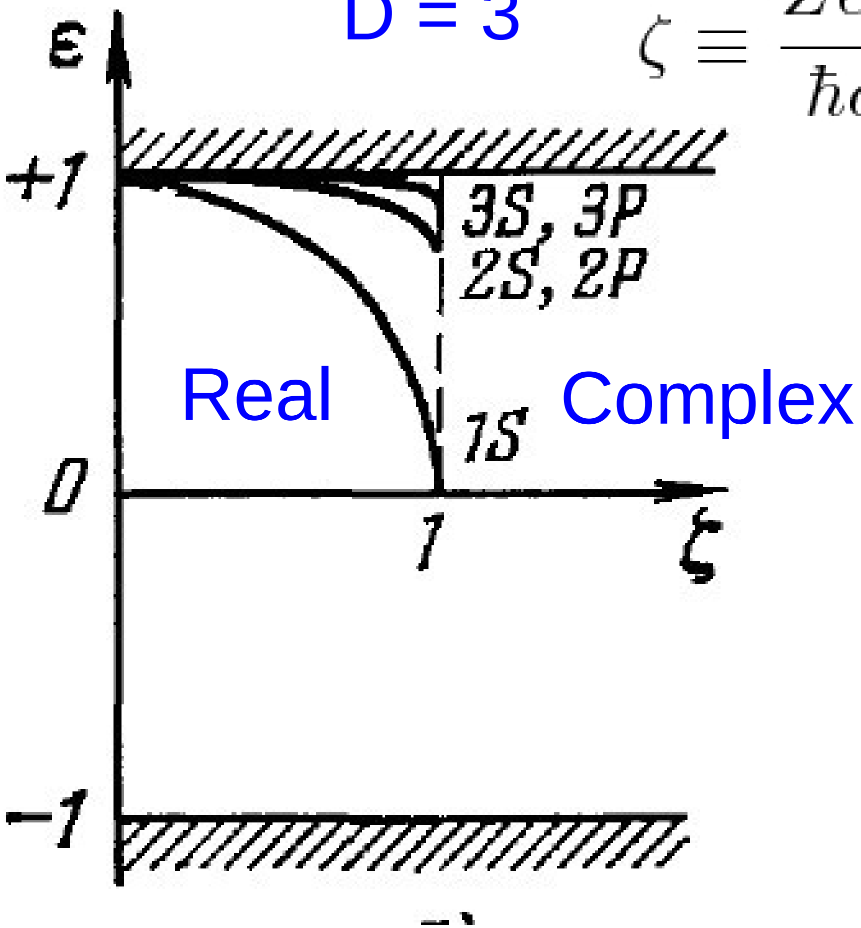
Dirac atoms can implode:

Subcritical (Z < 137)

$m \neq 0$ $\qquad \epsilon_1 = m\sqrt{1-\zeta^2}$

D = 3 $\qquad \zeta \equiv \dfrac{Ze^2}{\hbar c}$

Dirac (1929)



Complex energies at

$\zeta > 1$

Dirac atoms can implode:

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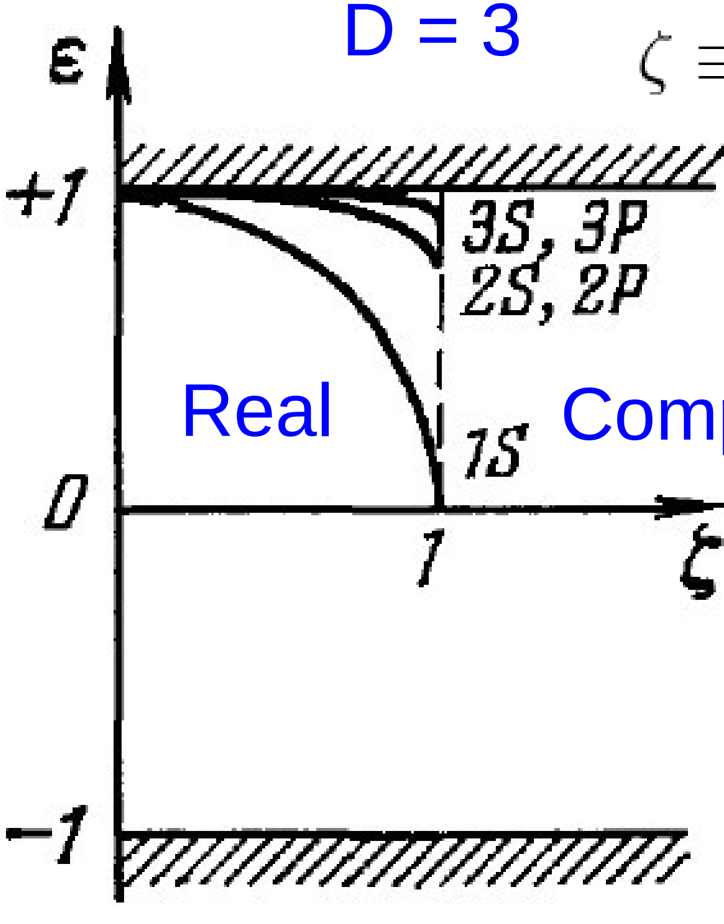
D = 3 $\zeta \equiv \dfrac{Ze^2}{\hbar c}$



Dirac (1929)

Complex energies at

$\zeta > 1$

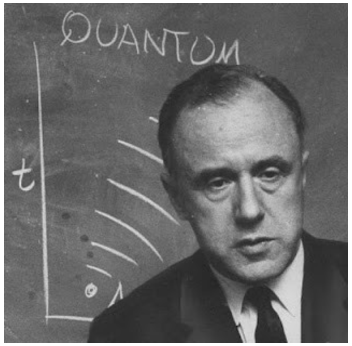
What happens at
Z > 137?

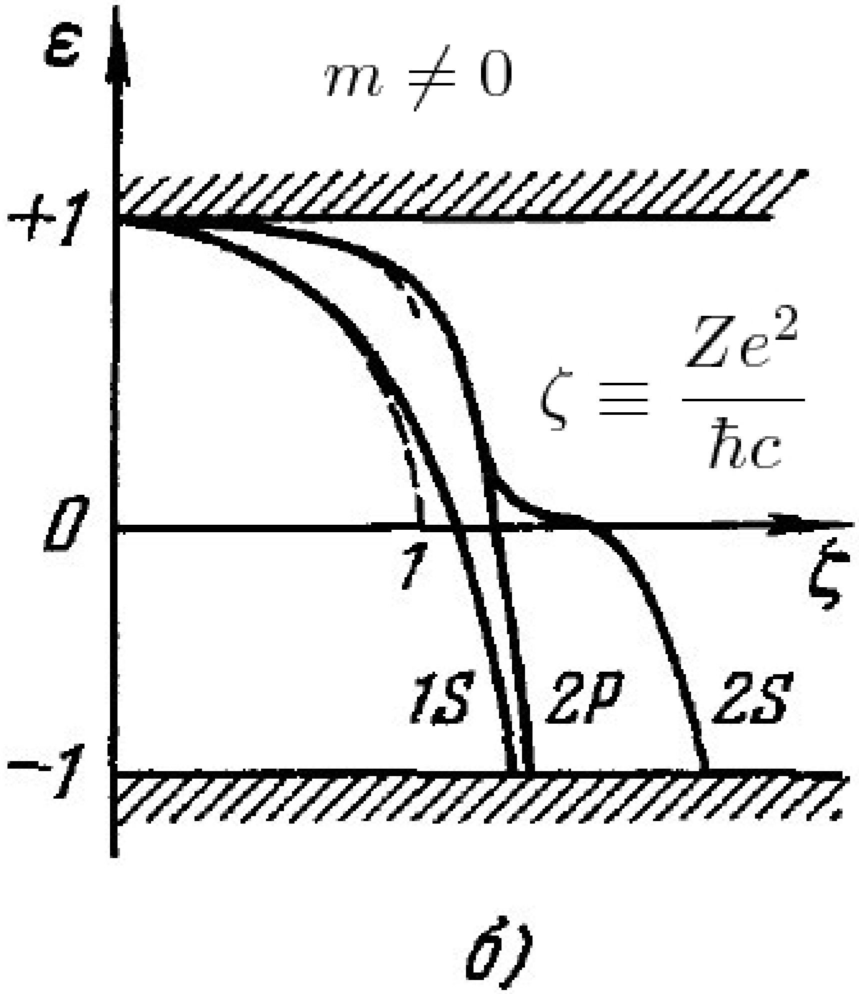
Supercritical atom

Pre-collapse (137 < Z < 170)

Pomeranchuk & Smorodinskii (1945); Werner and Wheeler (1957)







Finite size of nucleus
Pomeranchuk nuclear formfactor

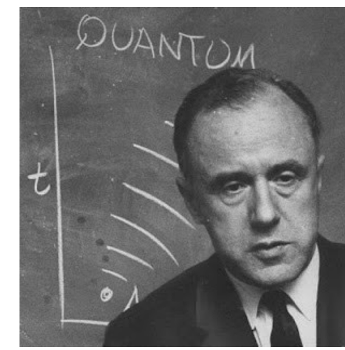
$$r_0 \approx 1.2 \cdot 10^{-12} cm$$

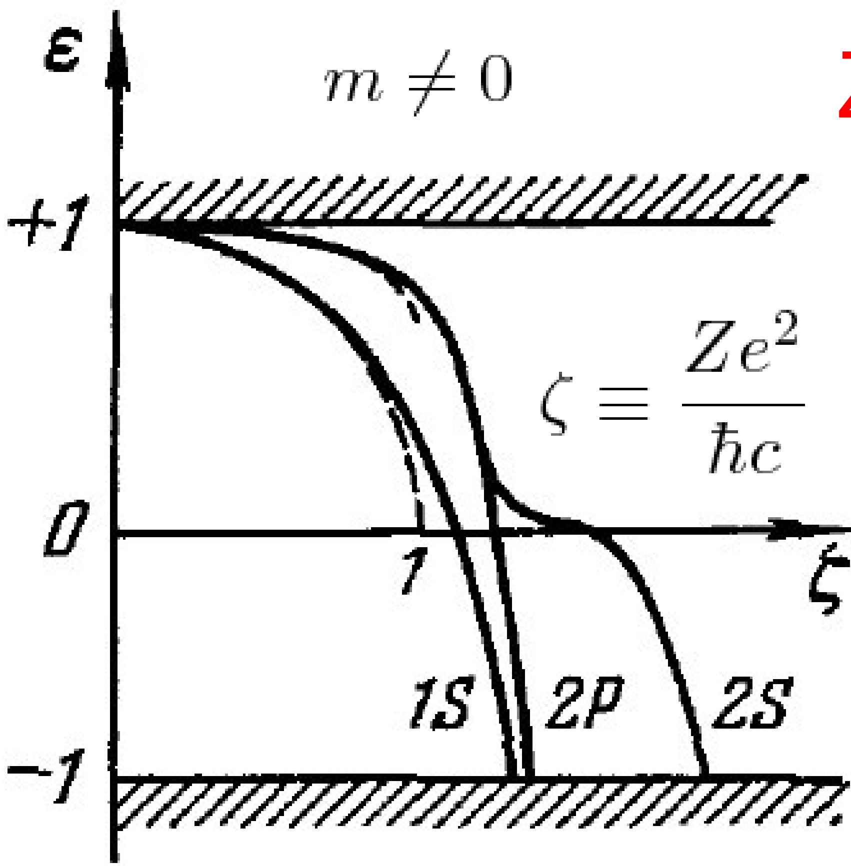
1S level dives into Dirac sea at Z = 170

Supercritical atom

Pre-collapse (137 < Z < 170)

Pomeranchuk & Smorodinskii (1945); Werner and Wheeler (1957)





0

б)

Z > 170?

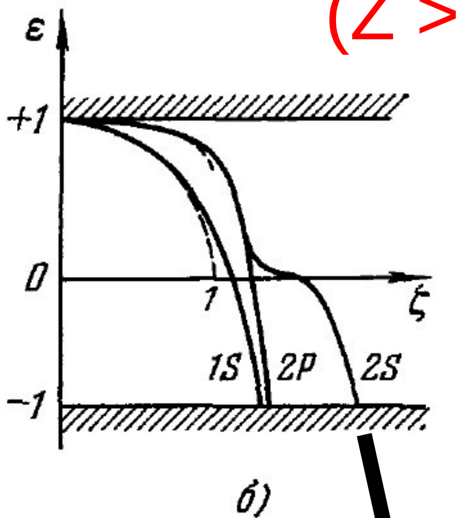
Finite size of nucleus
Pomeranchuk nuclear formfactor

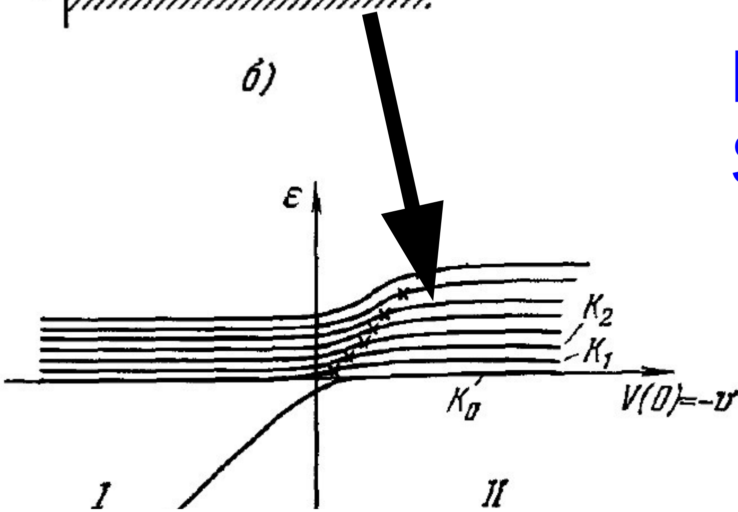
$$r_0 \approx 1.2 \cdot 10^{-12} cm$$

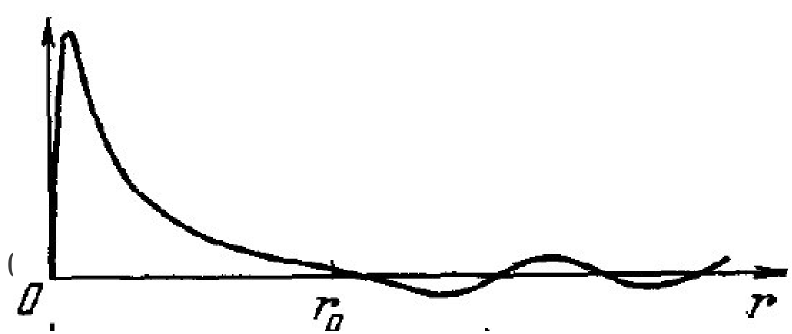
1S level dives into Dirac sea at Z = 170

Supercritical atom

Collapse, vacuum reconstruction
(Z > Zc = 170)







Gershteyn, Zeldovich (1969)
Popov (1970)

Resonance state in the Dirac sea
Screening by pair production?

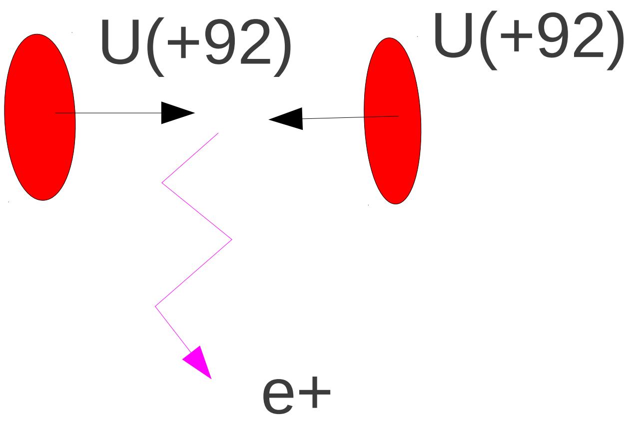
$$\epsilon = \epsilon_0 - i\gamma \quad \epsilon_0 = -m - a(Z - Z_c)$$

$$\gamma \sim \exp\left(-\frac{b}{\sqrt{Z - Z_c}}\right)$$

Quasilocalized spatial structure
of the resonance state

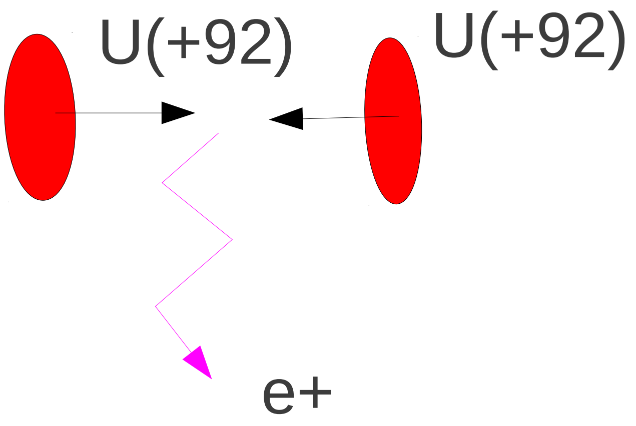
Signatures: supercritical e+ emission

Darmstadt experiment (1980s, revisited in 1990s)
UNILAC (EPOS, ORANGE)
3-6 MeV collisions

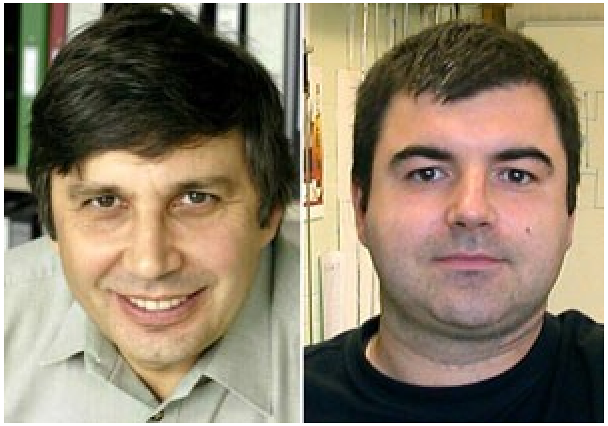


Signatures: supercritical e+ emission

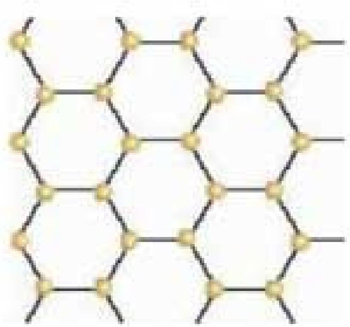
Darmstadt experiment (1980s, revisited in 1990s)
UNILAC (EPOS, ORANGE)
3-6 MeV collisions
No signatures of <u>supercritical</u> emission



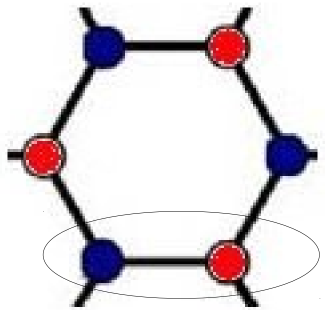
Relativistic massless electrons in graphene



Two sublattices

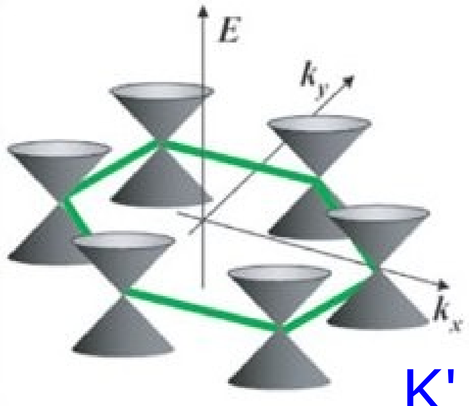


$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$



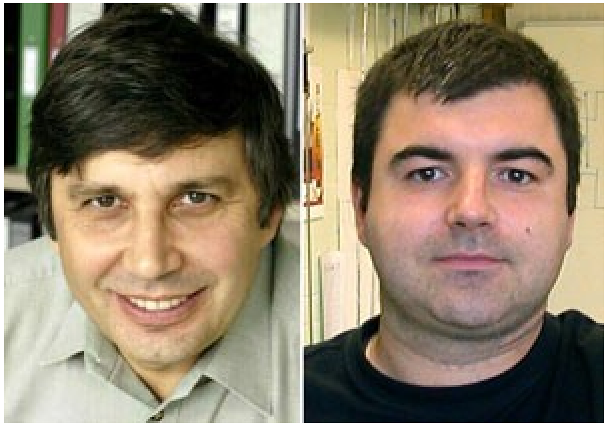
pseudo-spin (sublattice)

$$\hat{H} = v_F \hat{\boldsymbol{\sigma}} \mathbf{p}$$

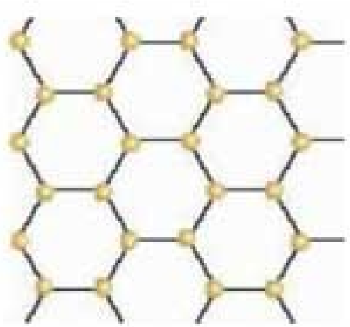


4-fold degeneracy spin&valley

Relativistic massless electrons in graphene



Two sublattices



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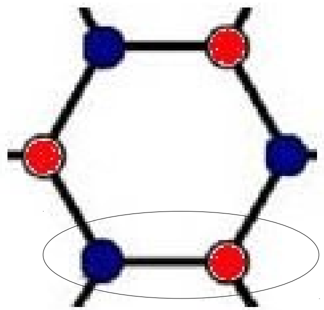
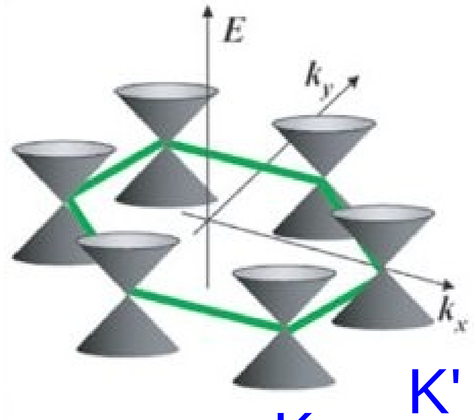


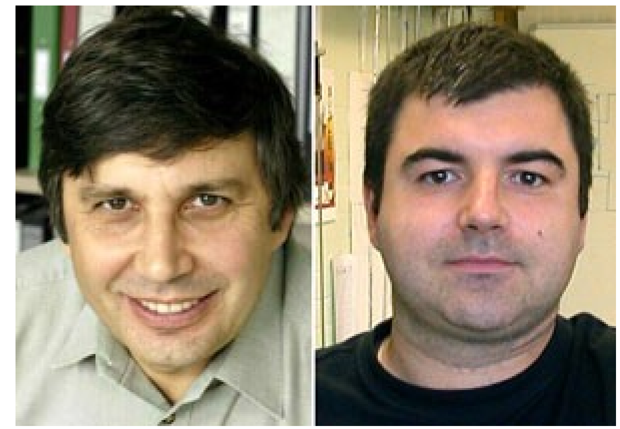
pseudo-spin (sublattice)

$$\hat{H} = v_F \hat{\boldsymbol{\sigma}} \mathbf{p} \qquad E = \sqrt{c^2 p^2 + m^2 c^4}$$



4-fold degeneracy spin&valley

Relativistic massless electrons in graphene



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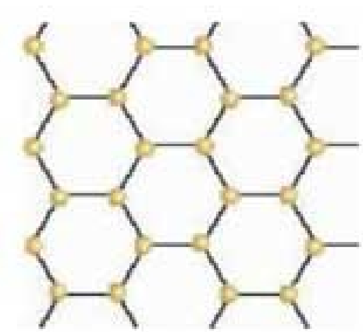

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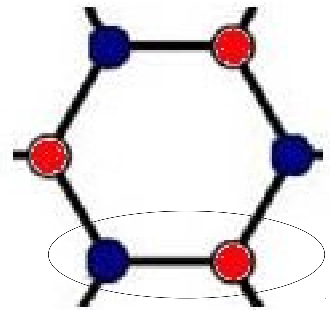
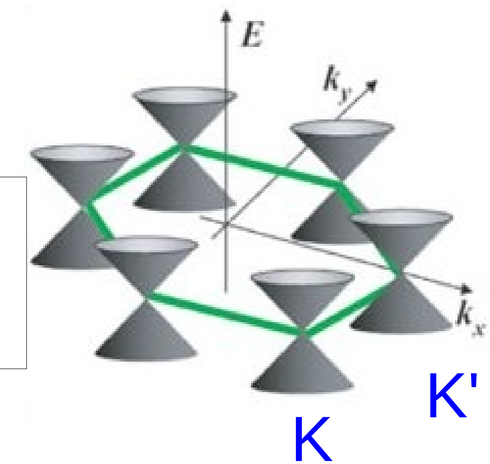


pseudo-spin (sublattice)

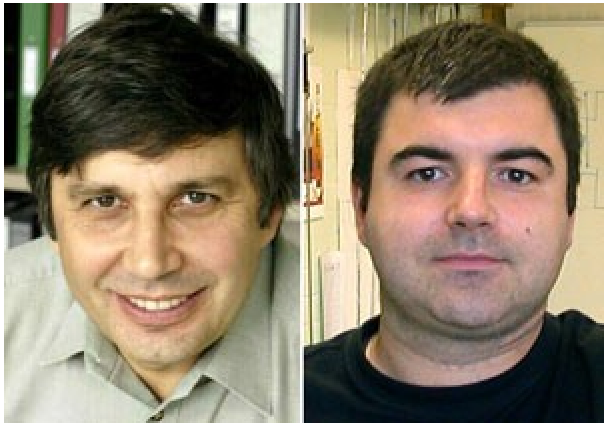
$$\hat{H} = v_F \hat{\boldsymbol{\sigma}} \mathbf{p} \qquad E = \sqrt{c^2 p^2}$$

No gap (cond-mat) ≡ No mass (hep)

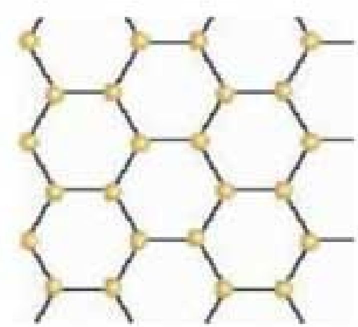


4-fold degeneracy spin&valley

Relativistic massless electrons in graphene



Two sublattices



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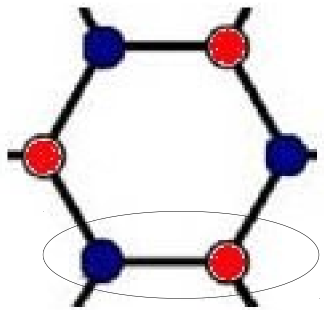

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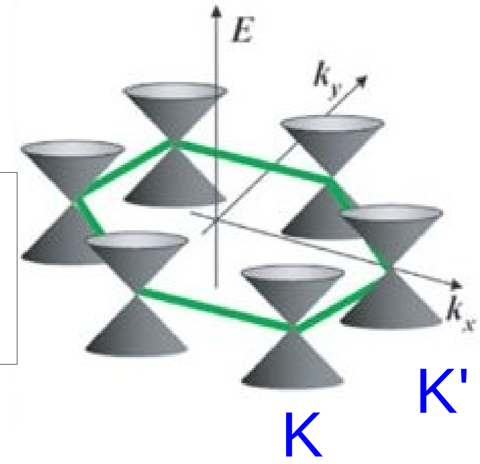


No gap (cond-mat) ≡ No mass (hep)

$$v_F \approx 10^6 m/s = \frac{c}{300} \qquad \alpha = e^2/\hbar v \approx 2.5$$

4-fold degeneracy spin&valley

Slow, but ultra-relativistic Dirac fermions

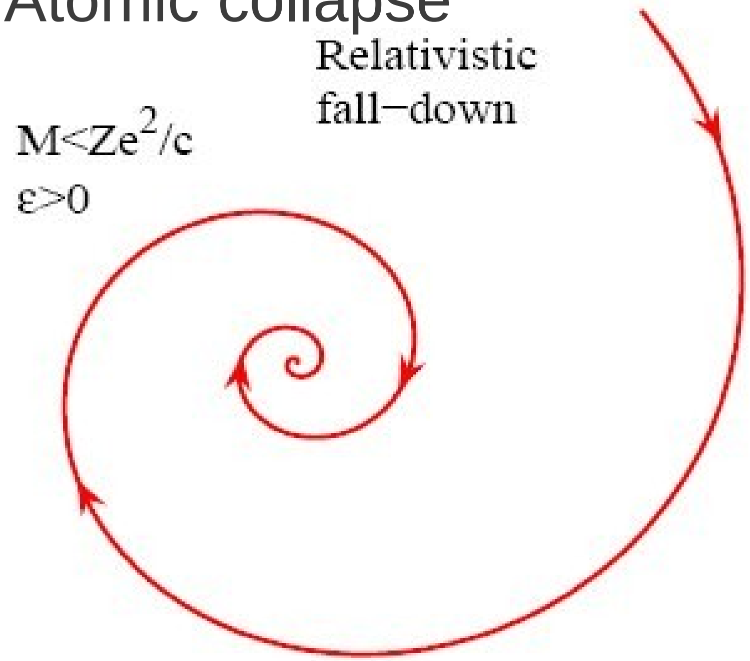
Atomic collapse for massless fermions

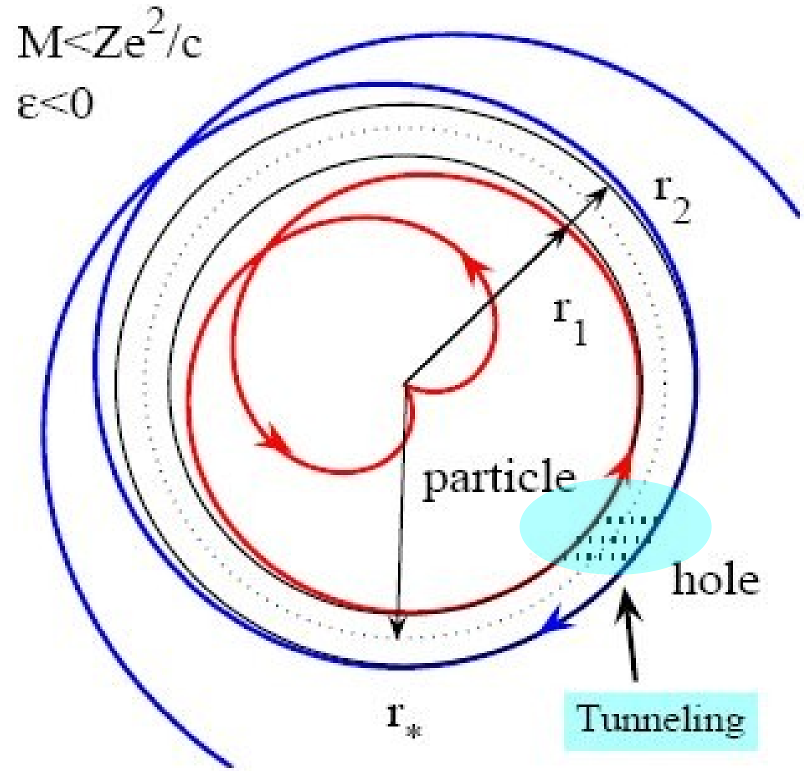
- Large fine structure constant, $\alpha = \dfrac{e^2}{\hbar v} \approx 2.5$, low collapse threshold Z~1
- Massless Dirac equation: continuum spectrum only, no discrete spectrum
- Manifestation of collapse: formation of resonances (infinite family, quasilocalized spatial structure)
- Strong effects in vacuum polarization: screening cloud may extend indefinitely (*cf.* cutoff at Compton wavelength for massive Dirac electron)

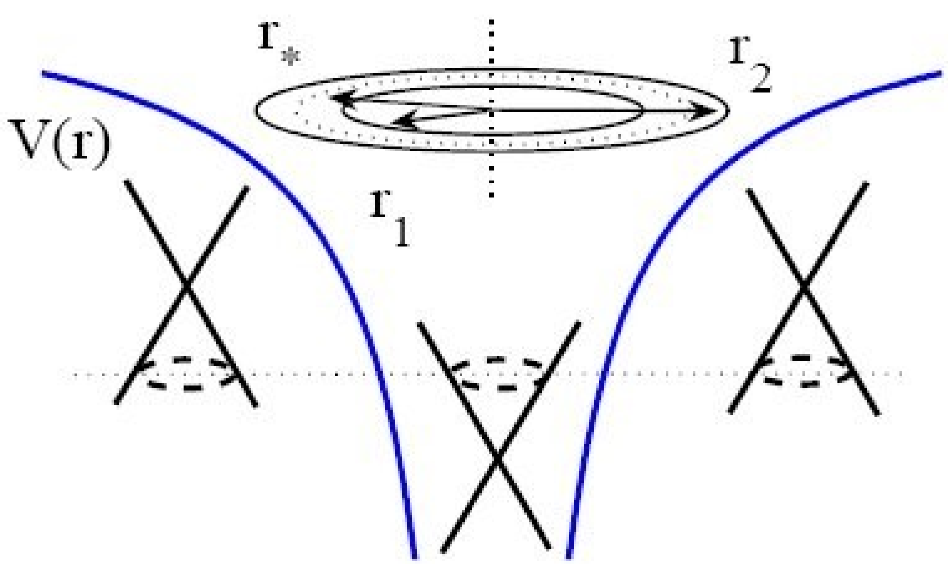
A. V. Shytov, M. I. Katsnelson, and L. S. Levitov,
PRL 99, 236801 (2007), PRL 99, 246802 (2007)

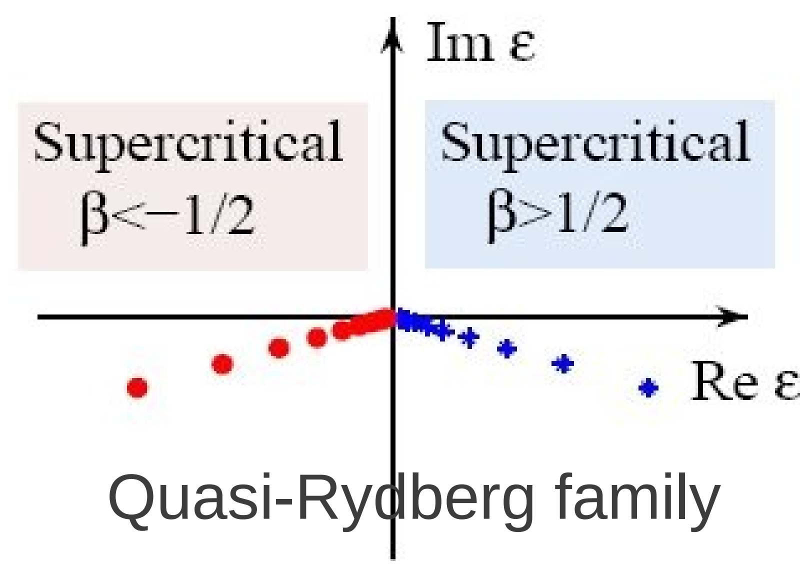
Quasistationary states

Atomic collapse





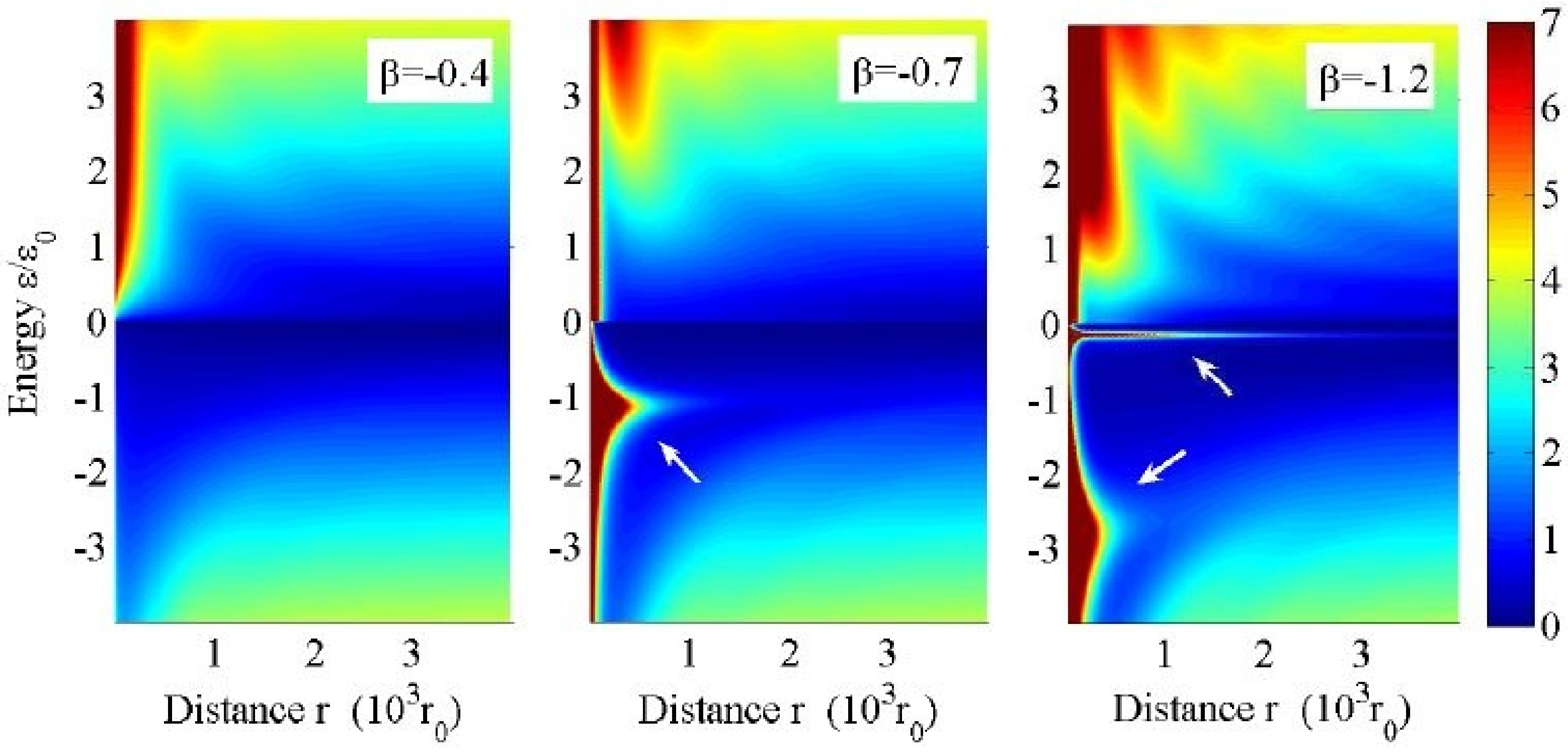




Quasi-Rydberg family

Resonances in the local density of states, can be probed by STM

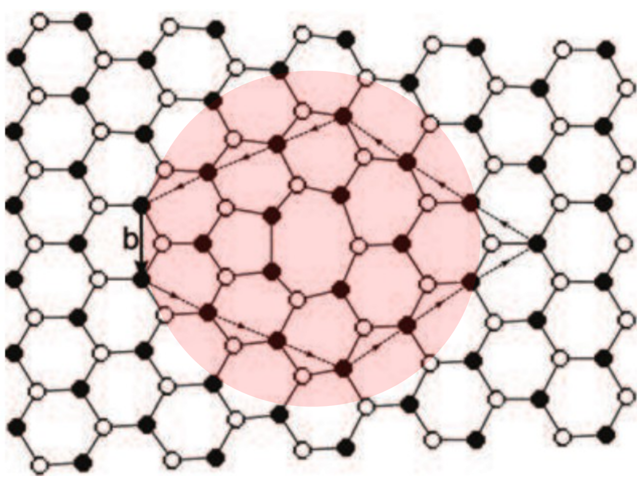
Tunneling spectroscopy $\beta = Ze^2/\kappa\hbar v$

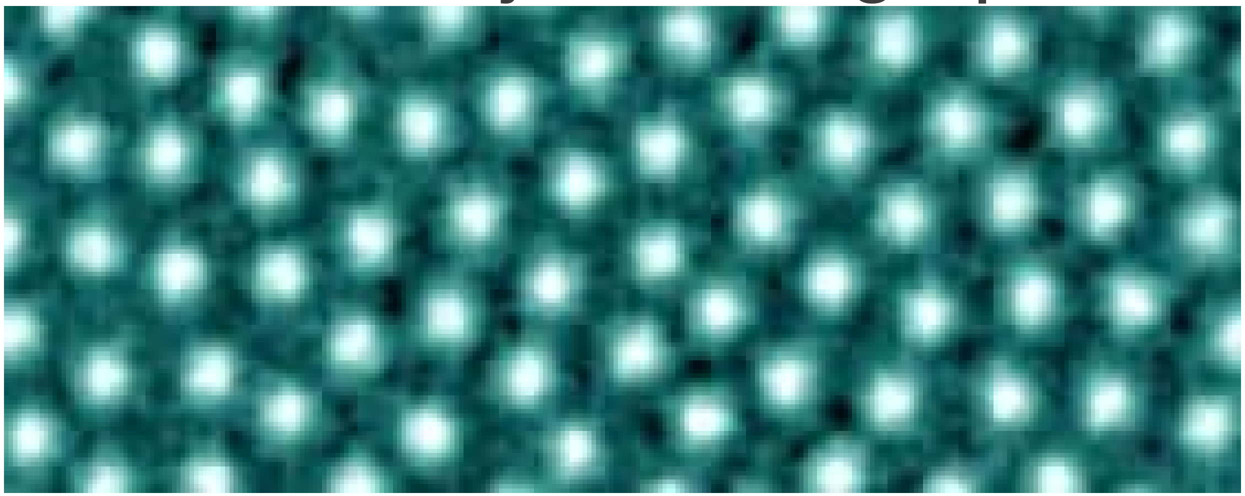


 Energy scales as the width Γ and as 1/(localization radius)

Reaching collapse (critical Z values)

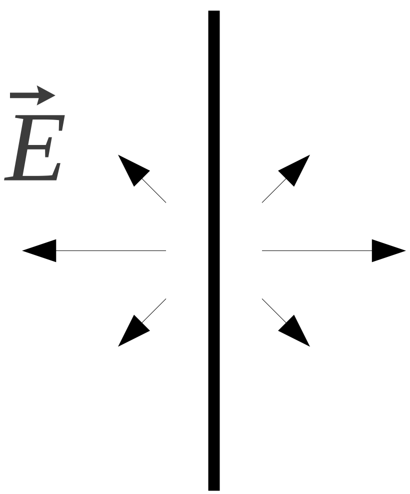
- Vary Z. Critical value affected by intrinsic screening. Accounting for self-consistent nonlinear screening challenging. Optimistic estimates yield 1<Z<2.
- Experiment (Crommie): Co trimers, Ca dimers
- Recent work: AB flux as a vehicle to control collapse, Z<<1
- Can be realized for dislocations (pseudo-B field), or vortices in a superconductor adjacent to graphene



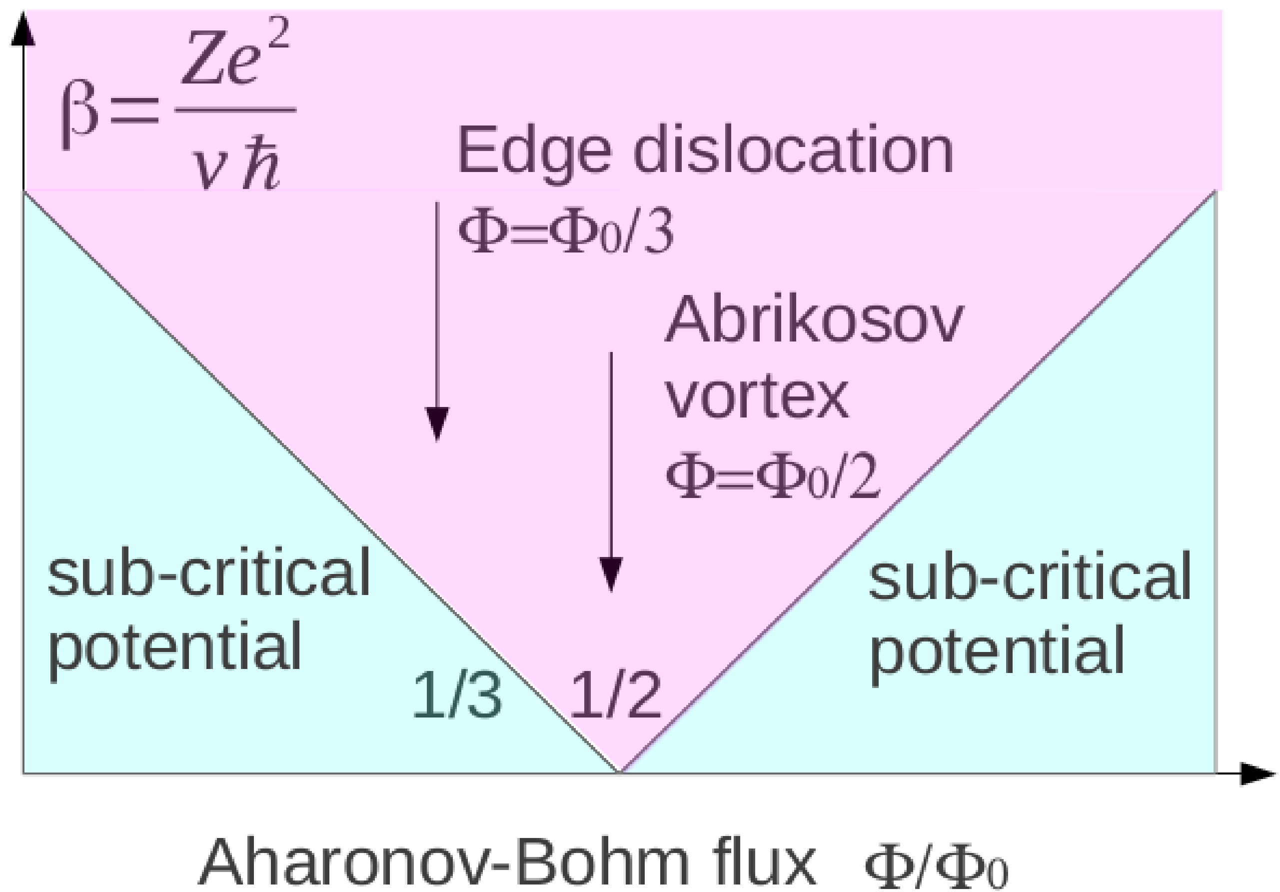


Atomic collapse in the presence of an Aharonov-Bohm solenoid

$$H = v\sigma(\vec{p} - \vec{A}(\rho)) - \frac{Ze^2}{\rho} \qquad \vec{A}(\rho) = \frac{\Phi}{2\pi\rho}(-y, x)$$



$$\vec{B} = \Phi\,\delta(\rho)$$



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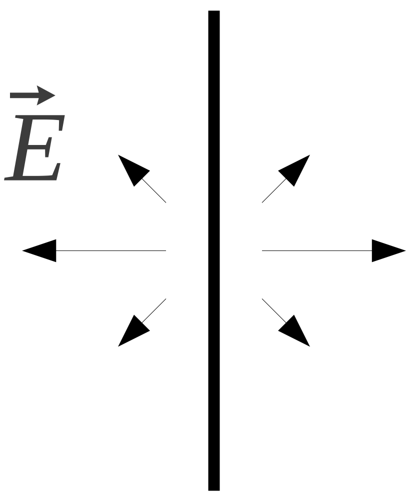

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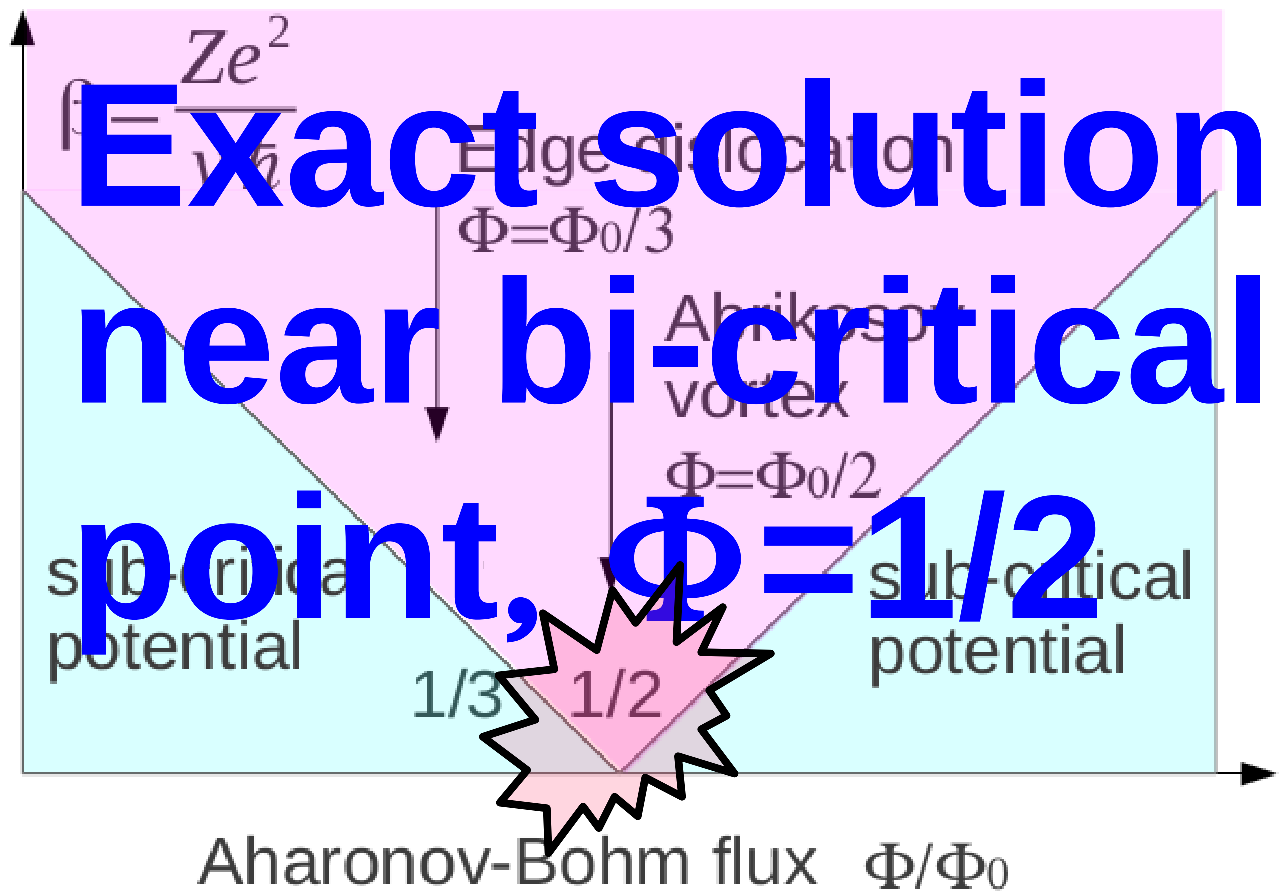


Exact solution near bi-critical point, Φ=1/2

Screening cloud: transition between dielectric and metallic behavior

- Subcritical regime: dielectric behavior, 1/r potential, screening charge concentrated on the lattice scale
- Supercritical regime: metallic behavior, large-scale screening cloud with a power law tale
- Predict scaling exponent value from exact solution

$$V(\rho \gg a) \sim \rho^{-\eta} \qquad 1 < \eta < 2$$

Take-home message

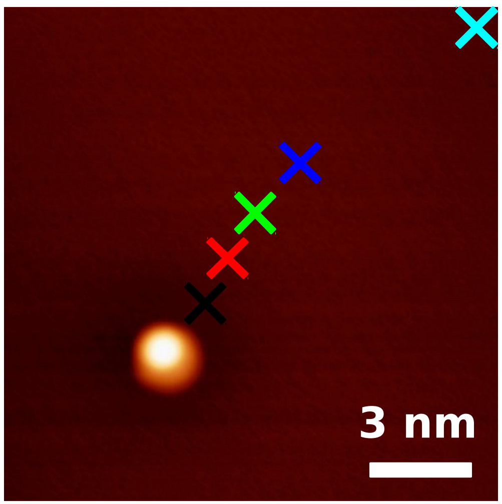
- AB flux a knob to tune collapse (in addition to Z)
- A bi-critical point, two regimes: “dielectric” and “metallic”
- Exact solution for vacuum polarization and the screening cloud structure
- Experimental search for supercritical potential.

Take-home message

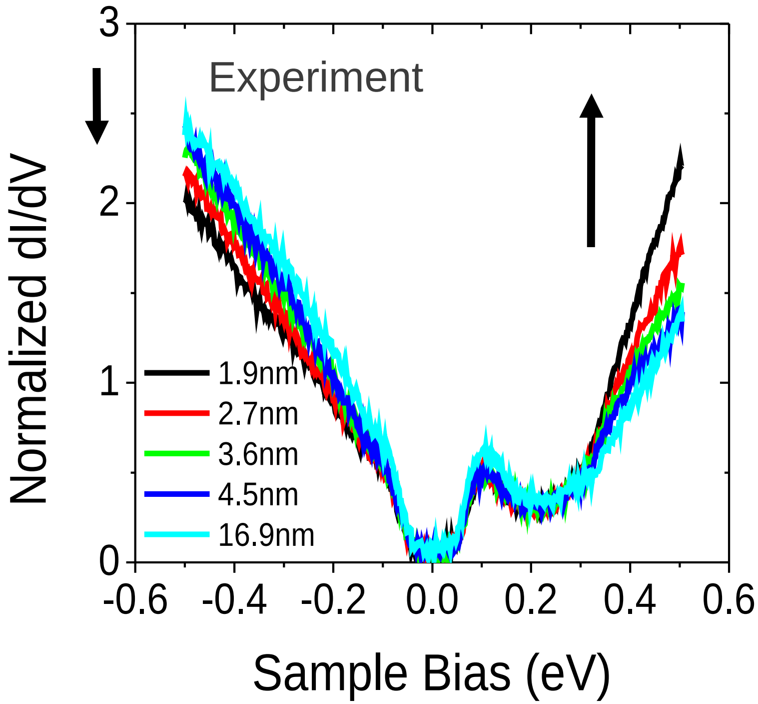
- AB flux a knob to tune collapse (in addition to Z)
- A bi-critical point, two regimes: “dielectric” and “metallic”
- Exact solution for vacuum polarization and the screening cloud structure
- Experimental search for supercritical potential. **Just found it**

How do we make a supercritical potential?
Ca dimers on G/BN, Crommie's Lab (Berkeley)

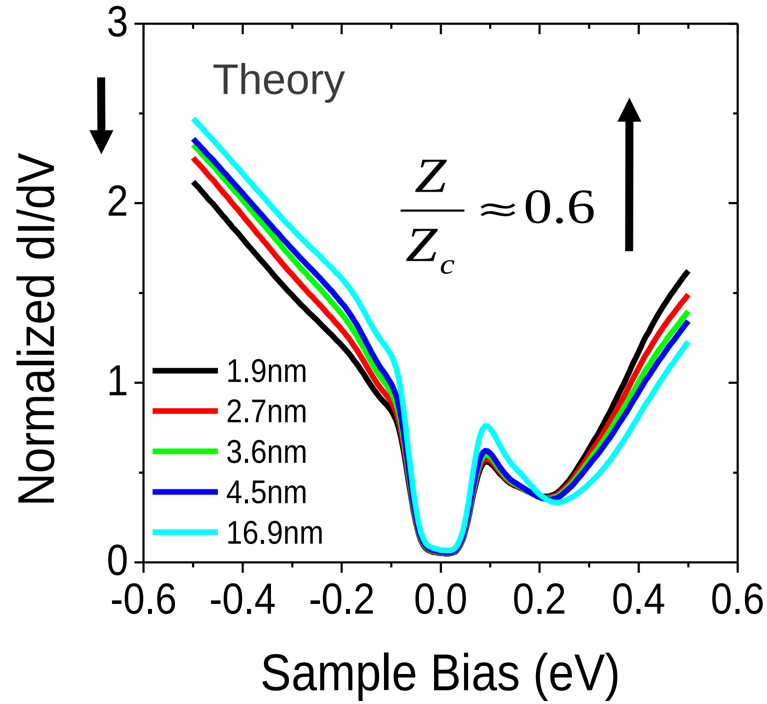
Ca dimer



STS for 1 Dimer



Simulation for 1 Dimer



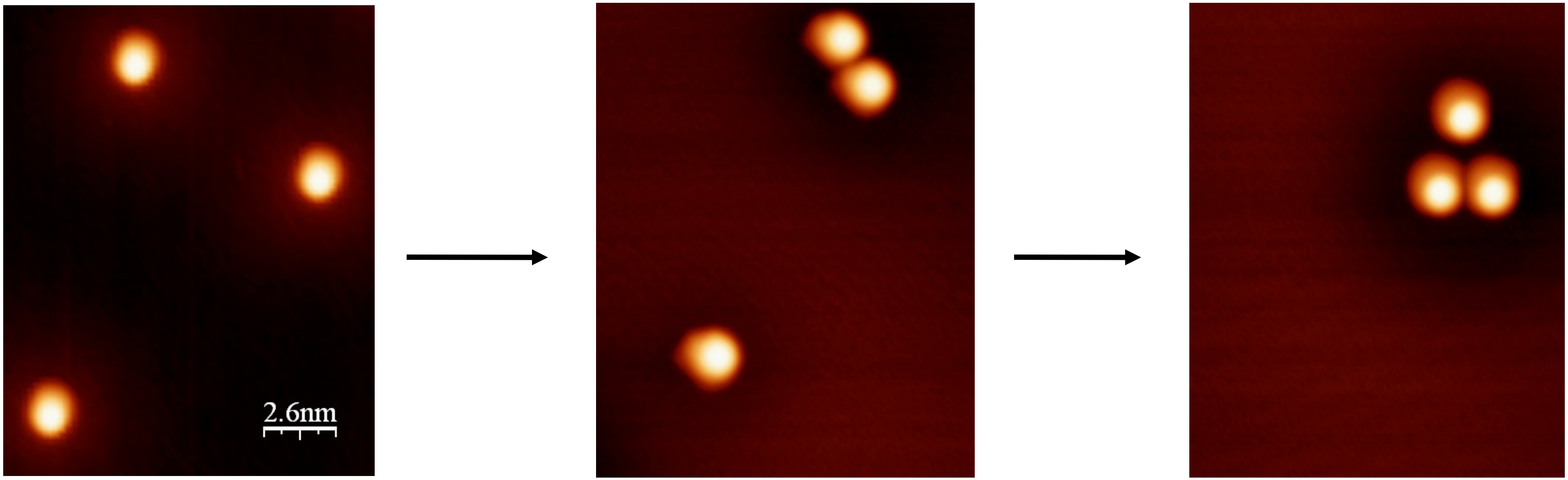
$Z < Z_c$ Subcritical...

Previous work (probed intrinsic screening for Co trimers)
Wang, Brar, Shytov, Wu, Regan, Tsai, Zettl, Levitov, Crommie, Nat Phys 8, 653 (2012)

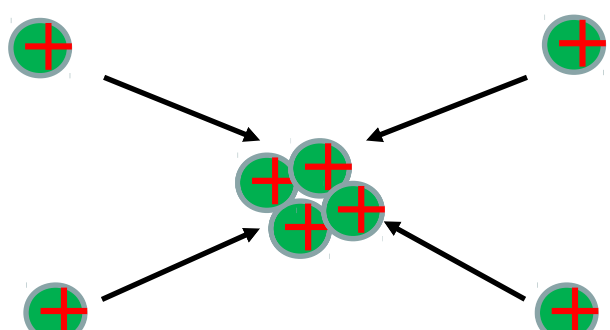
Ca Dimer Has an Advantage: Can Manipulate it

Ca Dimers are Moveable Charge Centers

n=3 dimer cluster

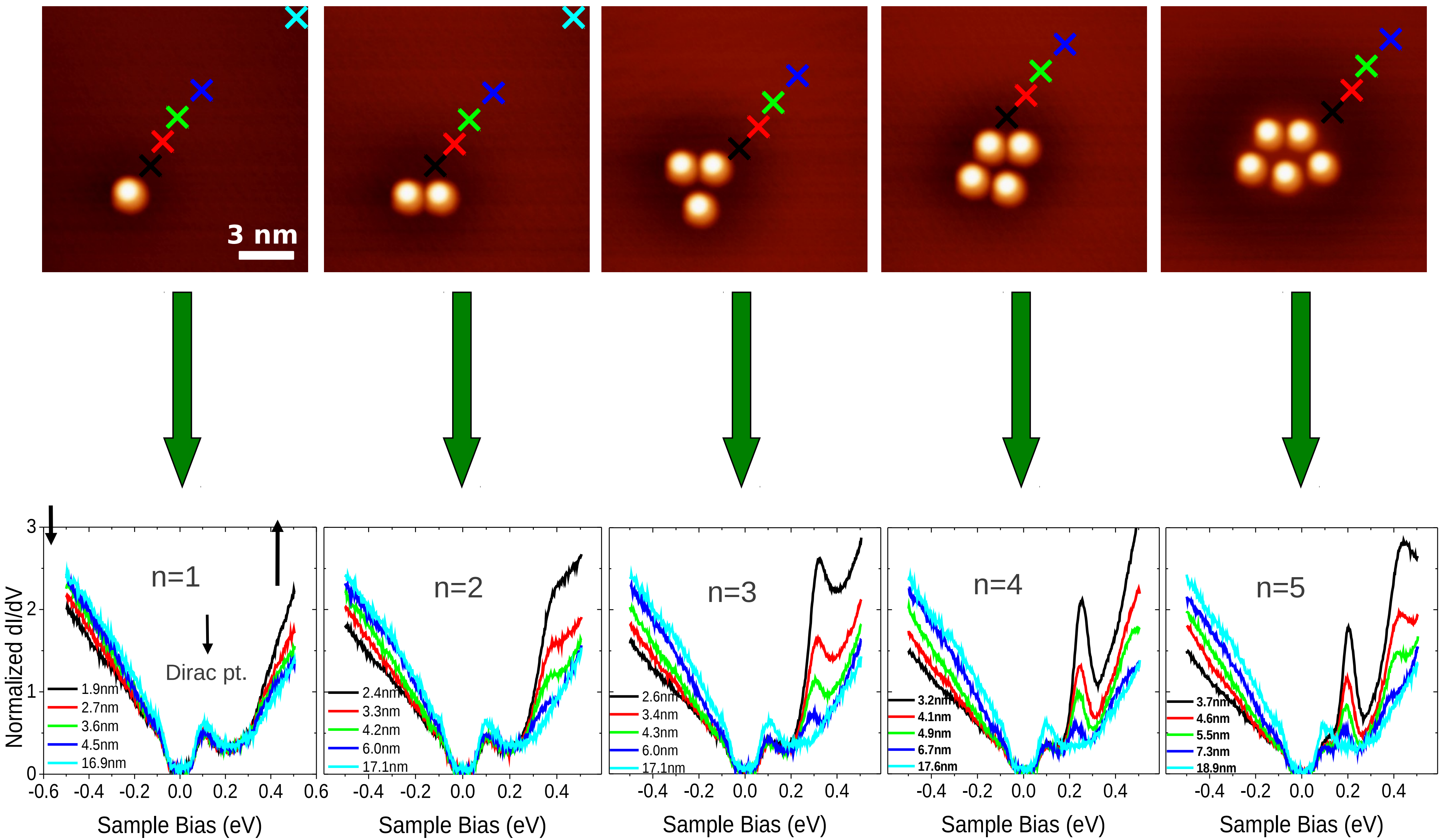


Fabricate Artificial Nuclei

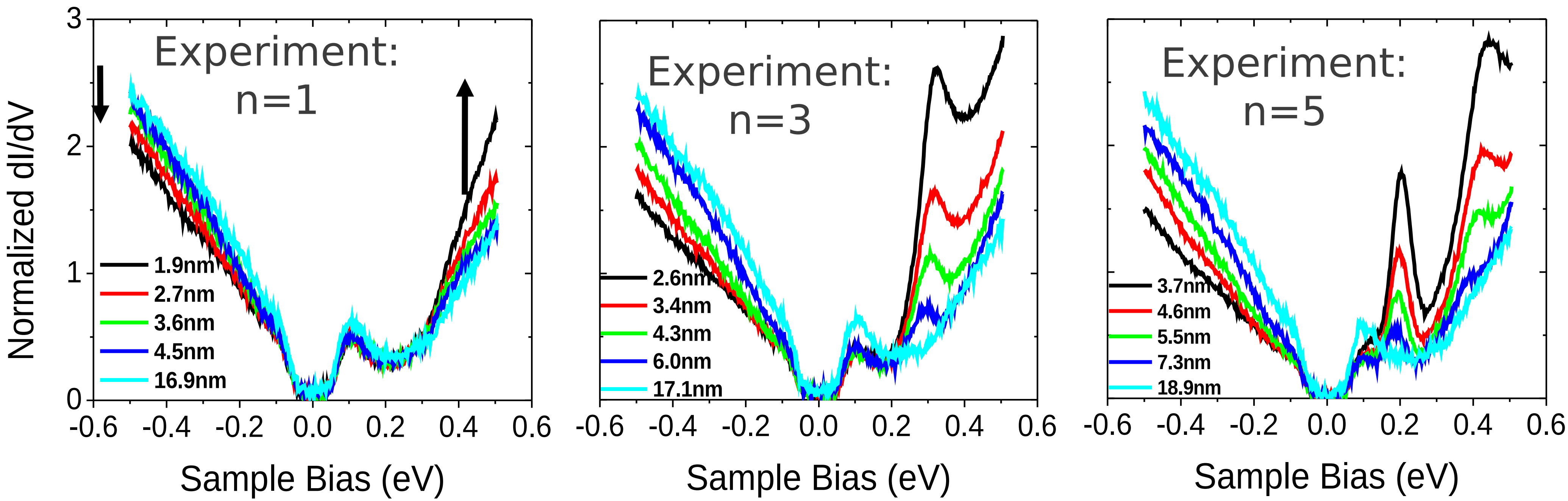


Tune Z above Z_c

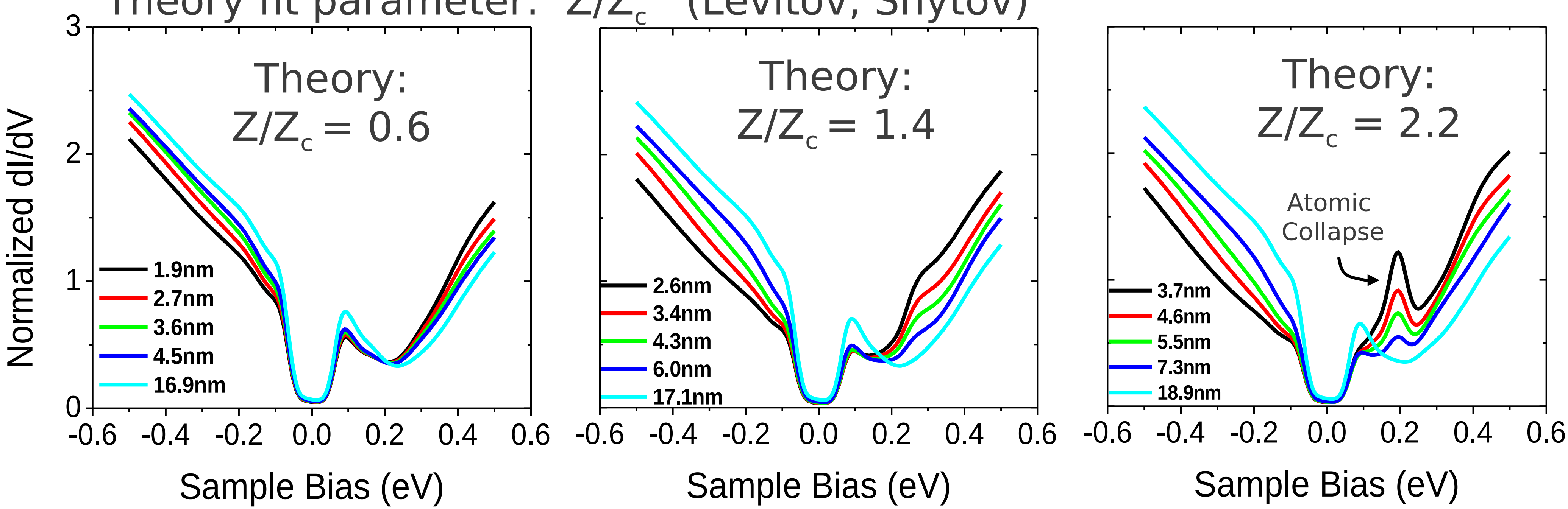
Tuning Z by Building Artificial Nuclei from Ca Dimers



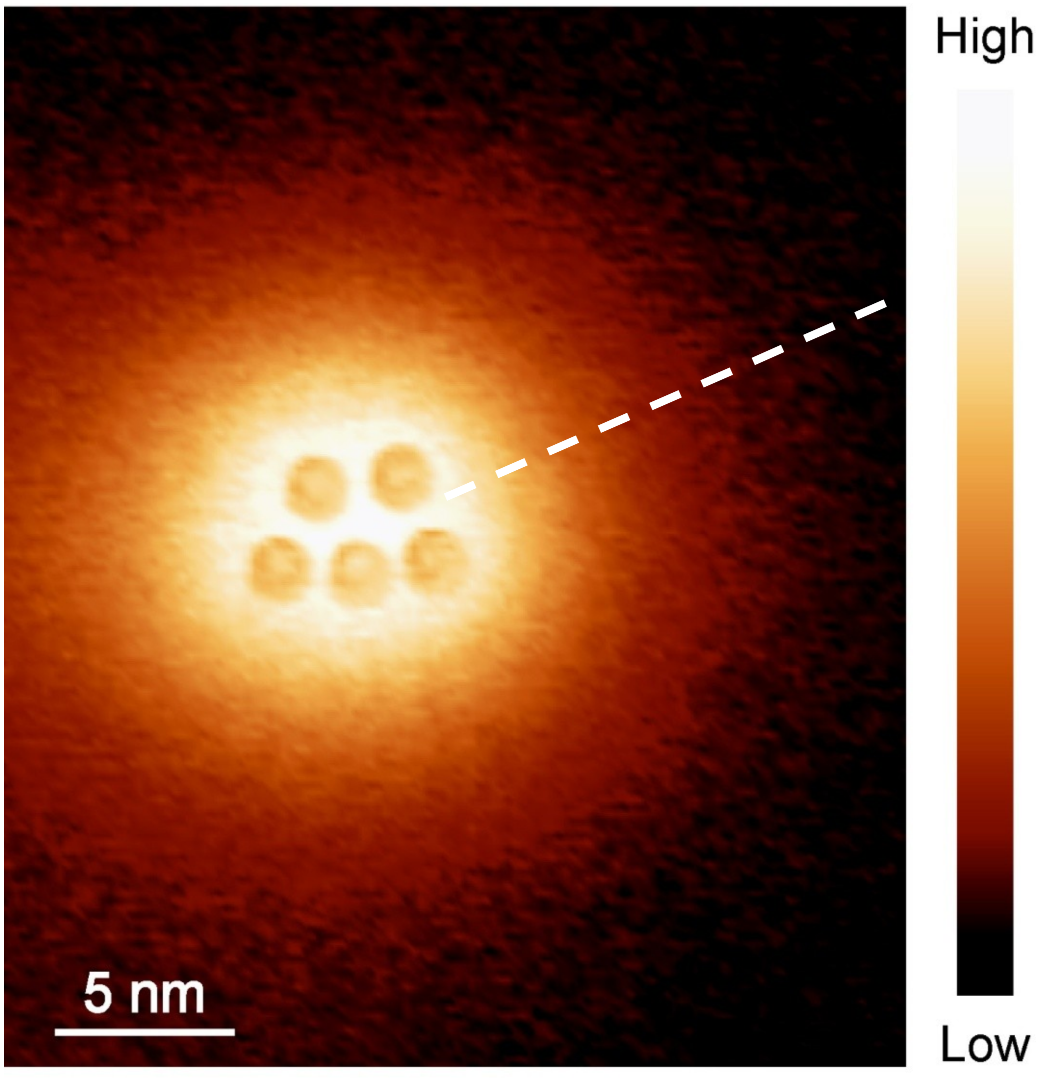
Compare to Simulated Behavior

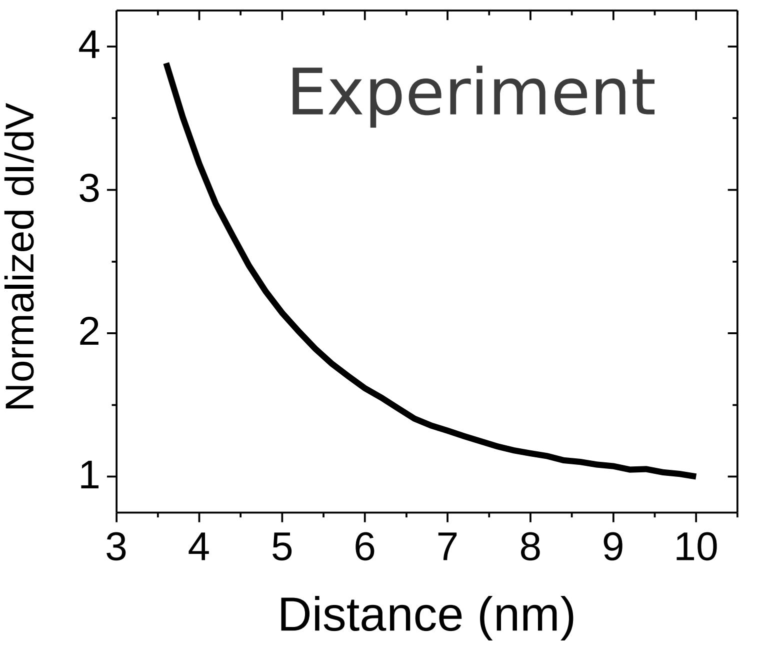


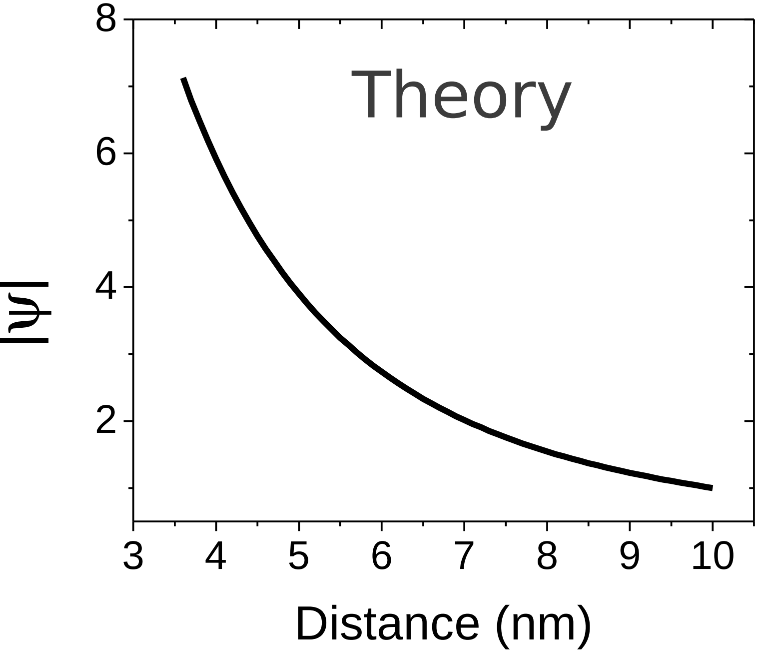
Theory fit parameter: Z/Z_c (Levitov, Shytov)



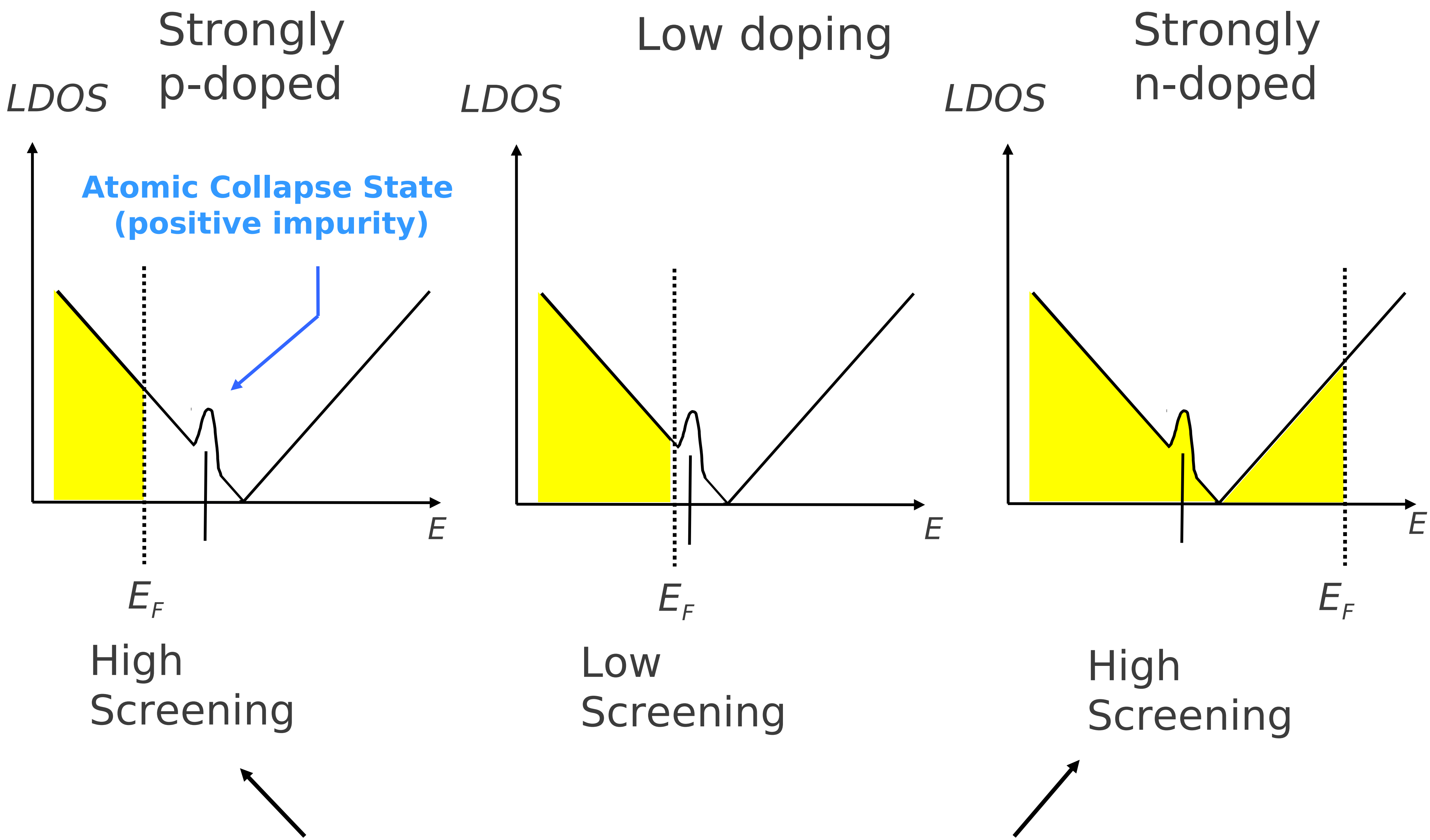
Atomic Collapse Spatial Dependence n=5 Cluster





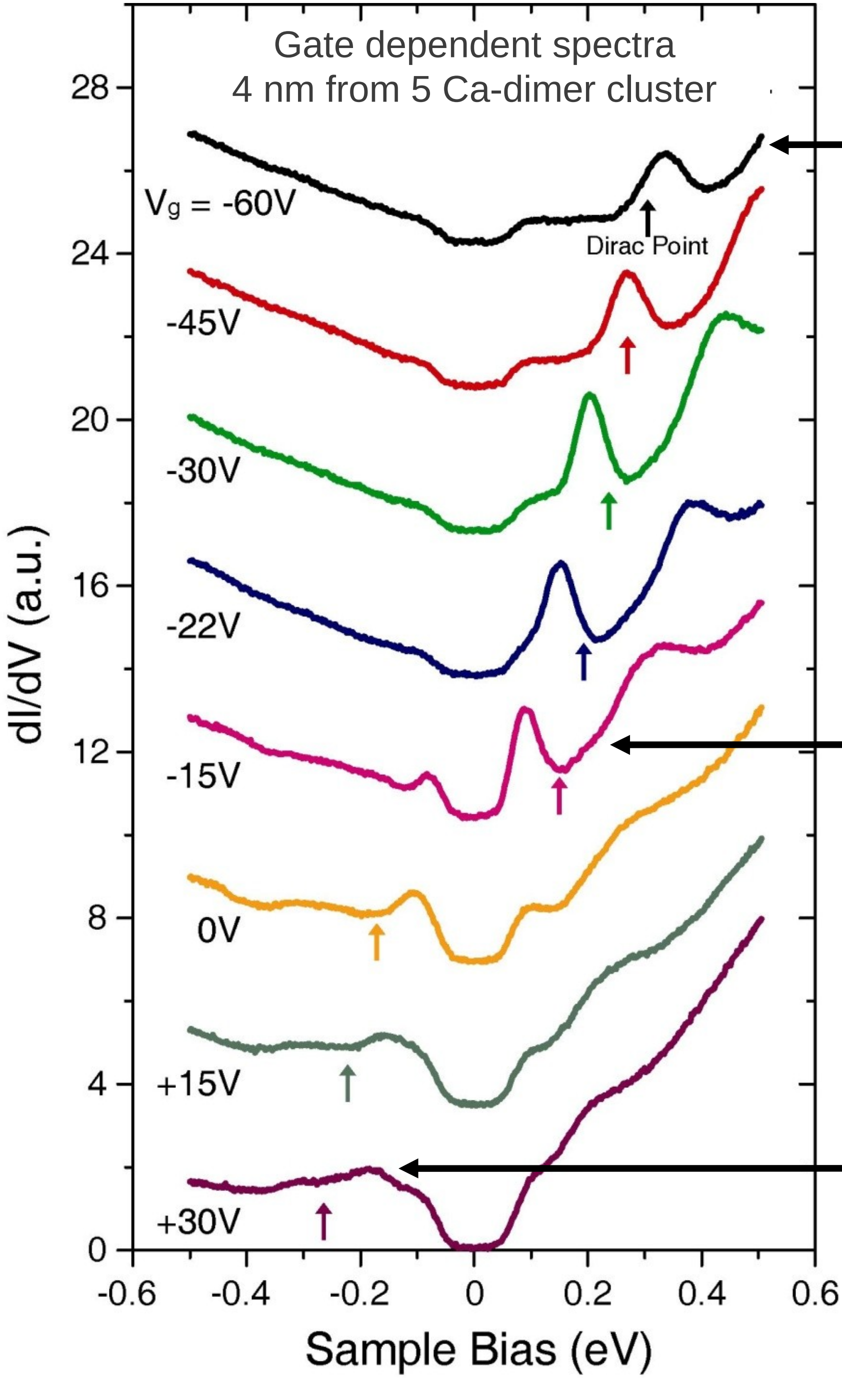


Dependence on Electron Occupation



Expect similar behavior in single-particle picture

Observed Density Dependence NOT Symmetric



High p-doped
$n \sim 3 \times 10^{12}$ cm^{-2}

Low p-doped
$n \sim 4 \times 10^{11}$ cm^{-2}

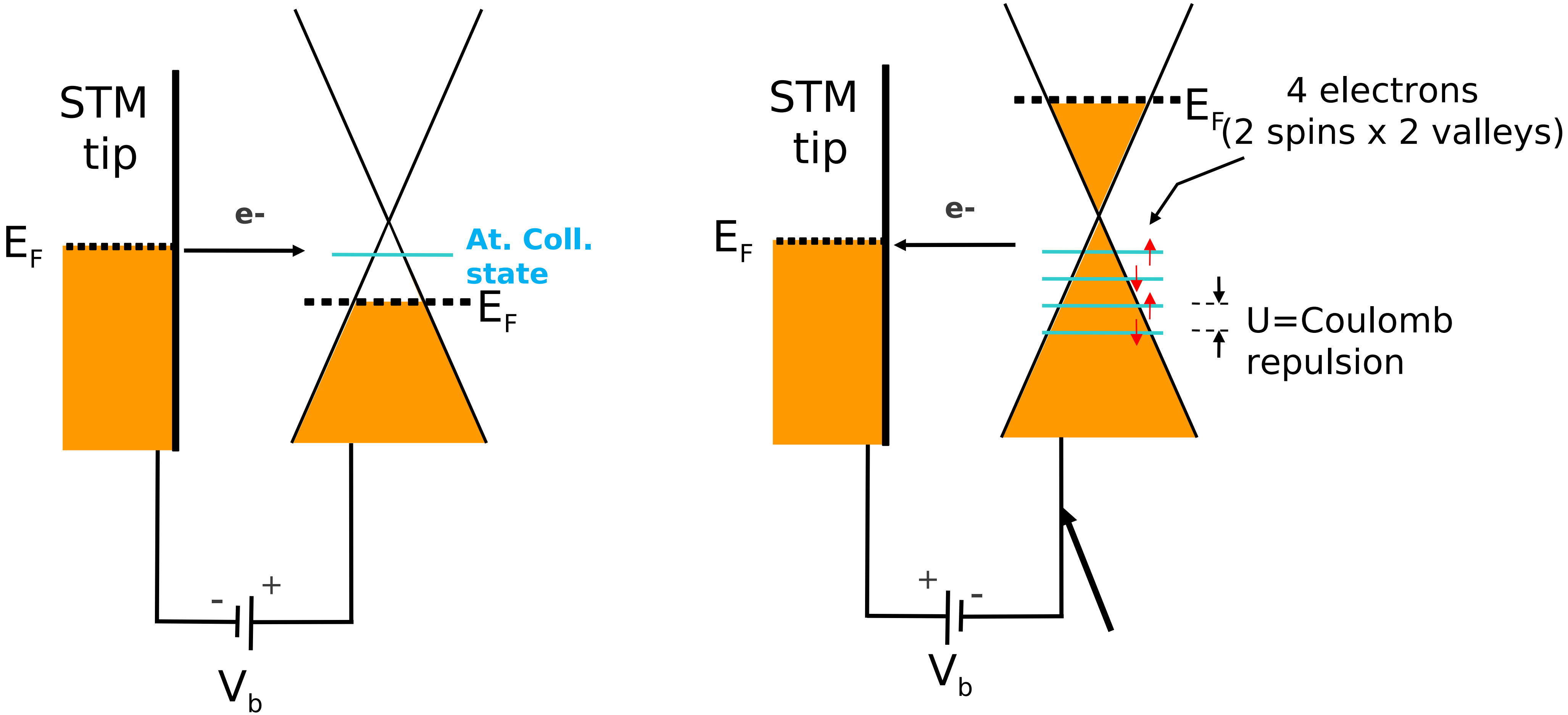
High n-doped
$n \sim 3 \times 10^{12}$ cm^{-2}

Strong Suppression

Why the Strong Suppression for Occupied Regime?

Possible explanation: <u>electron-electron interactions</u>

p-doped case   n-doped case (electron occupied)



<u>Suppression of Single-Particle Spectral Density</u>

Atomic collapse in graphene

1. Graphene: relativistic high-energy physics in a condensed matter system. Atomic collapse near charge impurities, Z~1.

2. Manifestations: formation of resonances and Dirac vacuum polarization. STM experiments with Z~1 impurities. Collapse observed on artificial nuclei (few-impurity clusters)

3. Use Aharonov-Bohm solenoid (B field or pseudo-B field) to bring collapse threshold from Z~1 down to Z<<1.
Exact solution for screening cloud near critical point.

Future:

(1) New electron-electron interaction effects

(2) Impurity-impurity interactions

(3) Supercriticality in other systems? (Topol. insulators; mass $\neq 0$)

(4) Spin Effects