Full Functional Verification of Linked Data Structures

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Goal

Verify full functional correctness of linked data structure implementations

What is Full Functional Correctness?

- Complete, precise formal specification
- Captures every property client needs (except resource consumption)
- Implementation satisfies specification

Benefits of Full Functional Correctness

- Complete, precise, unambiguous interfaces for linked data structures
- Enables sound reasoning with specification (can discard implementation when reasoning)
  - Human developers
  - Automated analyses of client code
- First complete realization of concept of abstract data types

Example

Hashtable Specification

```
class Hashtable {
    //: public ghost specvar content :: “(obj * obj) set” = “{}”;
    //: public ghost specvar init :: “bool” = “false”;

    public Object put(Object key, Object value)
    /*: requires “init ∧ key ≠ null ∧ value ≠ null”
        modifies content
        ensures “content = old content - {(key, result)} ∪ {(key, value)} ^
          (result = null → ¬(∃v. (key, v) ∈ old content)) ∧
          (result ≠ null → (key, result) ∈ old content)” */
     { … }
    …
}
```

- Specifications at class granularity
- Specifications appear as comments
- Can use standard Java compilers

Abstract State as Sets, Relations

```
class Hashtable {
    //: public ghost specvar content :: "(obj * obj) set" = "{}";
    //: public ghost specvar init :: "bool" = "false";

    public Object put(Object key, Object value)
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          (result ≠ null → (key, result) ∈ old content)" */
     { … }
    …
}
```

- Represent abstract state using specification variables
- Contents of hash table as set of key-value pairs

Method Preconditions, Postconditions

class Hashtable {
    //: **public ghost specvar** *content* :: “(obj * obj) set” = “{}”;
    //: **public ghost specvar** *init* :: “bool” = “false”;

    public Object put(Object key, Object value)
    /*: **requires** “init ∧ key ≠ null ∧ value ≠ null”
        modifies *content*
        ensures “content = old content - {(key, result)} ∪ {(key, value)} ^
            (result = null → ¬(∃v. (key, v) ∈ old content)) ∧
            (result ≠ null → (key, result) ∈ old content)” */
    { … }
    …
}

- Standard assume-guarantee reasoning for method interfaces
- Pre-, post-conditions in higher-order logic (HOL)

Requires Clause

```
class Hashtable {
    //: public ghost specvar content :: "(obj * obj) set" = "{}";
    //: public ghost specvar init :: "bool" = "false";

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          (result ≠ null → (key, result) ∈ old content)" */
     { … }
    …
}
```

Pre-condition requires that key and value be non-null

Modifies Clause

```
class Hashtable {
    //: public ghost specvar content :: “(obj * obj) set” = “{}”;
    //: public ghost specvar init :: “bool” = “false”;

    public Object put(Object key, Object value)
    /*: requires “init ∧ key ≠ null ∧ value ≠ null”
        modifies content
        ensures “content = old content - {(key, result)} ∪ {(key, value)} ^
           (result = null → ¬(∃v. (key, v) ∈ old content)) ∧
           (result ≠ null → (key, result) ∈ old content)” */
     { … }
    …
}
```

- Modifies clause gives frame condition
- **put** method modifies only *content*

Ensures Clause

```
class Hashtable {
    //: public ghost specvar content :: “(obj * obj) set” = “{}”;
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          (result ≠ null → (key, result) ∈ old content)” */
     { … }
    …
}
```

- Previous key-value binding is removed
- New binding is added

Ensures Clause

```
class Hashtable {
    //: public ghost specvar content :: “(obj * obj) set” = “{}”;
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    public Object put(Object key, Object value)
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          (result ≠ null → (key, result) ∈ old content)” */
     { … }
    …
}
```
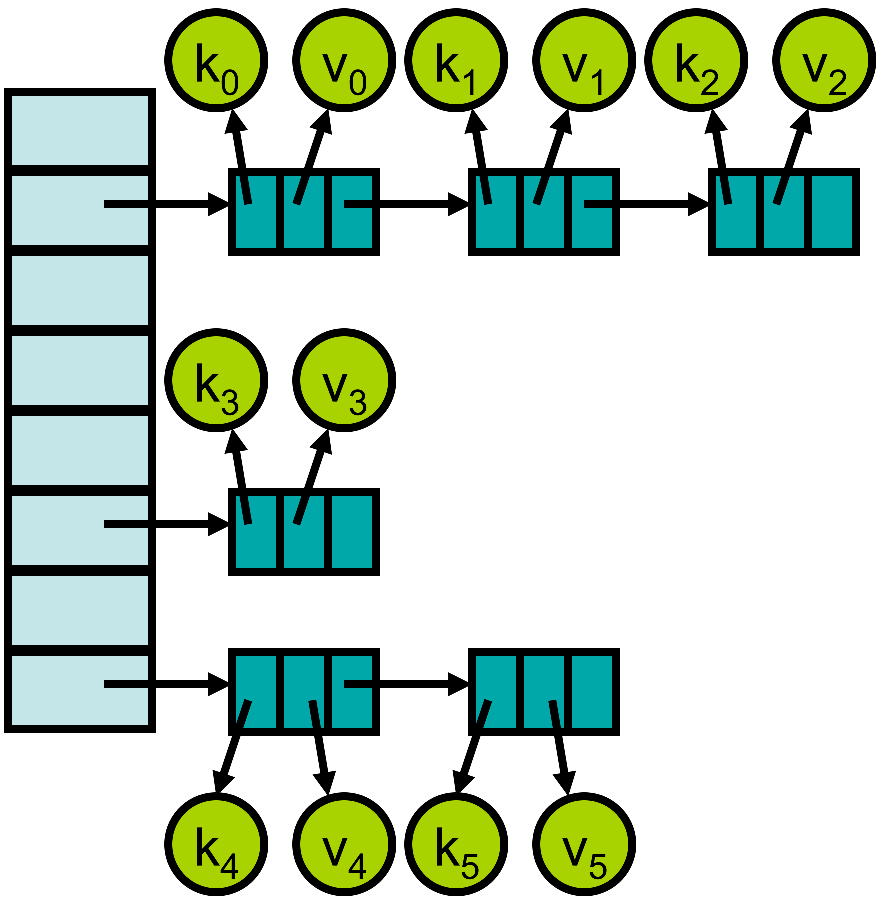
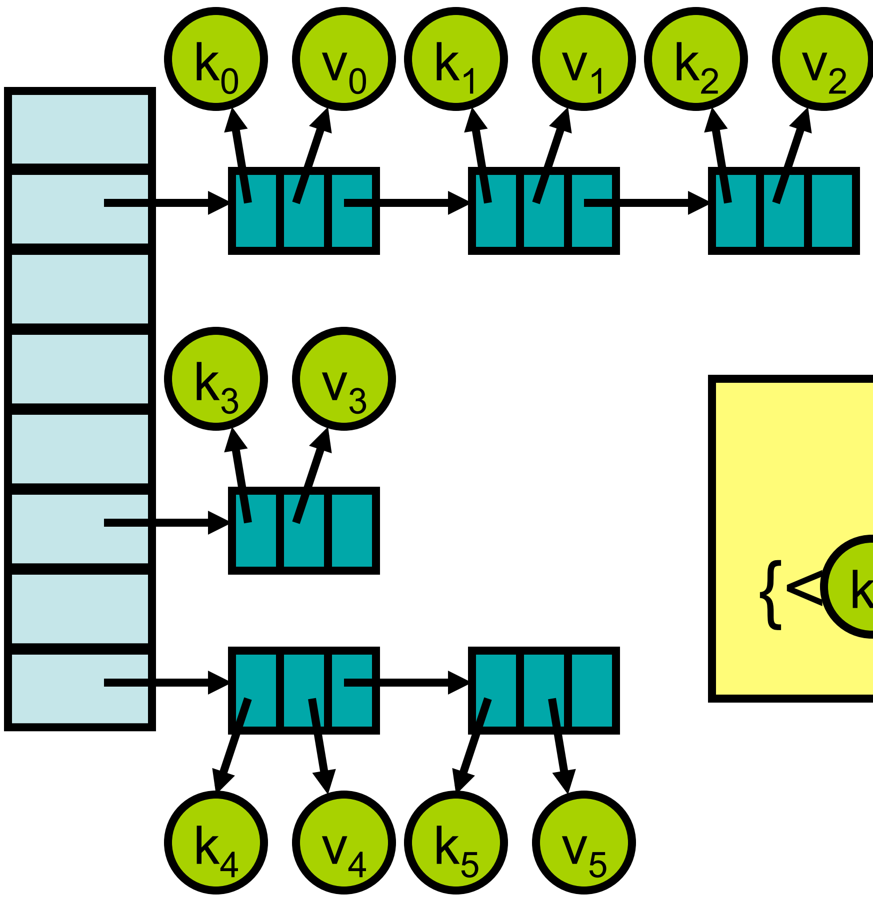
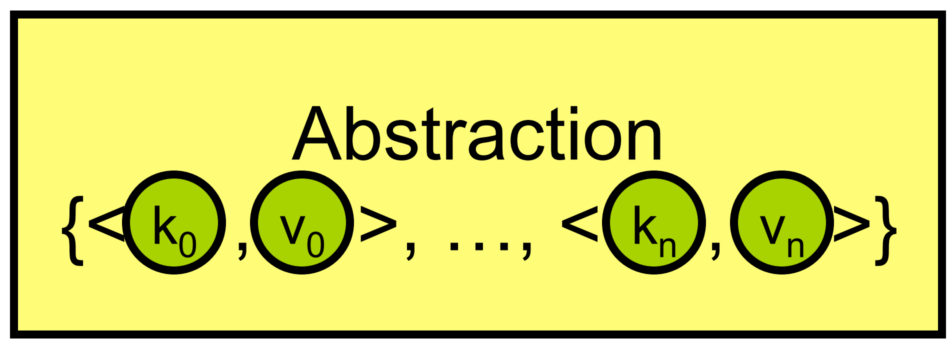

Returns previously-bound value or null

Hashtable Data Structure



Hashtable Data Structure





Abstraction Function

```
/*:  invariant ContentDef:
     "init → content = {(k,v). (∃ i. 0 ≤ i ∧ i < table.length ∧ (k,v) ∈ table[i].bucketContent)}"

     static ghost specvar bucketContent :: "obj ⇒ (obj * obj) set" = "λ n. {}"
     invariant bucketContentNull: "null.bucketContent = {}"
     invariant bucketContentDef: "∀x. x ∈ Node ∧ x ∈ alloc ∧ x ≠ null → x.bucketContent =
     {<x.key, x.value>} ∪ x.next.bucketContent ∧ (∀v. (x.key, v) ∉ x.next.bucketContent)"

     invariant Coherence:
     "init → (∀ i k v. (k,v) ∈ table[i].bucketContent → i = hash k table.length)"

     static specvar hash :: "obj ⇒ int ⇒ int"
     vardefs "hash == λ k. (λ n. (abs (hashFunc  k)) mod n)"

     static specvar abs :: "int ⇒ int"
     vardefs "abs == λ m. (if (m < 0) then -m else m)"
     …
*/
```

- Invariants
- Dependent specification variables

Abstraction Function

/*: **invariant** ContentDef:
“init → content = {(k,v). (∃ i. 0 ≤ i ∧ i < table.length ∧ (k,v) ∈ table[i].bucketContent)}”

static ghost specvar bucketContent :: “obj ⇒ (obj * obj) set” = “λ n. {}”
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*/

Hash table contents consists of the contents of all the buckets

Abstraction Function

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…
*/

- Contents of each bucket defined recursively over linked list
- Keys are unique

Abstraction Function

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"init → content = {(k,v). (∃ i. 0 ≤ i ∧ i < table.length ∧ (k,v) ∈ table[i].bucketContent)}"

static ghost specvar bucketContent :: "obj ⇒ (obj * obj) set" = "λ n. {}"
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…
*/

Every key is in the correct bucket

Abstraction Function

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/*:  invariant ContentDef:
     “init → content = {(k,v). (∃ i. 0 ≤ i ∧ i < table.length ∧ (k,v) ∈ table[i].bucketContent)}”

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     vardefs “abs == λ m. (if (m < 0) then -m else m)”
     …
*/
```

set expressions

quantifiers

lambda expressions

Loop Invariants

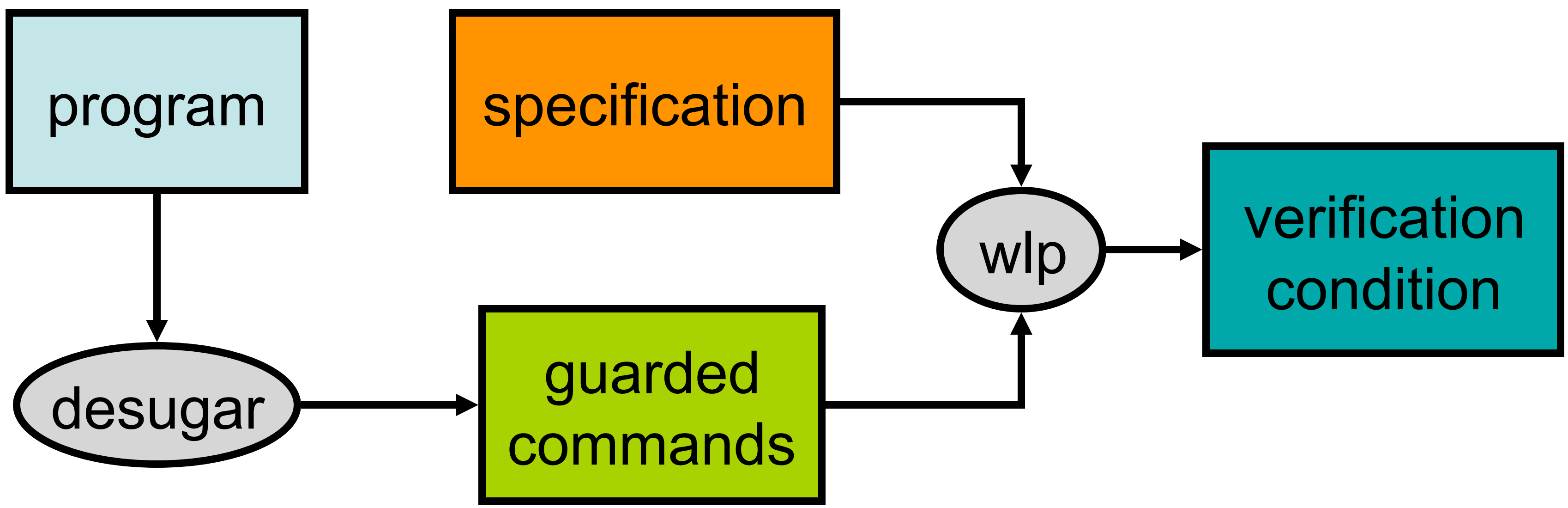
```
public Object get(Object k0)
/*:  requires "init ∧ k0 ≠ null"
     ensures "(result ≠ null → (k0, result) ∈ content) ∧
        (result = null → ¬(∃v. (k0, v) ∈ content))" */
{
     int hc = compute_hash(k0);
     Node current = table[hc];
     while /*: inv "∀v. ((k0, v) ∈ content) = ((k0, v) ∈ current.bucketContent)" */
          (current != null) {
        if (current.key == k0) { return current.value; }
        current = current.next;
     }
       return null;
}
```

Source of Invariants

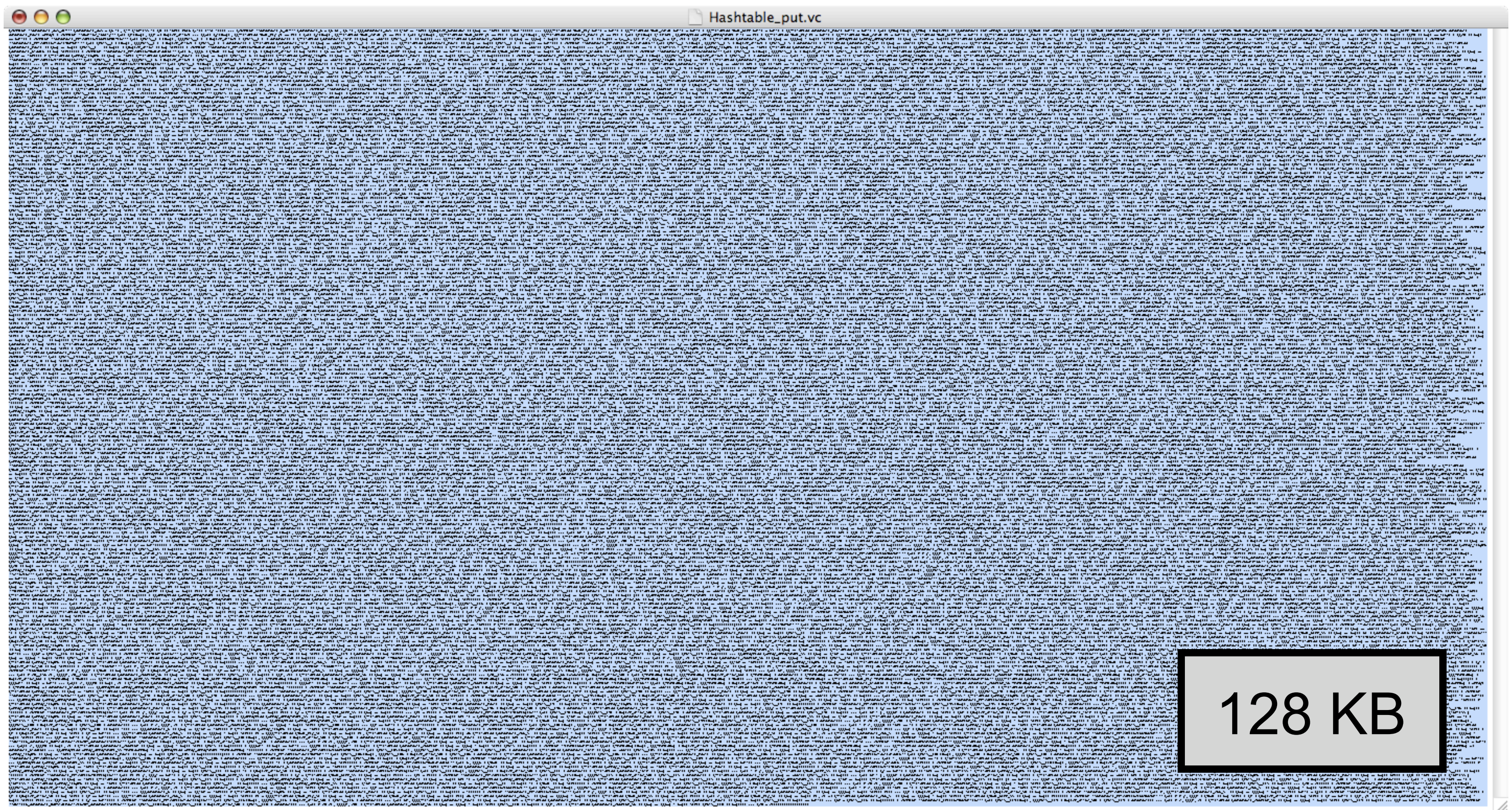
- Inferred by system
- Provided by developer
- Inferred by shape analysis

Generating Verification Conditions

- Convert to guarded commands
- Apply weakest liberal pre-conditions



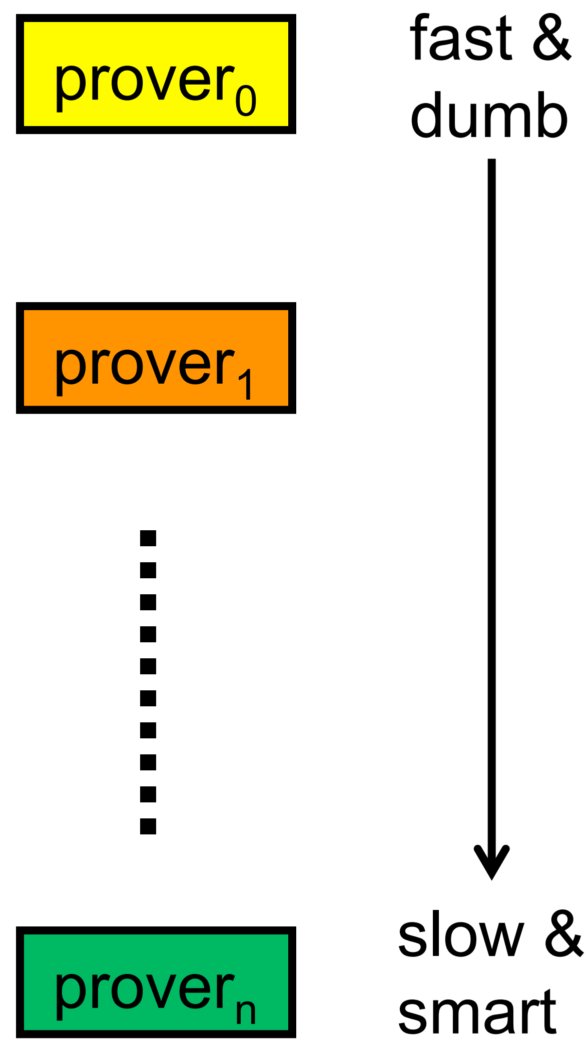
Verification Condition for Hashtable.put

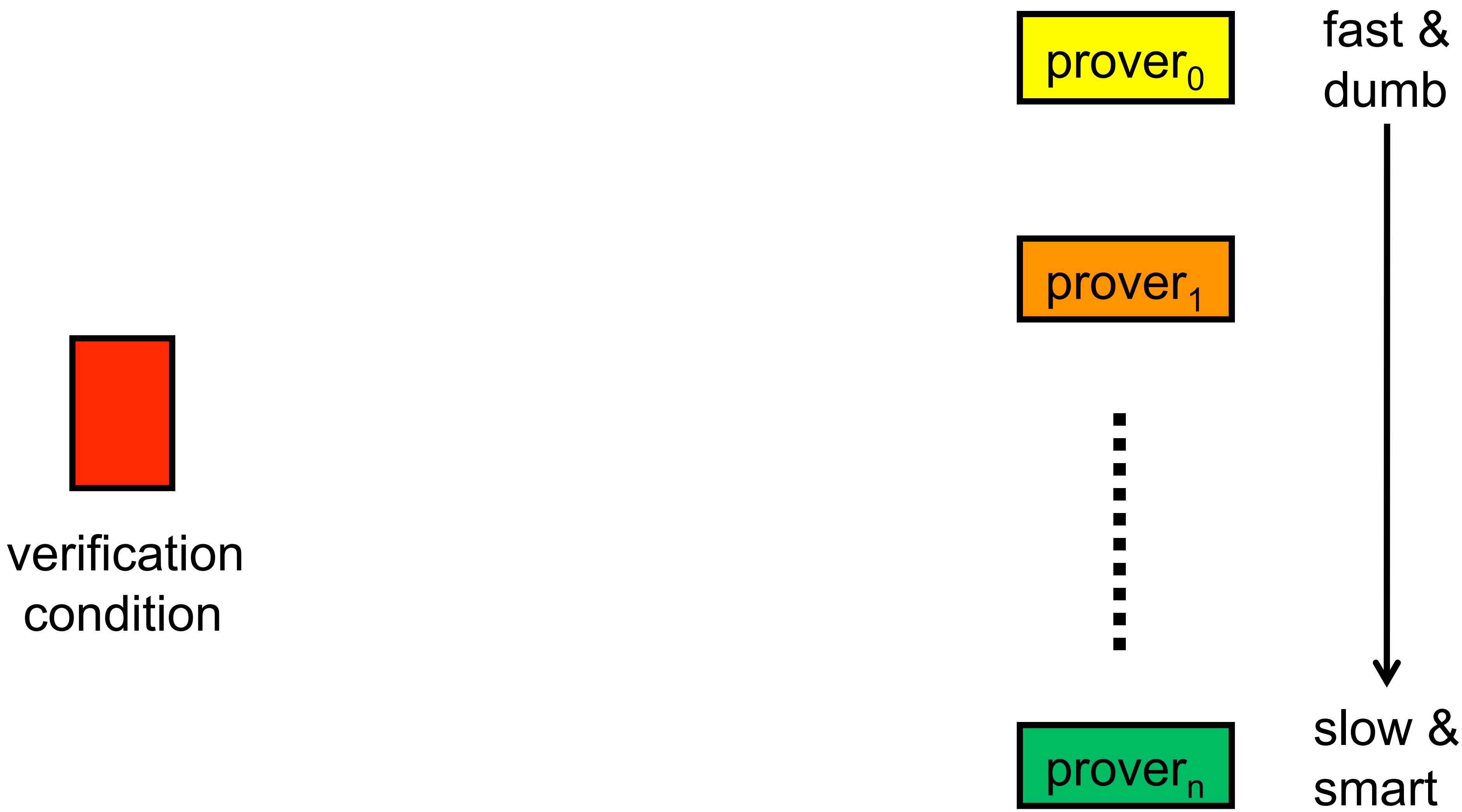


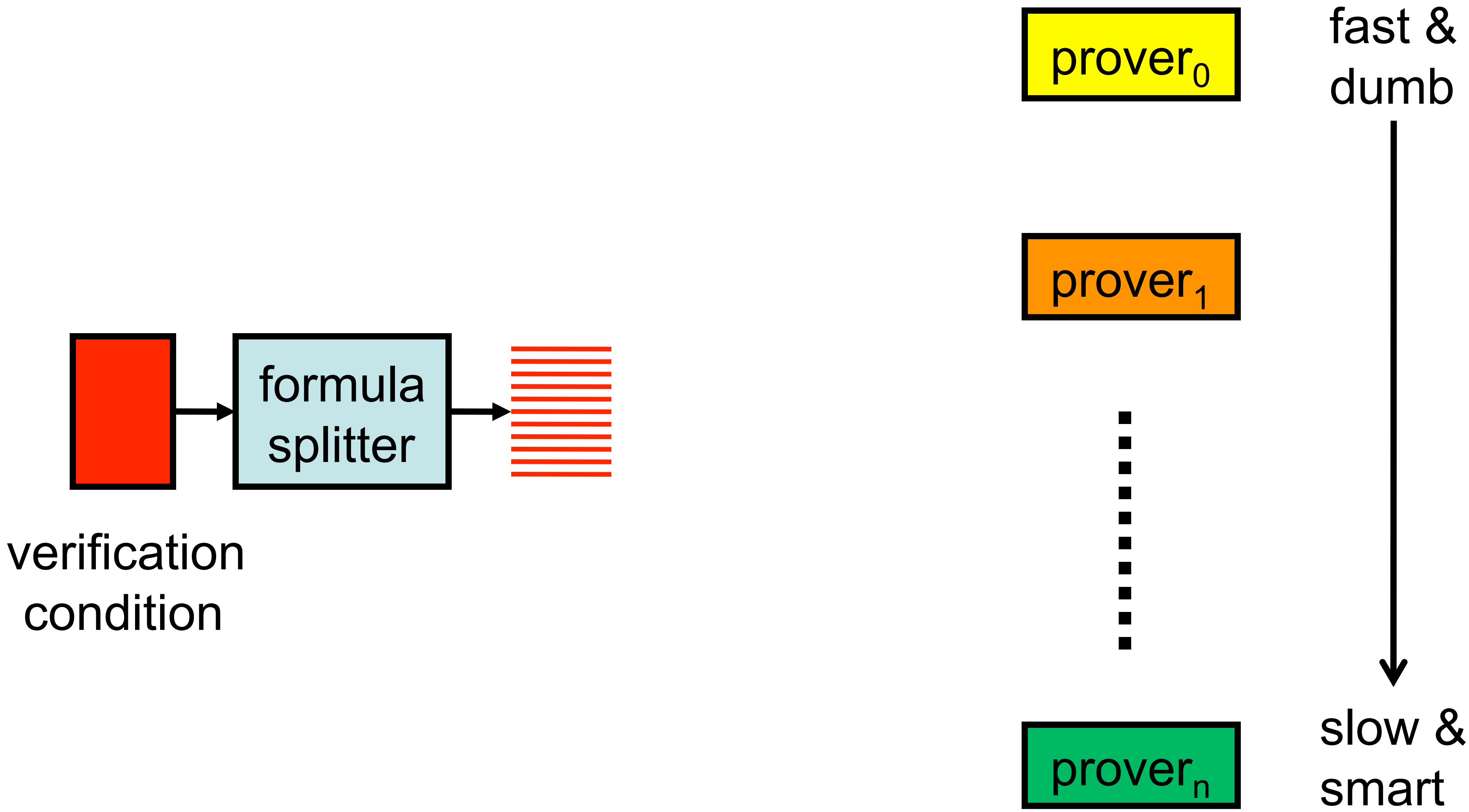
How to prove?

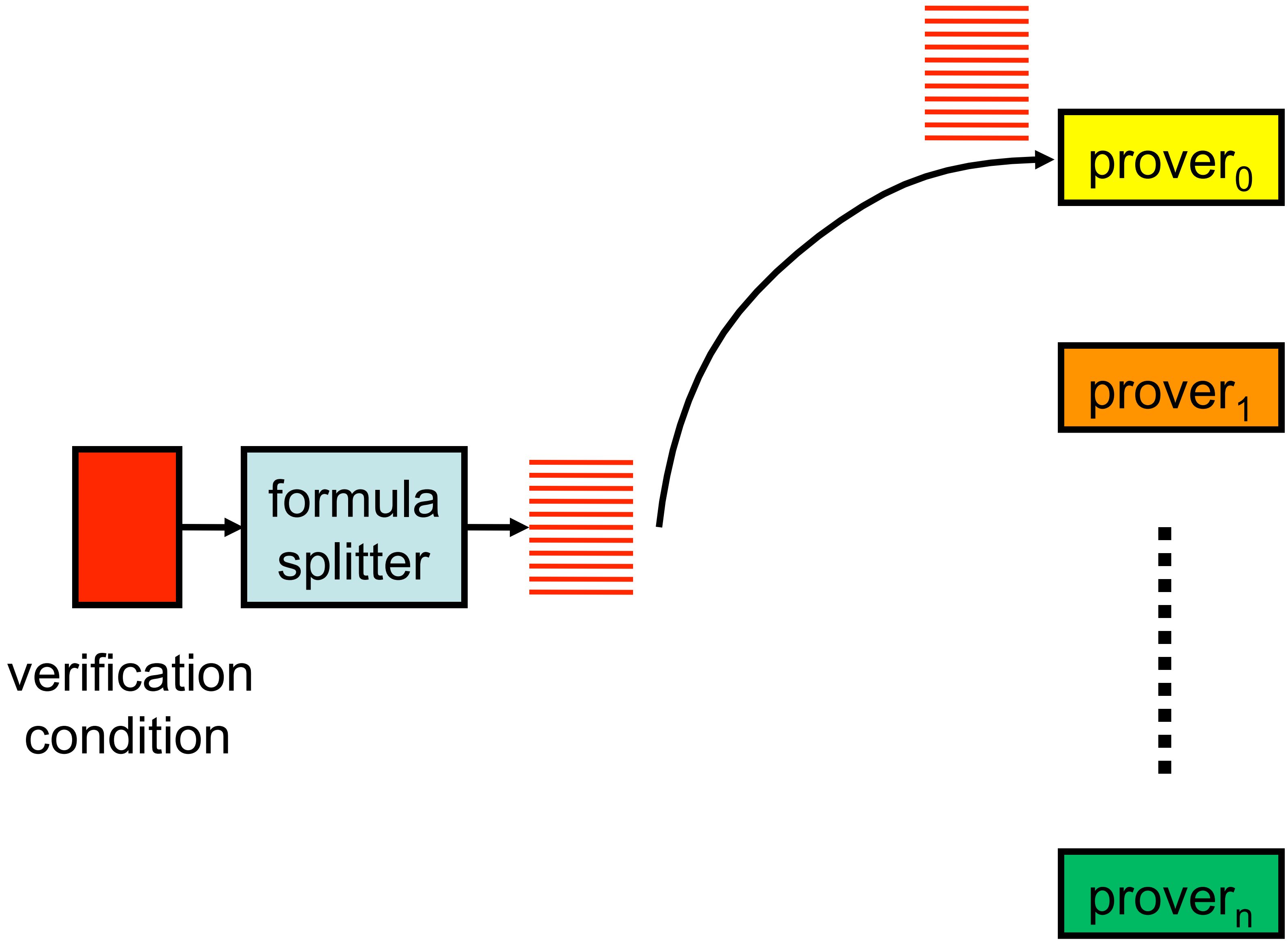
Available Provers

- Syntactic provers
- Nelson-Oppen provers
- Resolution-based provers
- Decision procedures
  - Monadic second-order logic
  - BAPA
- Proof assistants




prover0
prover1
provern
fast & dumb
slow & smart
verification condition


formula splitter
verification condition
$prover_0$
$prover_1$
$prover_n$
fast & dumb
slow & smart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prover0
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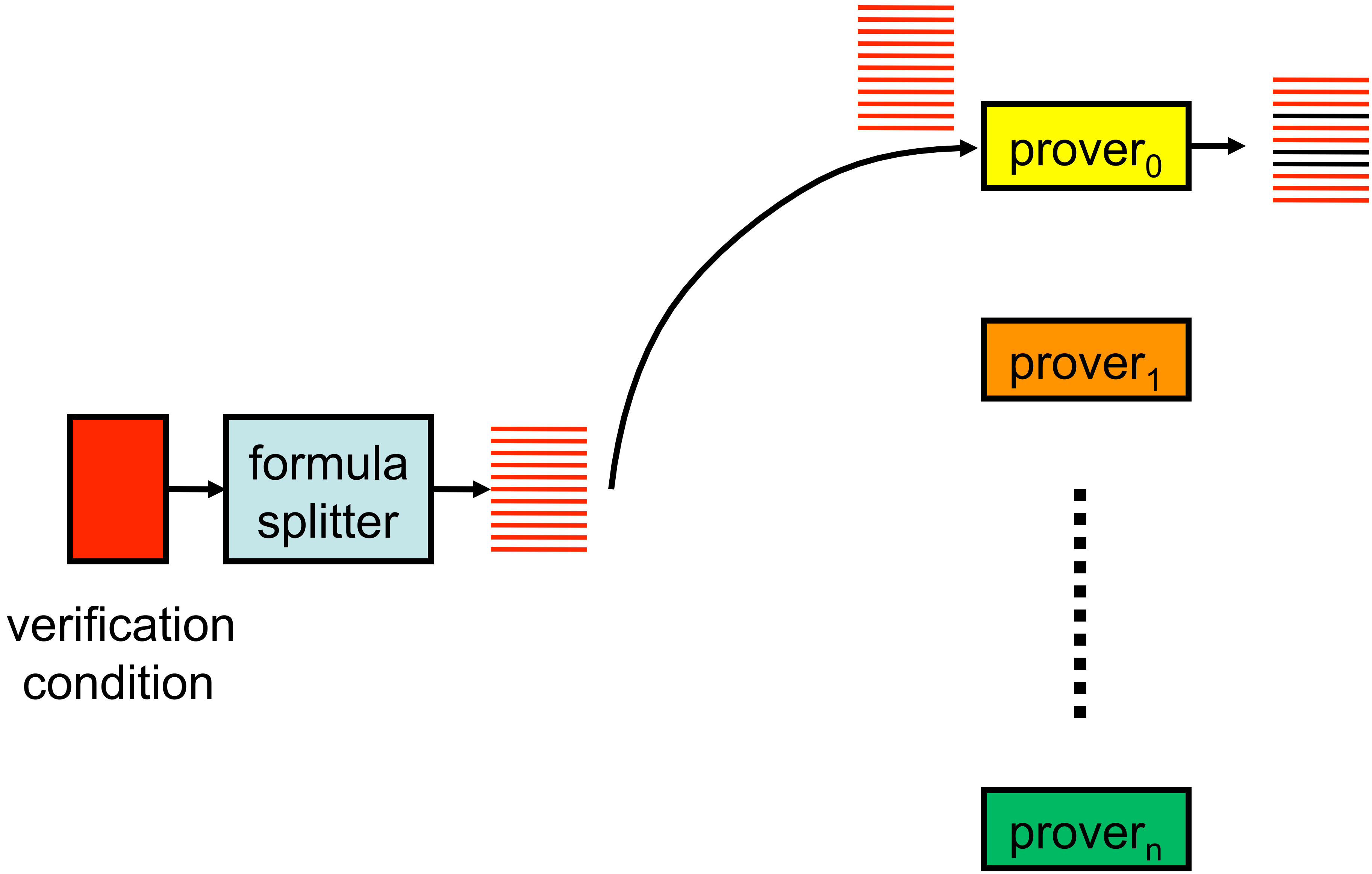
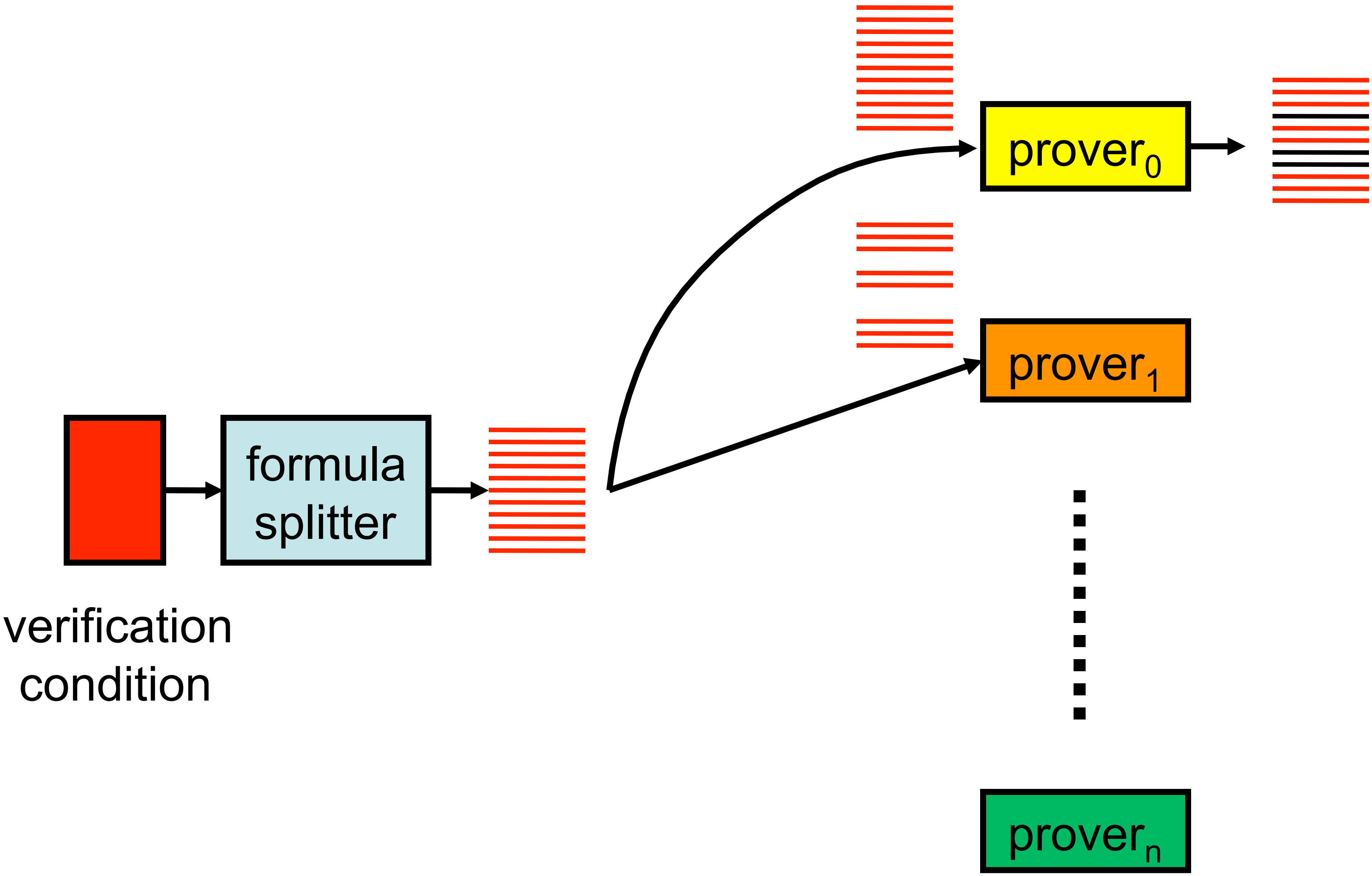

formula splitter
verification condition
prover$_0$
prover$_1$
prover$_n$


formula splitter
verification condition
prover0
prover1
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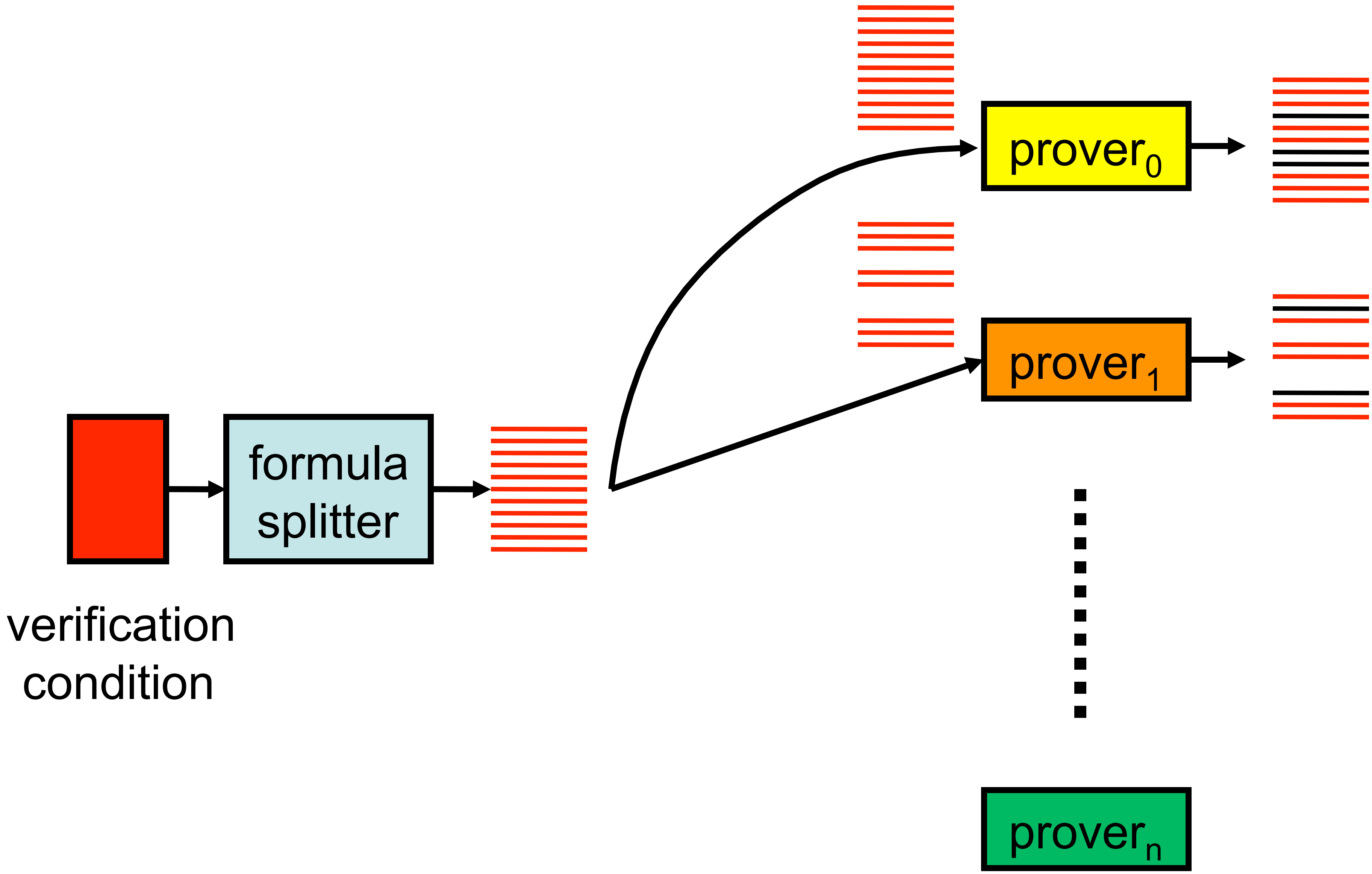
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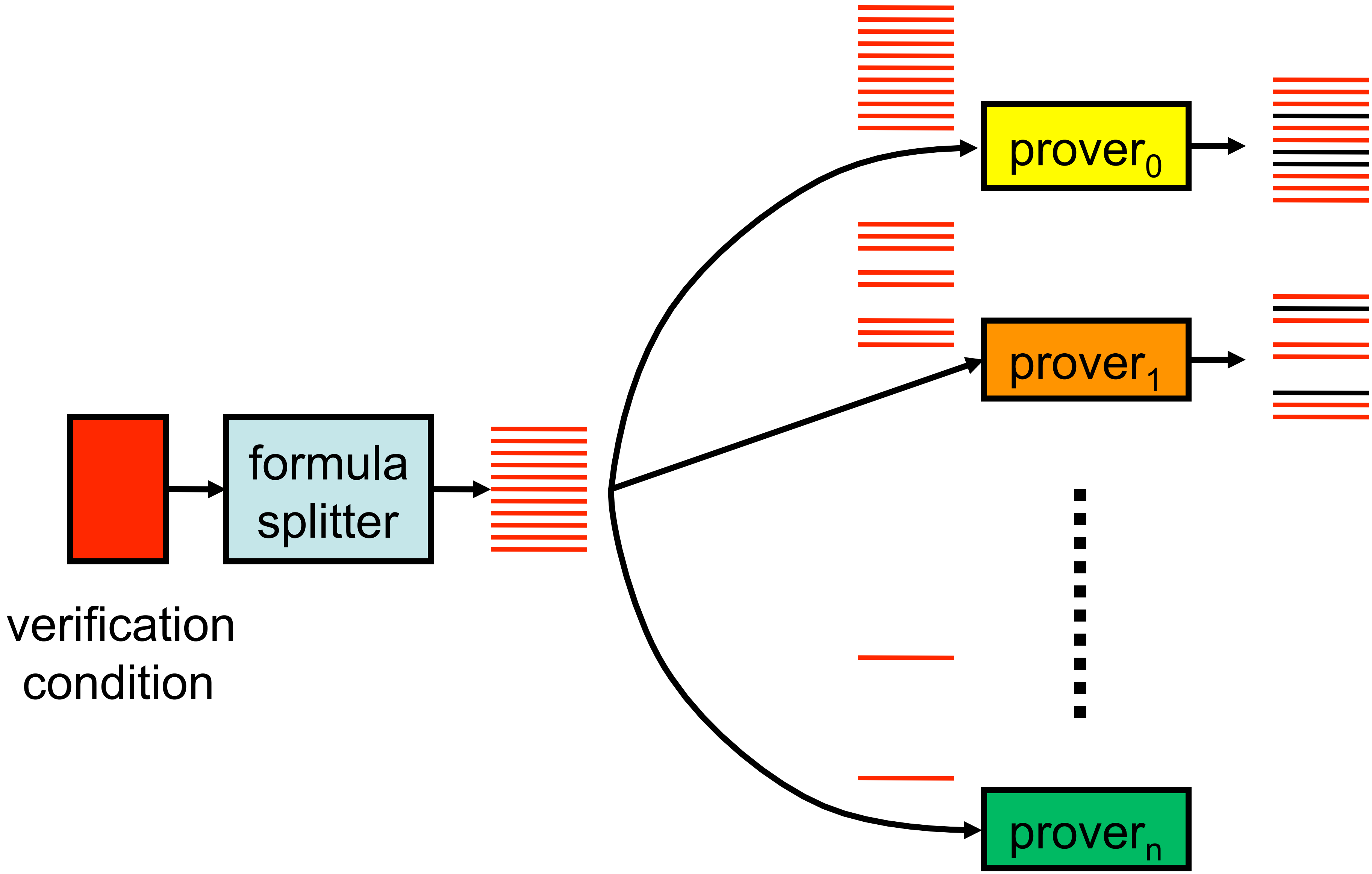
$prover_1$
$prover_n$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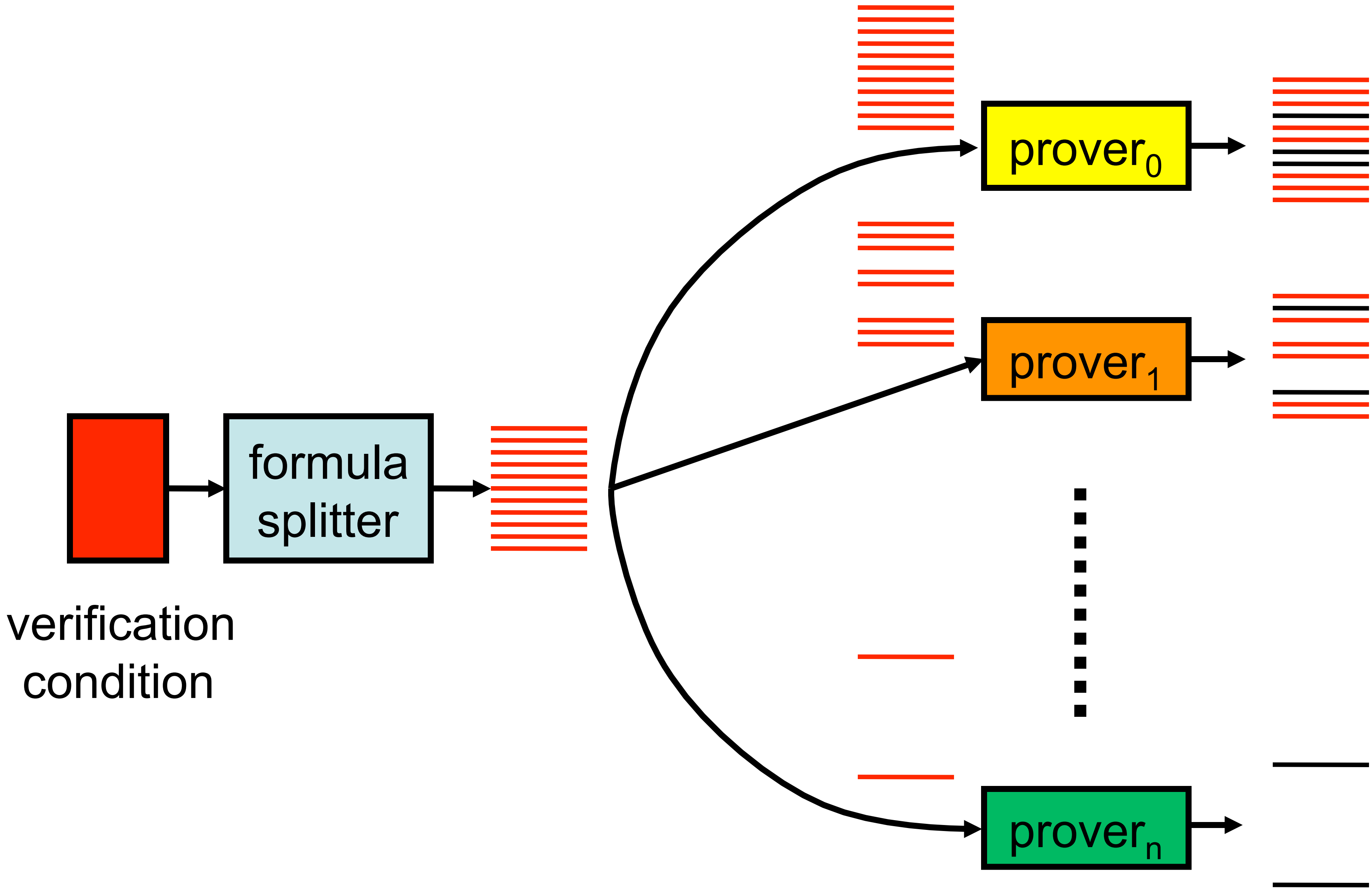
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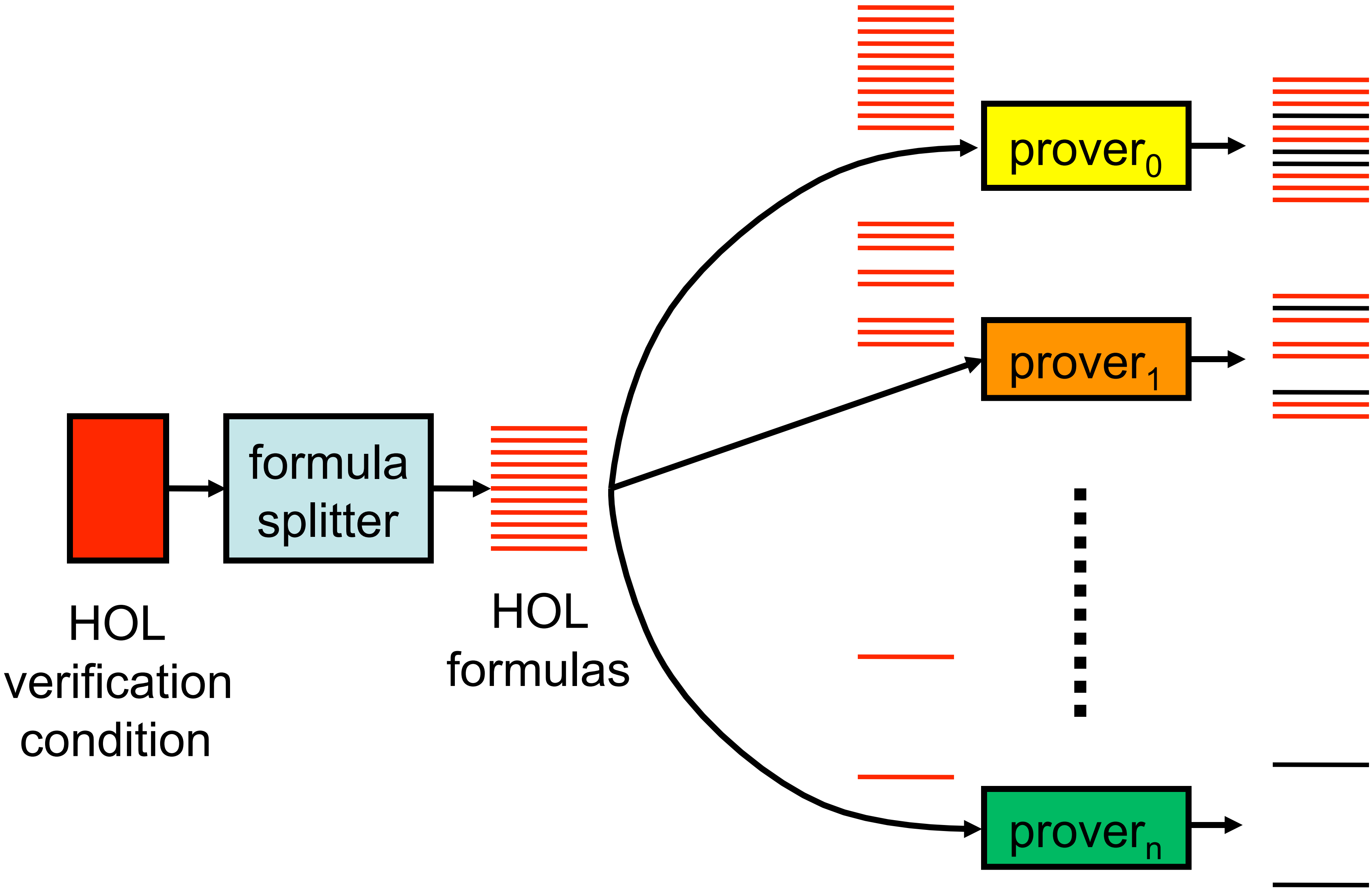
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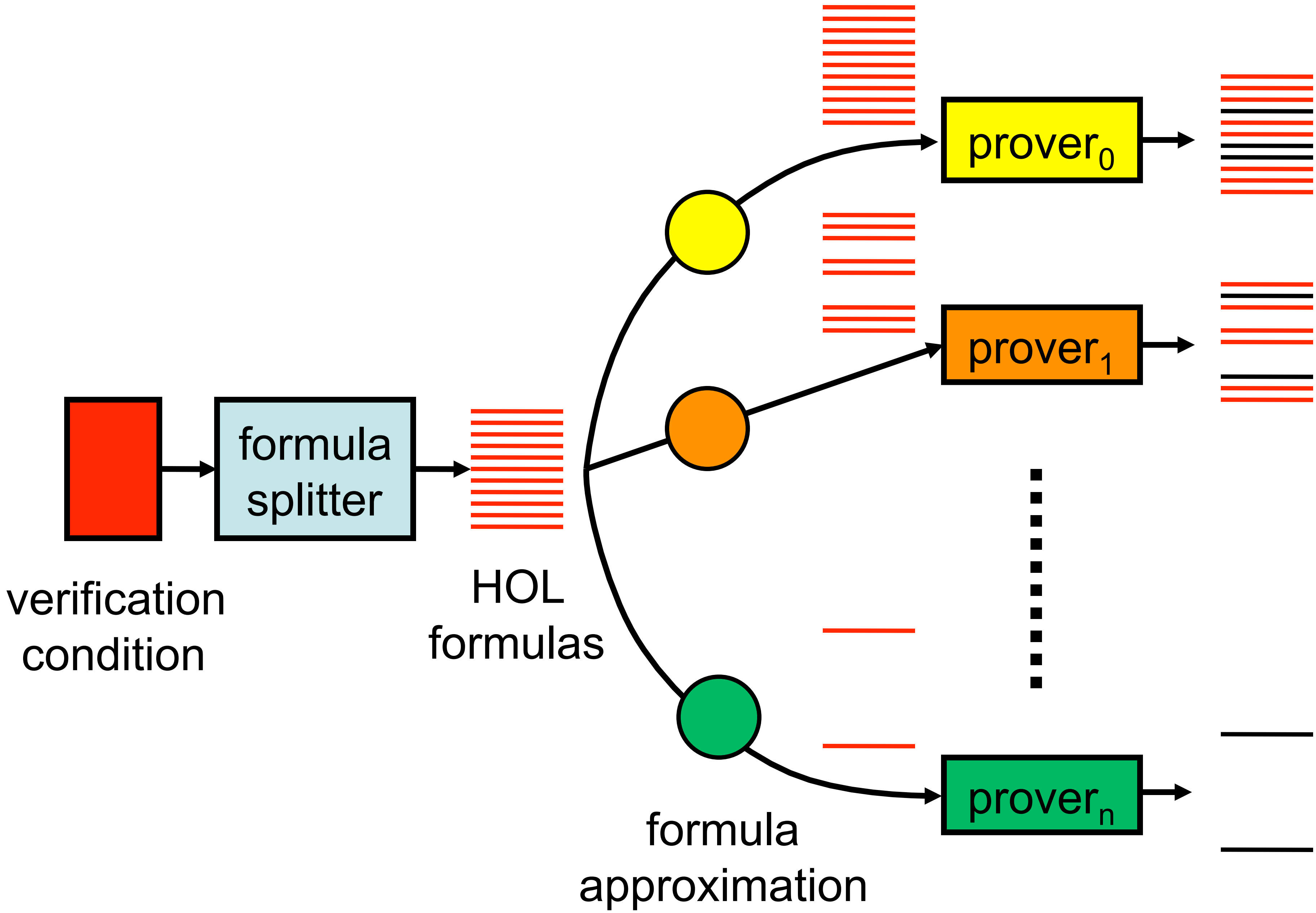
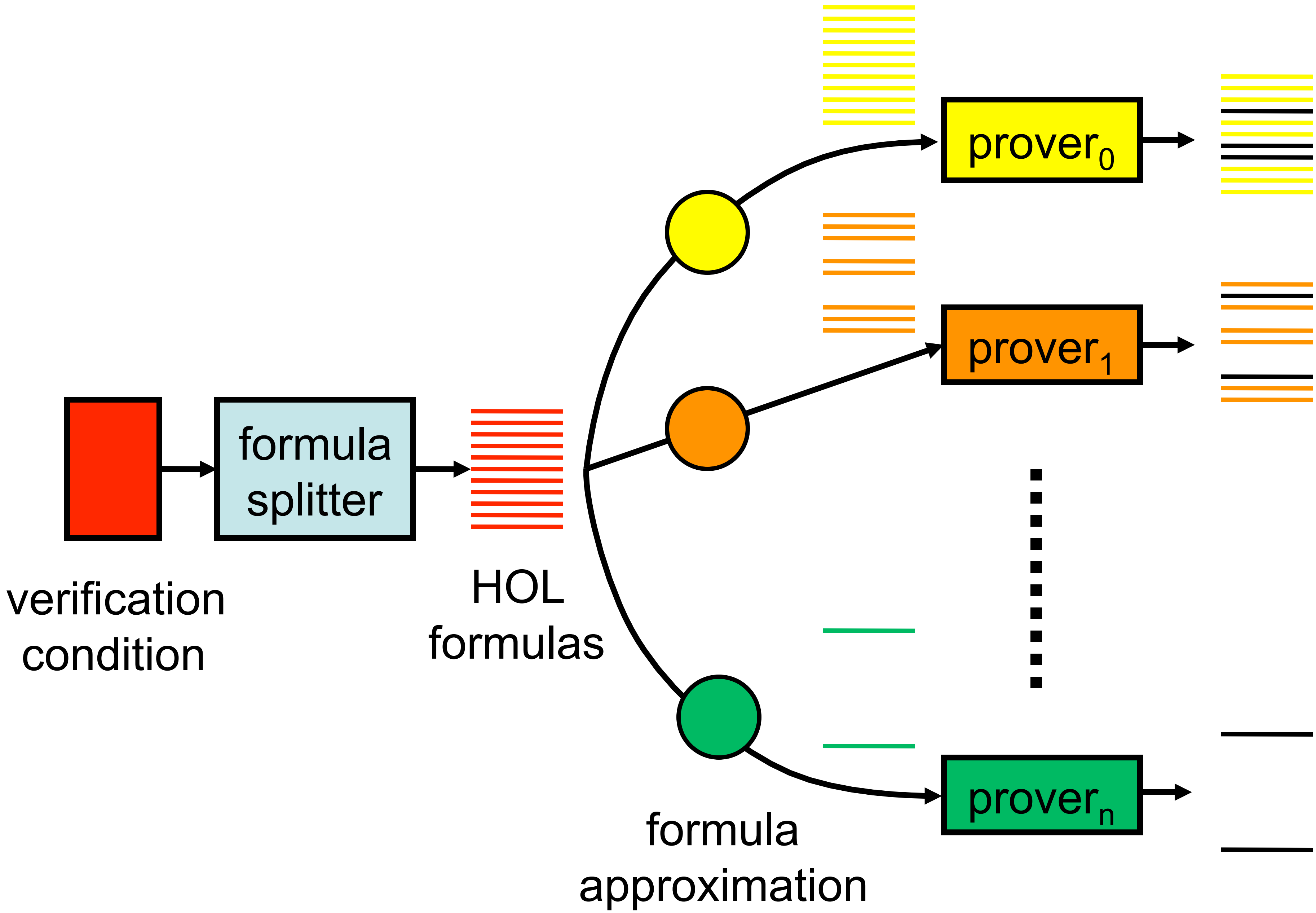
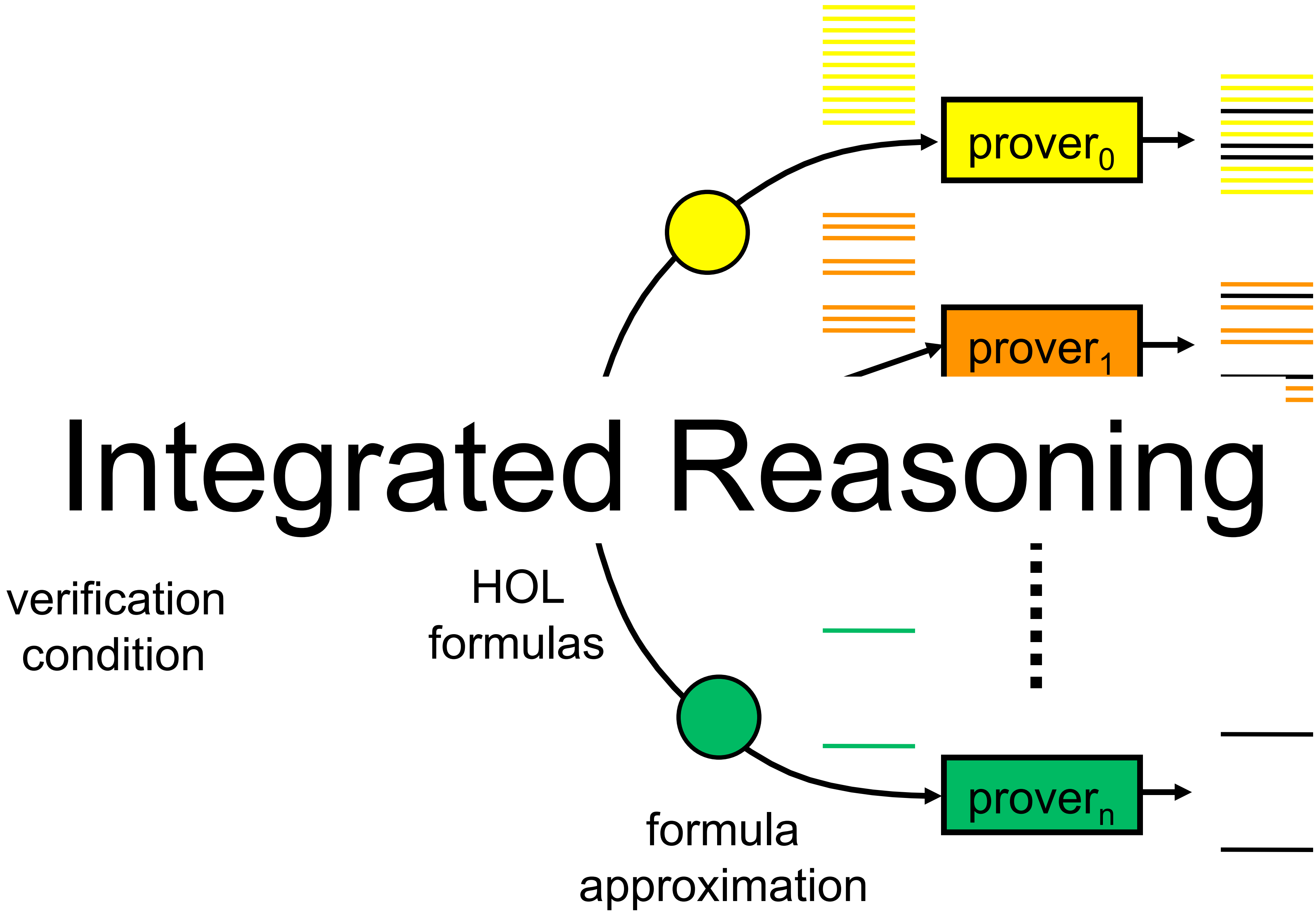

formula
splitter
verification
condition
HOL
formulas
formula
approximation
prover$_0$
prover$_1$
prover$_n$

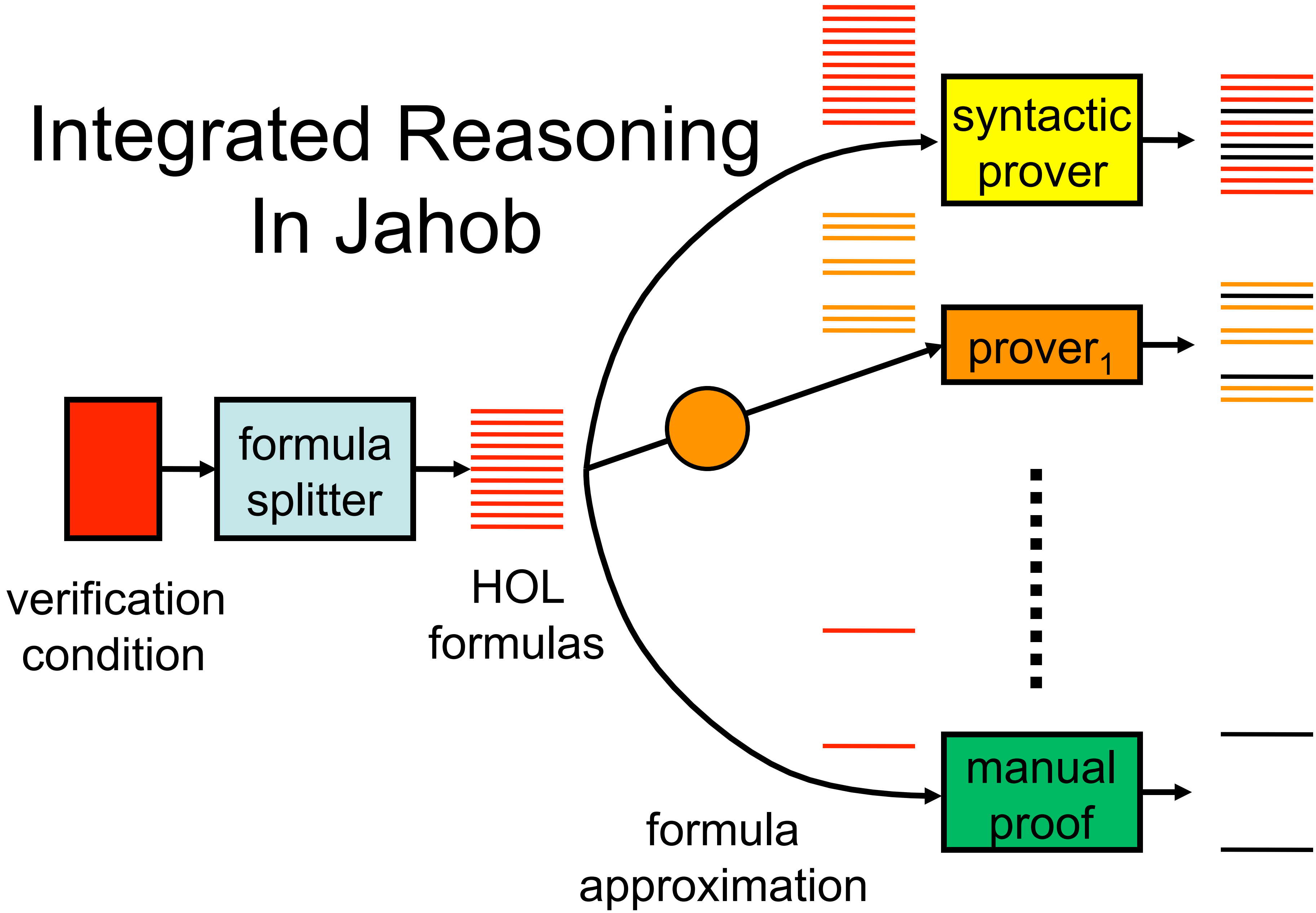


formula splitter
verification condition
HOL formulas
prover$_0$
prover$_1$
prover$_n$
formula approximation

Integrated Reasoning



Integrated Reasoning In Jahob



Formula Splitting

- Transform verification condition into equivalent conjunction of smaller formulas

$$A \rightarrow G_1 \wedge G_2 \quad \Rightarrow \quad A \rightarrow G_1,\ A \rightarrow G_2$$

$$A \rightarrow (B \rightarrow G^{[p]})^{[q]} \quad \Rightarrow \quad (A \wedge B^{[q]}) \rightarrow G^{[pq]}$$

$$A \rightarrow \forall x.G \quad \Rightarrow \quad A \rightarrow G[x:=x_{fresh}]$$

- Enables application of different solvers to solve different parts of verification condition

Formula Approximation

- Feed *any* formula to *any* prover or decision procedure
- Approximate given formula with a stronger formula in appropriate logic subset
- Translate constructs where possible
  - Transform set expressions to predicates
  - Apply beta-reduction to lambda expressions
  - Rewrite tuples into elements
  - Apply extensionality
- Approximate where necessary
  - Descend formula recursively
  - Approximate inexpressible subformulas according to arity

Formula Approximation Rules

$\alpha : (0,1) \times C$

$\alpha^p(f_1 \wedge f_2) \equiv \alpha^p(f_1) \wedge \alpha^p(f_2)$

$\alpha^p(f_1 \vee f_2) \equiv \alpha^p(f_1) \vee \alpha^p(f_2)$

$\alpha^p(\neg f) \equiv \neg \alpha^{\neg p}(f)$

$\alpha^p(\forall x.f) \equiv \forall x.\alpha^p(f)$

$\alpha^p(\exists x.f) \equiv \exists x.\alpha^p(f)$

$\alpha^0(f) \equiv$ false, for f not representable in C

$\alpha^1(f) \equiv$ true, for f not representable in C

$\alpha^p(f) \equiv$ e, for f directly representable in C as e

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Why Does Formula Approximation Work?

- Formula splitting preserves assumptions
- Proof of a given subformula depends only on a subset of assumptions

- Some HOL formulas directly translatable into formulas in simpler logics

Dealing With Proof Complexity

- Proof complexity issues
  - Provers overwhelmed by assumptions
  - Proof of a single subformula requires expertise of multiple provers
- Note statements

  //: note f: "…" from f_0, f_1, … f_n;
  - Tell provers which assumptions to use
  - Introduce intermediate lemmas into verification conditions
- In effect, developer guides proof decomposition

Note Example (get)

```
public Object get(Object k0)
/*:   requires "init ∧ k0 ≠ null"
      ensures "(result ≠ null → (k0, result) ∈ content) ∧
          (result = null → ¬(∃v. (k0, v) ∈ content))" */
{
      int hc = compute_hash(k0);
      Node current = table[hc];
      //: note ThisProps: "this ∈ old alloc ∧ this ∈ Hashtable ∧this.init";
      //: note HCProps: "0 ≤ hc ∧hc < table.length ∧ hc = hash key (table.length)";
      /*: note InCurrent: "∀v. ((k0, v) ∈ content) = ((k0, v) ∈ current.bucketContent)"
            from ContentDef, HCProps, Coherence, ThisProps, InCurrent; */
      while /*: inv "∀v. (k0, v) ∈ content) = ((k0, v) ∈ current.bucketContent" */
          (current != null) {
          if (current.key == k0) { return current.value; }
          current = current.next;
      }
       return null;
}
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```

Label known facts

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      /*: note InCurrent: “∀v. ((k0, v) ∈ content) = ((k0, v) ∈ current.bucketContent)”
          from ContentDef, HCProps, Coherence, ThisProps, InCurrent; */
      while /*: inv "∀v. (k0, v) ∈ content) = ((k0, v) ∈ current.bucketContent” */
          (current != null) {
          if (current.key == k0) { return current.value; }
          current = current.next;
      }
       return null;
}
```

State proof goal
(intermediate fact or final goal)

Note Example (get)

```
public Object get(Object k0)
/*:   requires "init ∧ k0 ≠ null"
      ensures "(result ≠ null → (k0, result) ∈ content) ∧
          (result = null → ¬(∃v. (k0, v) ∈ content))" */
{
      int hc = compute_hash(k0);
      Node current = table[hc];
      //: note ThisProps: "this ∈ old alloc ∧ this ∈ Hashtable ∧this.init";
      //: note HCProps: "0 ≤ hc ∧hc < table.length ∧ hc = hash key (table.length)";
      /*: note InCurrent: "∀v. ((k0, v) ∈ content) = ((k0, v) ∈ current.bucketContent)"
          from ContentDef, HCProps, Coherence, ThisProps, InCurrent; */
      while /*: inv "∀v. (k0, v) ∈ content) = ((k0, v) ∈ current.bucketContent" */
          (current != null) {
          if (current.key == k0) { return current.value; }
          current = current.next;
      }
       return null;
}
```

Identify a set of known facts

Note Example (get)

```
public Object get(Object k0)
/*:   requires "init ∧ k0 ≠ null”
      ensures "(result ≠ null → (k0, result) ∈ content) ∧
          (result = null → ¬(∃v. (k0, v) ∈ content))" */
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      int hc = compute_hash(k0);
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          (current != null) {
          if (current.key == k0) { return current.value; }
          current = current.next;
      }
       return null;
}
```

- Instruct prover to use set to prove goal
- Proven fact inserted into assumption base

Note Example (get)

```
public Object get(Object k0)
/*:    requires "init ∧ k0 ≠ null”
       ensures "(result ≠ null → (k0, result) ∈ content) ∧
            (result = null → ¬(∃v. (k0, v) ∈ content))" */
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       int hc = compute_hash(k0);
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            (current != null) {
            if (current.key == k0) { return current.value; }
            current = current.next;
       }
        return null;
}
```

Proved trivially by syntactic prover

Constructs for Directly Controlling Proof

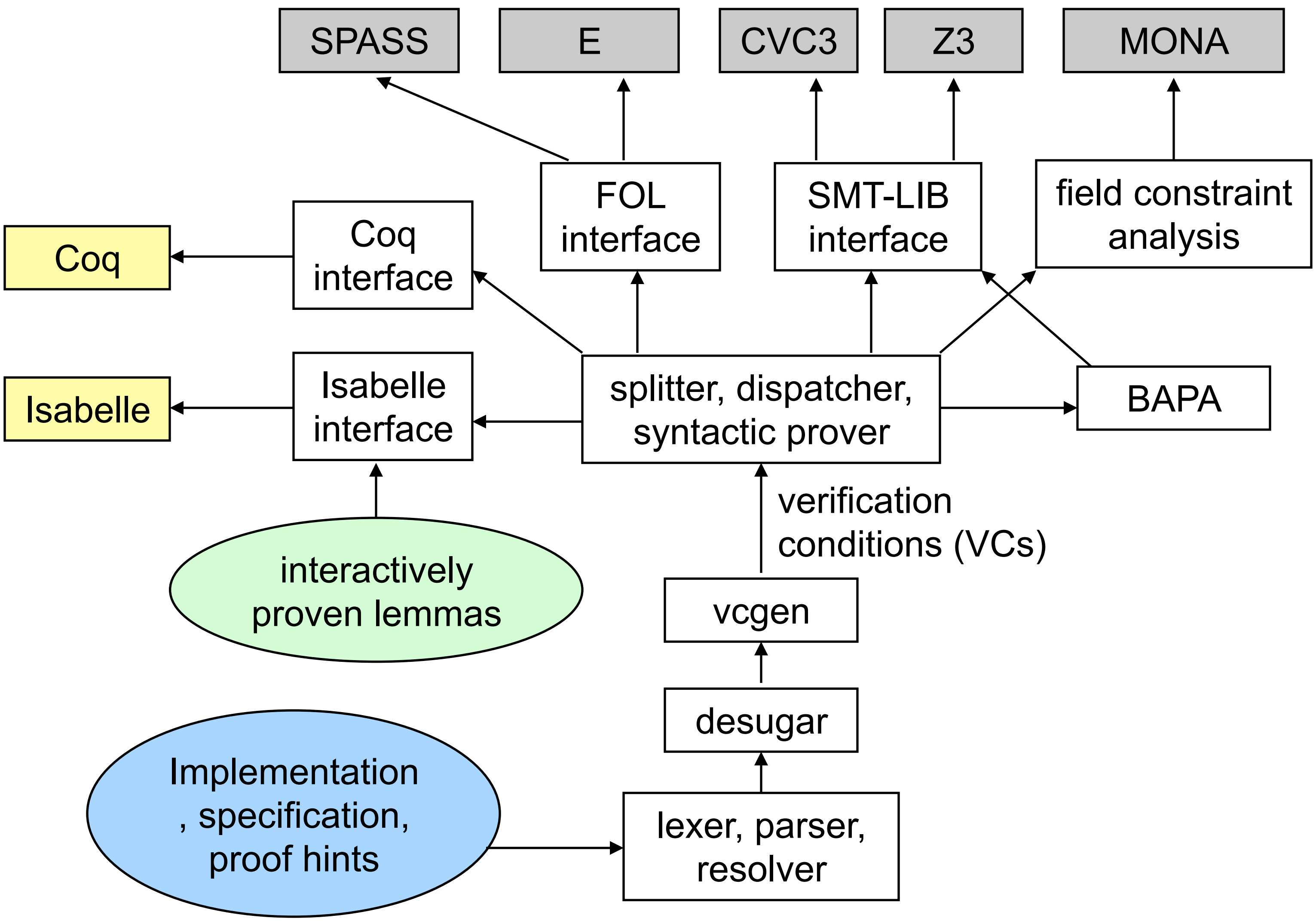
- havoc $x_0,\ldots,x_n$ suchThat f

  Instantiates $\exists x_0,\ldots,x_n.f$
- pickAny $x_0,\ldots,x_n$ in (c; note g)

  Prove $\forall x_0,\ldots,x_n.g$
- assuming f in (c_{pure}; note g)

  Prove $f \rightarrow g$
- Desugar into standard guarded commands

Jahob System Diagram



Experimental Results

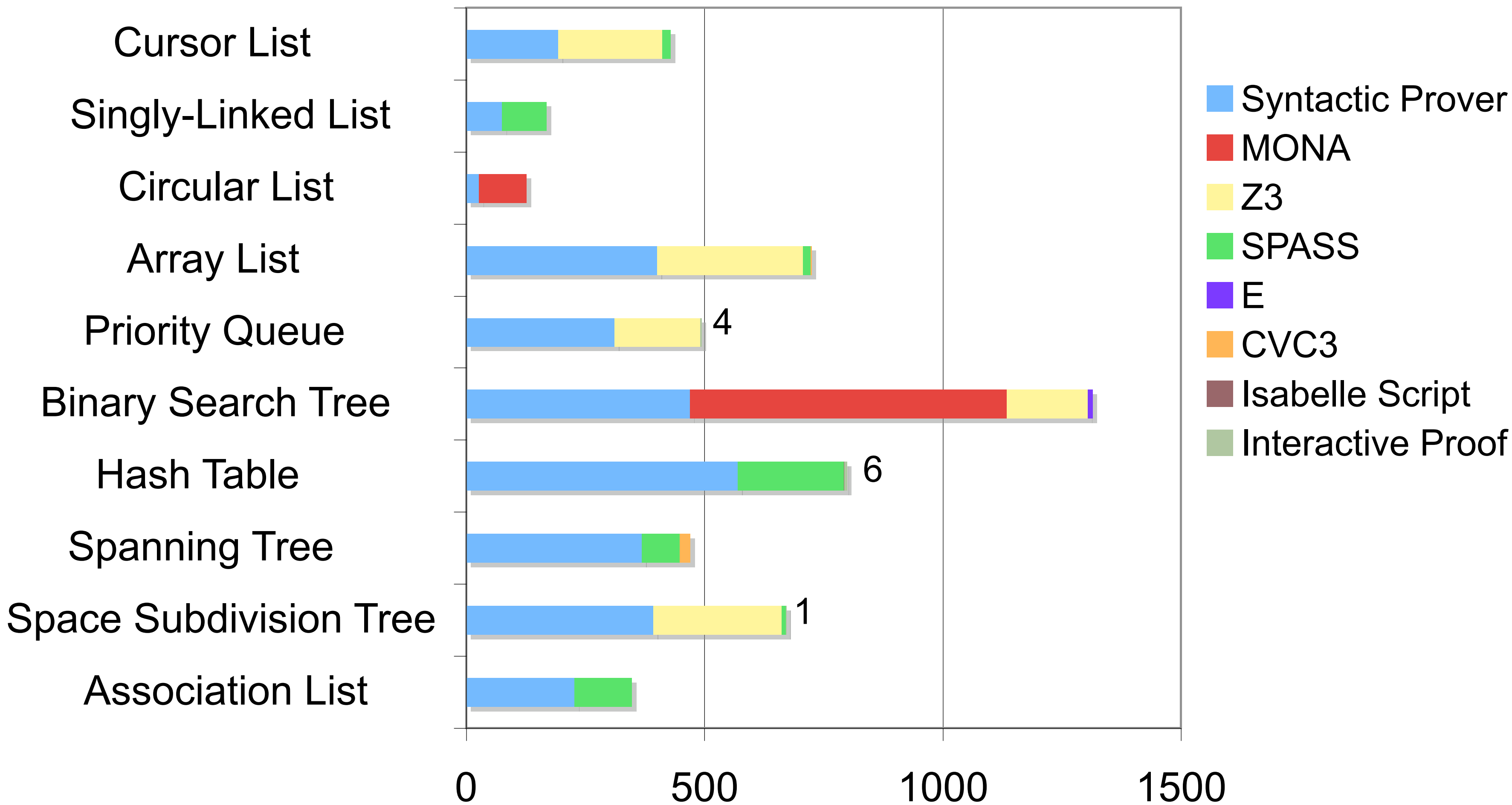
Verified Data Structures

- Cursor List
- Singly-Linked List
- Circular List
- Array List
- Priority Queue
- Binary Search Tree
- Hash Table
- Spanning Tree
- Space Subdivision Tree
- Association List

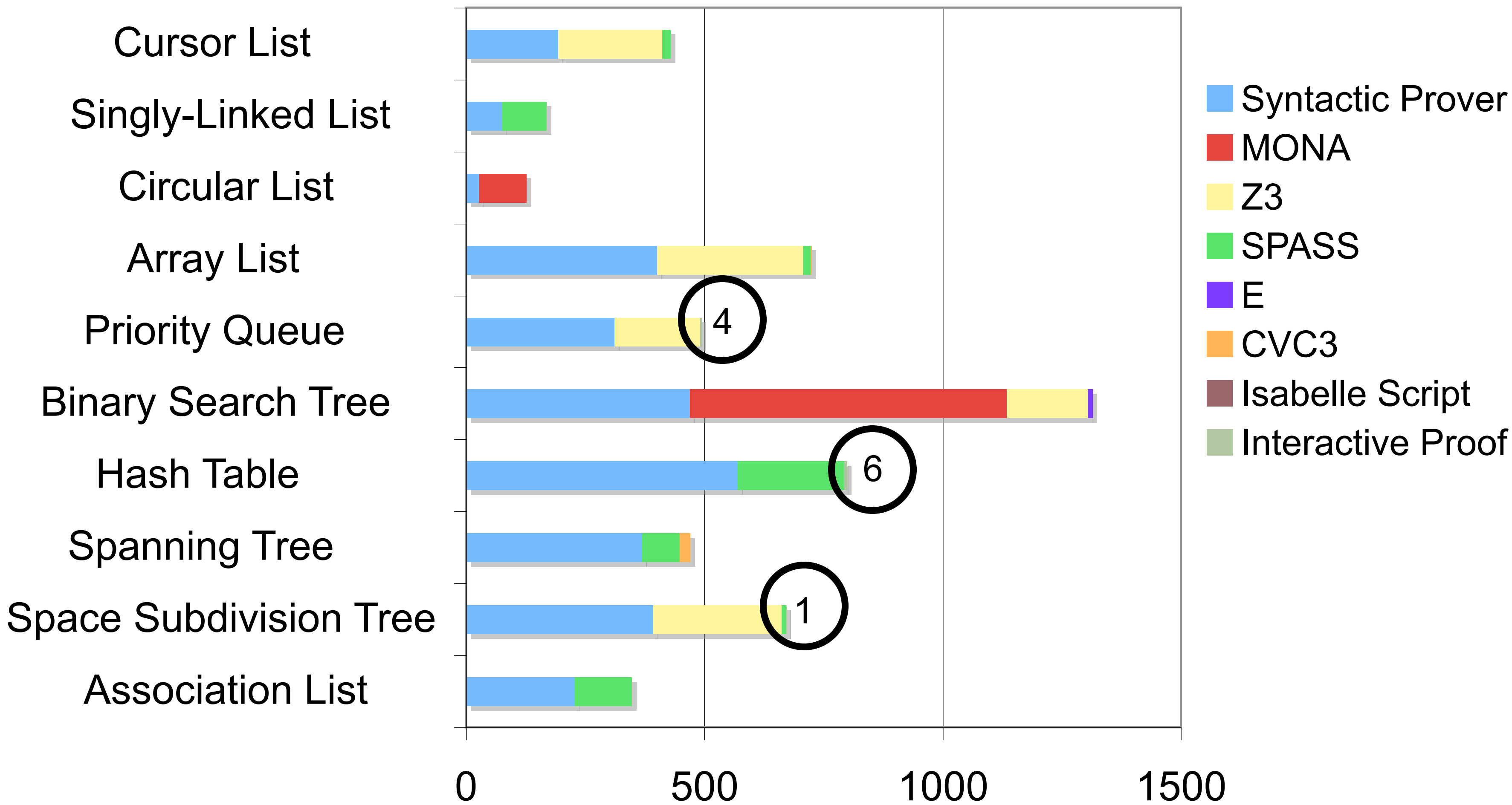
Provers

- Syntactic prover
- MONA
- Z3
- SPASS
- E
- CVC3
- Isabelle (simplifier)
- Isabelle proof assistant

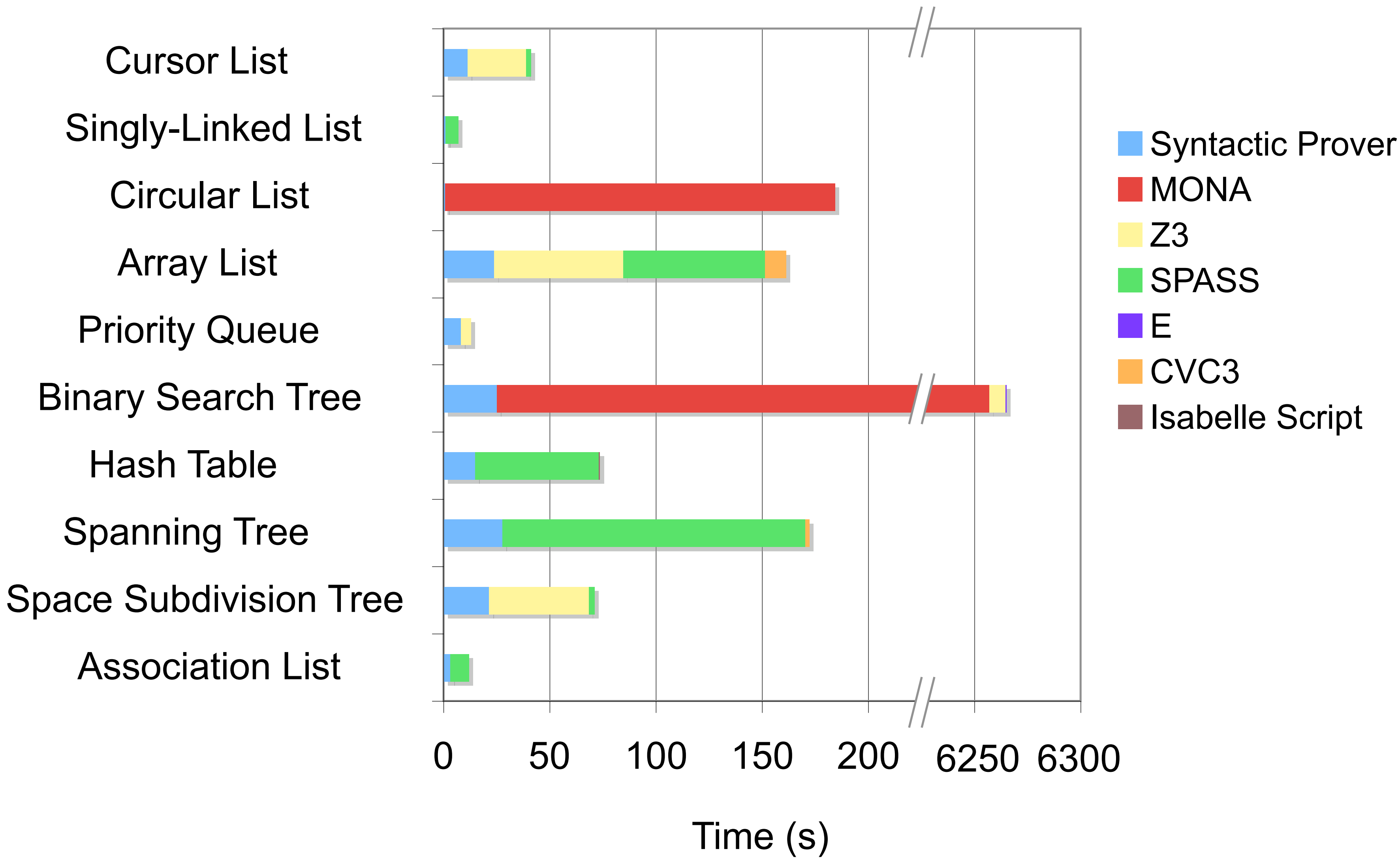
Formulas Verified



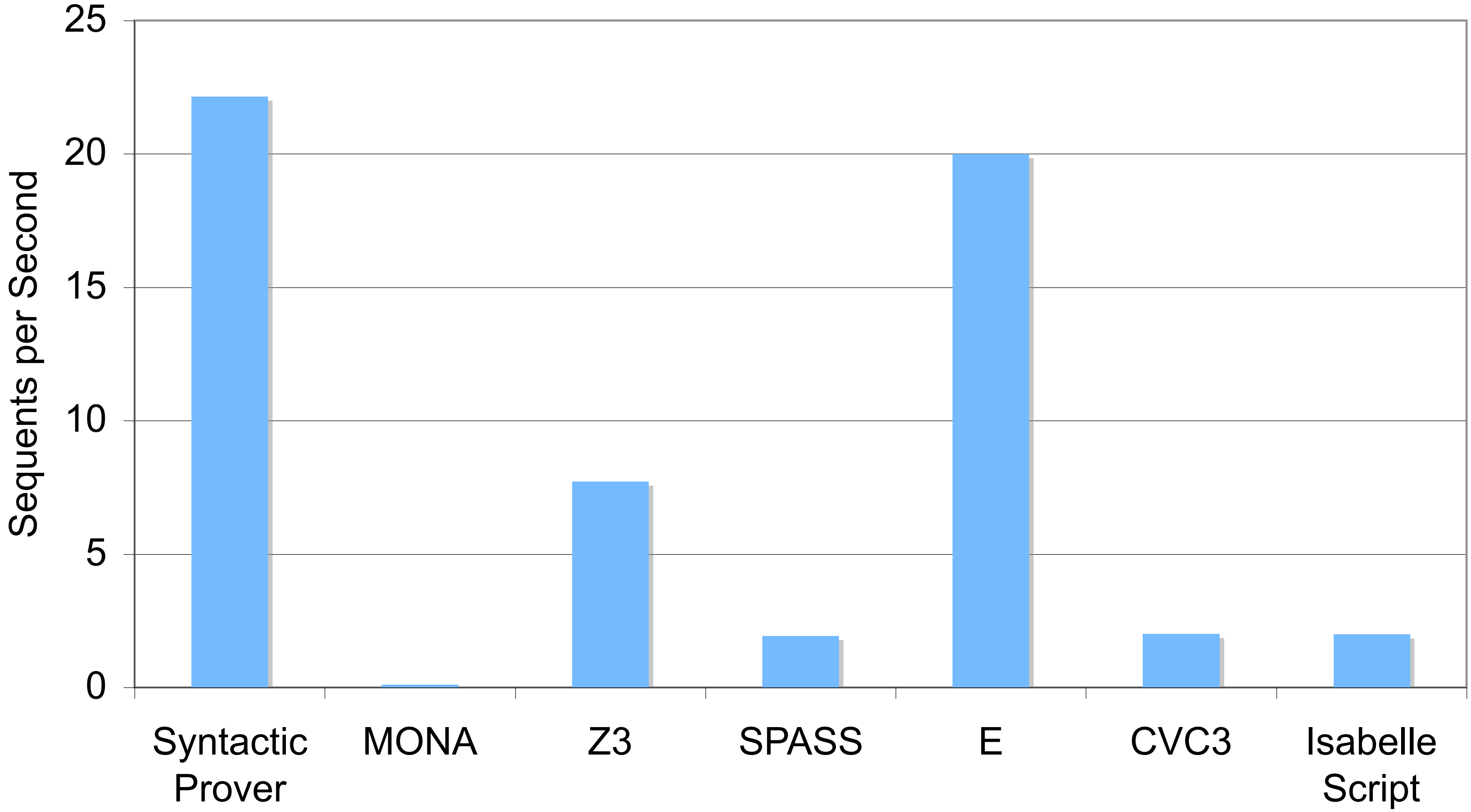
Formulas Verified



Verification Time



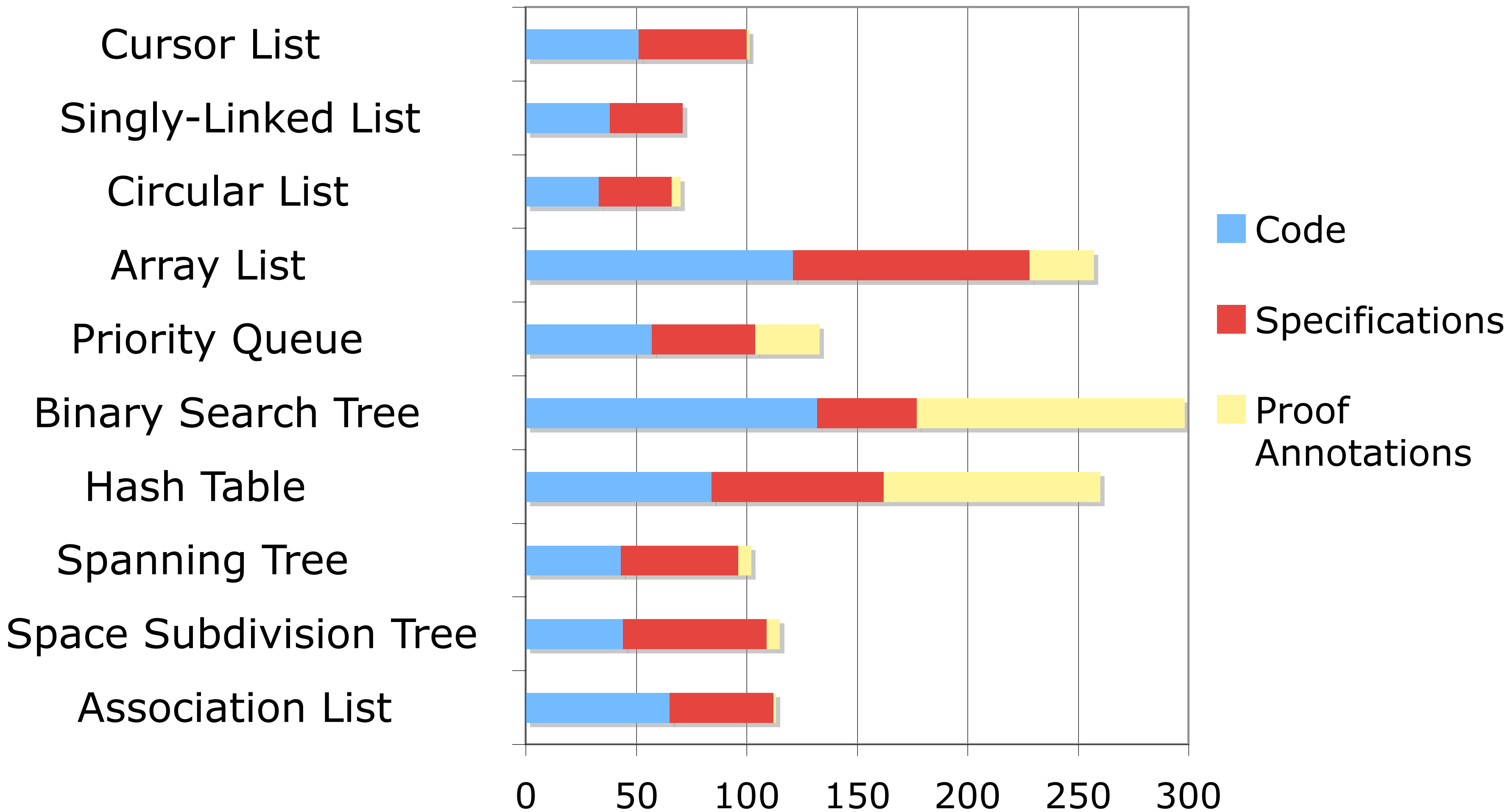
Prover Efficiency



Manual Proofs

- Space Subdivision Tree
  - 1 proof, 4 lines of proof script
  - 2 case splits, 1 quantifier instantiation
- Priority Queue
  - 2 proofs, 78 + 4 lines
  - Inductive proof with modulo arithmetic
- Hashtable
  - 1 proof for add, 7 lines, 3 case splits
  - 4 proofs for remove, 19 + 2 + 2 + 10 lines, case splits

Logical Lines of Code



Observations

- Implementation easier than specification
  (but more current expertise with implementation)
- Easy to prove – callers, observer methods
- Difficult to prove
  - Destructive update (constructors*, add, remove*)
  - Leaf methods (reasoning about concrete + abstract state)
- Incentive to decompose larger methods
- Incentive to reuse existing methods
- Must understand why implementation is correct to obtain proof

Implications

- Verified data structure libraries
- Integrated reasoning in other contexts
- New program analysis techniques
  - Client analyses based on sets and relations (no more reasoning about pointers)
  - More precise data structure analyses
- Semantic commutativity analysis for parallel programs

Related Work: Hob

- Kuncak, Lam, Zee, and Rinard [TSE 2006]; Lam [PhD. Thesis MIT 2007]
- Sets summarize data structure state
- Full functional verification only for data structures with set interface
- Multiple decision procedures (early form of integrated reasoning)

Related Work (cont’d)

Software Verification Tools

- Spec#: Barnett, DeLine, Fähndrich, Leino, and Schulte [J. Obj. Tech. 2004]
- ESC/Modula-3: Detlefs, Leino, Helson, Saxe [TR159 COMPAQ SRC 1998]
- ESC/Java: Flanagan, Leino, Lilibridge, Nelson, Saxe and Stata [PLDI 2002]
- ESC/Java: Chalin, Hurlin, and Kiniry [VSTTE 2005]
- Krakatoa: Filliatre [J. Func. Programming 2003]; Marche, Paulin-Mohring, and Urbain [J. Logic & Alg. Prog. 2003]
- KIV: Balser, Reif, Schellhorn, Stenzel, and Thums [FASE 2000]
- KeY: Ahrendt, Baar, Beckert, Bubel, Giese, Hähnle, Menzel, Mostowski, Roth, Schlager, and Schmitt [Soft. & Sys. Modeling 2005]
- LOOP: van der Berg and Jacobs [TR CSI-R0019 U. Nijmegen 2000]

Related Work (cont'd)

Shape Analysis

- Chong and Rugina [SAS 2003]
- Role analysis: Kuncak, Lam, and Rinard [POPL 2002]
- Grammar-based shape analysis: Lee, Yang, and Ki [ESOP 2005]
- TVLA: Sagiv, Reps, and Wilhelm [TOPLAS 2002]
- Symbolic shape analysis: Podelski and Wies [SAS 2005]
- Guo, Vachharajani, and August [PLDI 2007]

Separation Logic

- Smallfoot: Berdine, Calcagno, and O'Hearn [FMCO 2005]
- Nguyen, David, Qin, and Chin [VMCAI 2007]
- Nguyen and Chin [CAV 2008]

- Yang, Lee, Berdine, Calcagno, Cook, Distefano, and O'Hearn [CAV 2008]

Unrelated Work

Bounded model checking

- Bogor: Robby, Rodríguez, Dwyer and Hatcliff [STTT 2006]
- JACK: Bouali, Gnesi and Larosa [EATCS 1994]
- Forge: Dennis, Chang and Jackson [ISSTA 2006]
- J-Sim: Sobeih, Mahesh, Marinov and Hou [IPDPS 2007]

Testing

- Korat: Boyapati, Khurshid and Marinov [ISSTA 2002]
- TestEra: Khurshid and Marinov [Autom. Soft. Eng. 2004]
- Cute: Sen, Marinov and Agha [FSE 2005]

Conclusions

- Full functional correctness for linked data structure implementations
  - Formula splitting
  - Formula approximation
  - Integrated reasoning
  - Constructs for guiding proofs
- Complete realization of abstract data types
  - Precise, complete, and verified specifications
  - Enables new, more precise and scalable client program analyses

Questions?