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$T < D$ → GLV (low diversity)

$T > D$ → θ-GLV (high diversity iff $\boldsymbol{T > 2D}$)

The $D \gg r$ adiabatic limit

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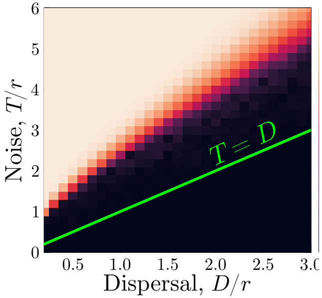
Valid whenever $D \gg r$ | Valid only for $T < D$

Effective GLV dynamics for $T < D$

- **Effective equation for means** $(D - T \gg r)$:

$$\partial_t \langle N_i \rangle = r \langle N_i \rangle \left[1 - \frac{D}{D-T} \langle N_i \rangle - \sum_{j \neq i} A_{ij} \langle N_j \rangle \right]$$



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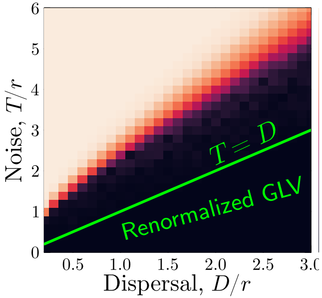
$\Rightarrow$ *Standard GLV with renormalized carrying capacity*

$\Rightarrow S^* \in \mathcal{O}(1), \phi = 0$ ✓



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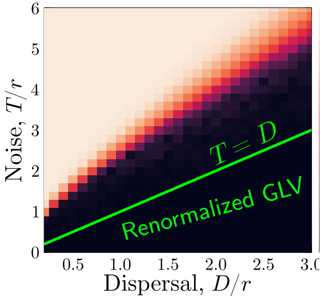
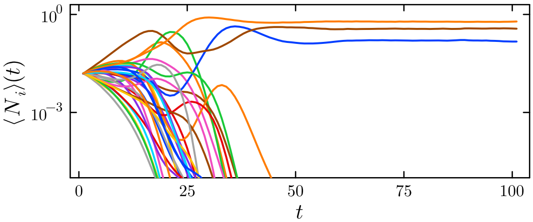


Metacommunity $(D = 1, T = 0.1)$**:**



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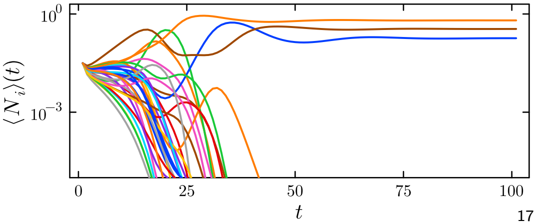

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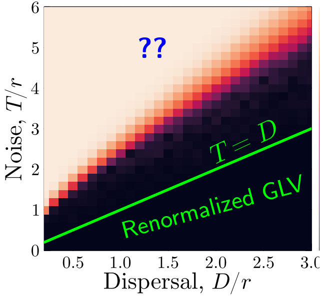

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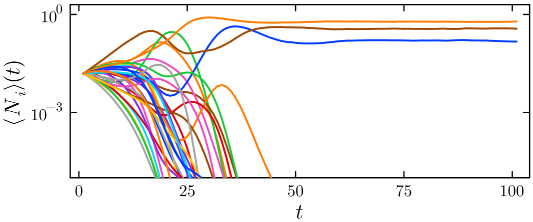

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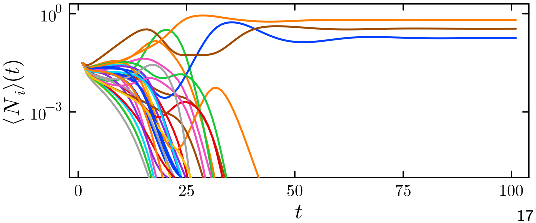

Stationary abundance distribution in coexistence phase

$$\dot{N}_i = N_i\left[1 - N_i - \xi_i\right] + D[\langle N_i\rangle - N_i] + \sqrt{2T}N_i\eta_i(t), \qquad \xi_i(t) \equiv \sum_{j\neq i} A_{ij}N_j(t)$$

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- **Claim**: for $D \gg r$, fluctuations of ξ_i vanish at large S:

$$\xi_i(t) \xrightarrow{S\to\infty} \langle\xi_i\rangle \equiv \sum_{j\neq i} A_{ij}\langle N_j\rangle$$

To show this:

1. Assume $\xi = \langle\xi\rangle$, find $\langle\xi\rangle$ self-consistently
2. Compute $\left\langle N_i^2\right\rangle$
3. Show $\lim_{S\to\infty}\left\langle\xi^2\right\rangle_c = 0$

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- Expand for small $\langle N_i \rangle \propto 1/S$ and solve for $\langle \xi \rangle$:

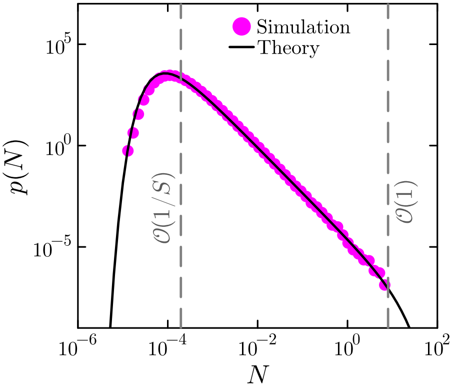
$$\boxed{\langle \xi_i \rangle = 1 + \mathcal{O}(S^{-\beta D})} \text{ for all } i.$$

Power law distribution of abundances

- **Result:**
  All species follow the same power law

$$p_i(N) \propto e^{-\beta N} e^{-\beta D \langle N_i \rangle / N} N^{-2-\beta D},$$

- Power laws observed in natural ecosystems:
  (Locey and Lennon, 2016; Ser-Giacomi, 2018; etc.)



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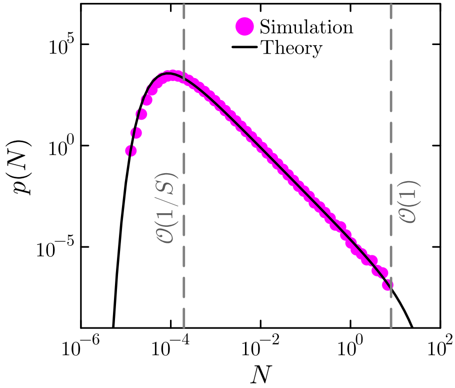
- **Consequence:**
  Anomalous scaling of moments



Anomalous scaling of moments

- For small $\langle N \rangle \propto 1/S$:

$$\left\langle N^2 \right\rangle \simeq \frac{D}{D-T} \langle N \rangle^2 \left[1 + (\dots) \langle N \rangle^{\beta D - 1} \right] + \mathcal{O}\left(\langle N \rangle^2 \right)$$

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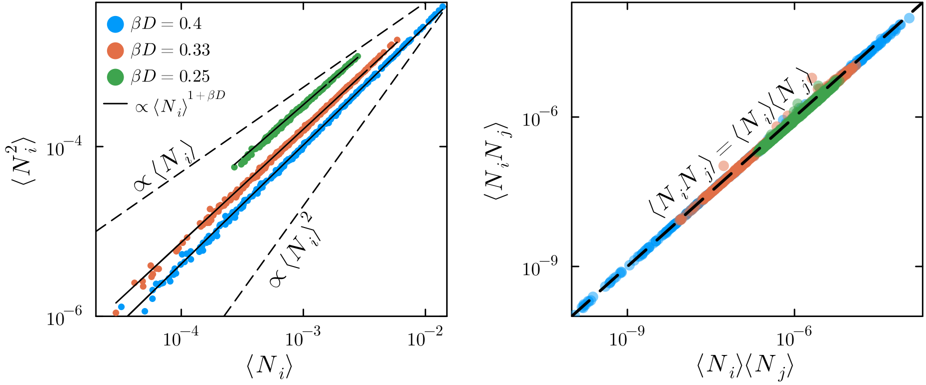
- **Taylor's power law (1961)**: $\langle N_i^2 \rangle_c \propto \langle N_i \rangle^{1+\theta}$
  Widely verified in ecological data, nonuniversal exponent (Taylor 1961; Eisler et al. 2008)

Anomalous scaling of moments in the coexistence phase



Simulations support $\langle N_i^2 \rangle \propto \langle N_i \rangle^{1+\beta D}$ in coexistence phase.

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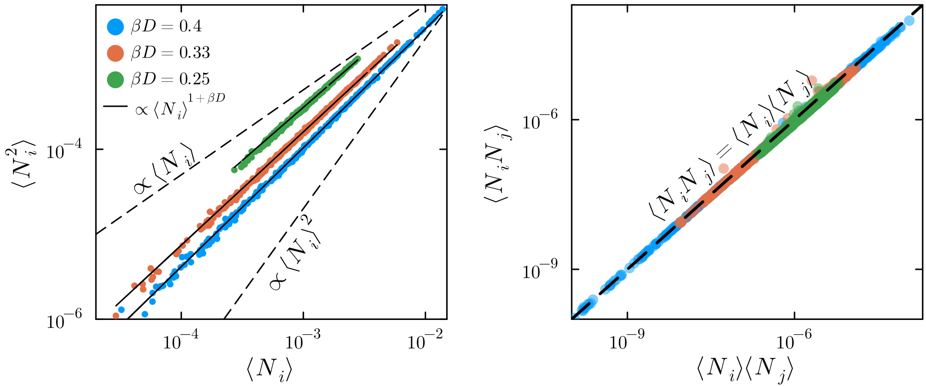

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$$\Longrightarrow \boxed{\langle N_i \rangle - \langle N_i^2 \rangle - \sum_j A_{ij} \langle N_i N_j \rangle} \xrightarrow{D \gg r} \boxed{\langle N_i \rangle - \langle N_i \rangle^{1+\theta} - \sum_j A_{ij} \langle N_i \rangle \langle N_j \rangle}$$

θ-logistic Generalized Lotka-Volterra Equation

- **Effective equation for means** ($\theta \equiv \beta D$)

$$\langle \dot{N}_i \rangle = \langle N_i \rangle \left[1 - \langle N_i \rangle^\theta - \sum_{j \neq i} A_{ij} \langle N_j \rangle \right], \qquad (\theta\text{-GLV}).$$

- Sometimes called $\boldsymbol{\theta}$**-logistic** self-regulation (Hatton et al.).

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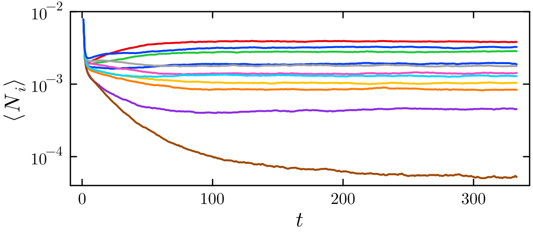
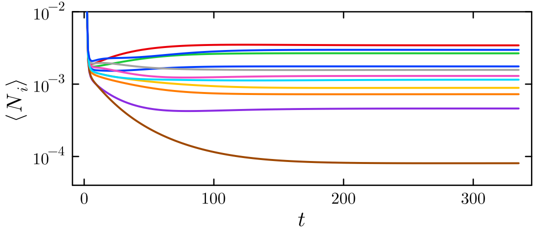


Effective θ-GLV



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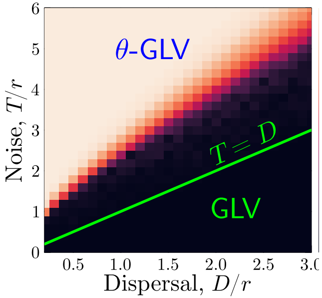

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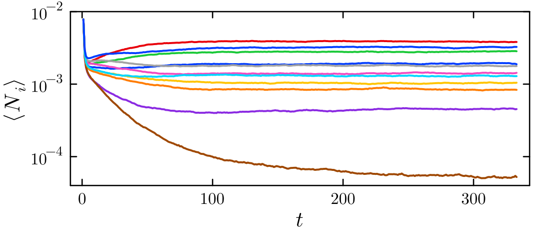

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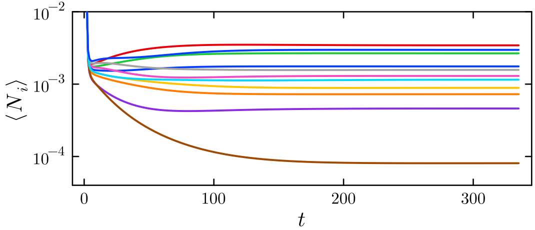

Fixed points of the θ-GLV model

$$0 = \langle N_i \rangle \left[1 - \langle N_i \rangle^\theta - \sum_{j \neq i} A_{ij} \langle N_j \rangle \right], \qquad \overline{A_{ij}} = \mu, \qquad \overline{A_{ij} A_{k\ell}} = \sigma^2 \delta_{ik} \delta_{j\ell}.$$

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Fixed points of the θ-GLV model

$$0 = \langle N_i \rangle \left[1 - \langle N_i \rangle^\theta - \sum_{j \neq i} A_{ij} \langle N_j \rangle \right], \qquad \overline{A_{ij}} = \mu, \qquad \overline{A_{ij} A_{k\ell}} = \sigma^2 \delta_{ik} \delta_{j\ell}.$$

- Two solutions: $\langle N_i \rangle = 0$ or $\langle N_i \rangle = (1 - \sum_j A_{ij} \langle N_j \rangle)^{1/\theta}$

- Cavity method: replace $\sum_{j \neq i} A_{ij} \langle N_j \rangle$ with mean + Gaussian fluctuations:

$$\langle N \rangle = \max \left(0, 1 - \mu S \overline{\langle N \rangle} - \sigma \sqrt{S \overline{\langle N \rangle^2}}\, Z \right)^{1/\theta}, \qquad Z \sim \mathcal{N}(0, 1).$$

  with $\overline{\langle N \rangle}, \overline{\langle N \rangle^2}$ determined self-consistently.

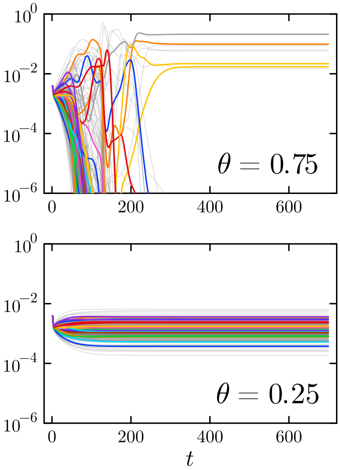
- Survival fraction $\phi = \mathbb{P}[\langle N \rangle > 0]$
- Solve using large-S ansatz: $\overline{\langle N \rangle} \sim S^{-\alpha}, \overline{\langle N \rangle^2} \sim S^{-\gamma}, \phi \to 1$

Cavity solution of the θ-GLV model

$$0 = \langle N_i \rangle \left[1 - \langle N_i \rangle^{\theta} - \sum_{j \neq i} A_{ij} \langle N_j \rangle \right]$$

Results:

- **There is a coexistence solution** $(\phi = 1)$ **iff** $\boldsymbol{\theta < \frac{1}{2}}$



Cavity solution of the θ-GLV model

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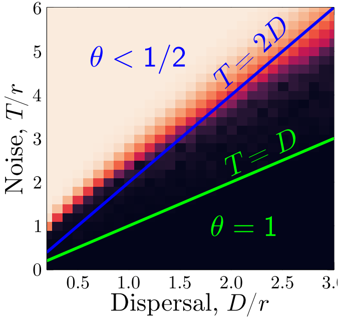
Results:

- **There is a coexistence solution** $(\phi = 1)$ **iff** $\boldsymbol{\theta < \frac{1}{2}}$
  Using $\theta = \beta D$, this implies $\boldsymbol{T_c = 2D}$



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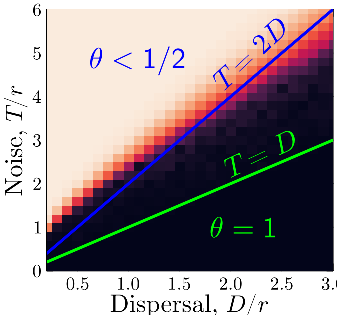
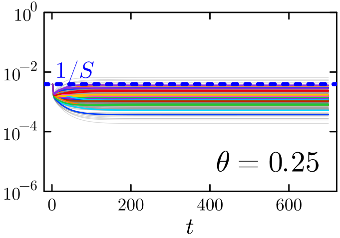
- In coexistence phase:

$$\overline{\langle N \rangle} \simeq \frac{1}{\mu S}, \qquad \overline{\langle N \rangle^2} \simeq \frac{1}{\mu^2 S^2}.$$

$\Rightarrow$ **mean abundances** $\in \mathcal{O}(1/S)$**.**





Cavity solution of the θ-GLV model

$$0 = \langle N_i \rangle \left[1 - \langle N_i \rangle^\theta - \textstyle\sum_{j \neq i} A_{ij} \langle N_j \rangle \right]$$

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  $\Rightarrow$ **mean abundances** $\in \mathcal{O}(1/S)$**.**

- As $S \to \infty$, $\overline{\langle N \rangle^2} \to \overline{\langle N \rangle}^2$
  $\Rightarrow$ **disorder becomes irrelevant**.