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INTRODUCTORY REMARKS ON QUANTUM ELECTRODYNAMICS

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The theory of the electron of Lorentz starts out by assuming a definite charge distribution inside the electron, for instance a homogeneous surface distribution on a spherical shell with radius R. A number of results tend towards a definite limit when $R \to 0$ and are not materially modified if R is of the order of 10^{-13} cm instead of 0. Such results are said to be shape-independent and the permanent value of Lorentz' theory may be said to be largely due to the existence of the shape-independent results.

If an electron is moving there is an additional field energy which for low velocities is of the form $\frac{1}{2} m_e v^2$; m_e is the electro-magnetic mass and the experimental mass is the sum of m_e and the mechanical mass. It was a consequence of electromagnetic theory that the electro-magnetic mass varies with velocity and the agreement between the variation of the mass of the electron found experimentally and the predictions of theory was originally regarded as a powerful argument in favour of the idea that the mass of the electron is entirely electromagnetic. Today this argument would appear to be no longer valid: a relativistically invariant theory must necessarily always lead to the same law of variation of mass. For a spherical shell the electromagnetic mass at low velocities is given by $\frac{2}{3} e^2/Rc$. In order to maintain the electron in equilibrium Poincaré has postulated a surface tension $e^2/8\pi R^4$ leading to an additional energy $e^2/6R$; since the electrostatic energy is $e^2/2R$ the relation $E = mc^2$ is fulfilled.

In quantum theory there are again a number of results that are shape-independent. The way in which these results are usually obtained is somewhat less satisfactory than in classical theory since one starts from a Hamiltonian based on the motion of a point electron which is strictly speaking meaningless. The self-energy corresponds to the electromagnetic mass. In the theory of holes the divergence is partly compensated but a logarithmic term remains.

In renormalisation theory a negative mass term is added to the Hamiltonian to compensate the infinite terms. Although this procedure is not satisfactory from a mathematical point of view it can be carried out in an unambiguous and invariant way. The resulting corrections to energy levels may be said to be due to a difference in electromagnetic mass in different states and these differences are again shape-independent.

If the electromagnetic field is resolved into Fourier components every wave should have a zero point energy of $\frac{1}{2}h\nu = \frac{1}{2}\hbar kc$. The total zero point energy of empty space is of course infinite and usually discarded without much comment. If we cut off at a wave number k_m a volume V contains an energy $\hbar c k_m^4 V/16\pi^2$.

As a curiosity I should like to point out that the existence of zero point energy might afford an explanation of Poincaré stresses; if we assume that inside the electron there is no electromagnetic field and cut off at k_m we obtain a surface tension $\hbar c k_m^4/16\pi^2$ and the Poincaré stresses may be said to be due to the zero point pressure of the electromagnetic field.

It is even possible to avoid the introduction of an arbitrary quantity k_m if we assume that there exists also an electromagnetic field inside the electron but that the shell acts as an infinitely conducting separation between the internal and the external field. There is then still a reduction of total zero point energy due to the discreteness of the states inside the sphere. This reduction of zero point energy will be of the form $C.\hbar c/R$, an expression that can be understood either by starting from the known result for the attraction of two infinitely conducting plates or by remarking that a contribution comes only from those waves for which $k \sim 1/R$. The value of the contact C has not yet been calculated. Comparing this reduction of zero point energy with the electrostatic self-energy we find that for $e^2/\hbar c = 2C$ a compensation is possible and it is an attractive feature that this compensation is now independent of the value of R.

Although these considerations are very crude they suggest that it might be possible to find within the realm of quantum electrodynamics a formalism in which zero point energy and self-energy compensate one another for one definite value of $e^2/\hbar c$ in a shape-independent way.

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PAIS: The suggestion of Casimir to compensate the electrostatic energy by a tension due to vacuum fluctuations looks quite different if one considers the effect of negaton positon pairs. Positon theory leads to an expression W for the electrostatic energy which is

$$W \sim (e^2/hc) . me^2 \lg \sqrt{(1 + \hbar/mcR)}$$

where R is a cut off. $W \sim e^2/R$ for $R \gg \hbar/mc$, thus Casimir's radius should be large compared to the Compton wave length, which is hard to believe. A relativistic version of the idea seems difficult to find. A main objection is perhaps that non observable quantities are introduced in this picture.

ROSENFELD: The analogy between the classical relationship pointed out by Casimir and the quantal self-energy contributions is perhaps closer in the case of a boson: both contributions diverge in the same way in that case.

As regards the idea of compensating self-energies of different fields, it has hitherto been found impracticable because the equations of condition between the masses of the fields cannot be satisfied with positive masses only.

PEIERLS: 1) In Casimir's model one might still obtain the right logarithmic divergence for the zero point energy, if one is allowed to offset the positively infinite zero point energy of the electromagnetic field against the negatively infinite volume energy of the "sea" of electrons (and other particles) in negative-energy states.

2) In discussing renormalisation it is important not to think of it as a subtraction. If the divergent electromagnetic mass is allowed to stand, also the energy differences between different states diverge. This is avoided only by carrying the divergent mechanical mass which compensates the self-mass so that the actual mass in any stage of the calculation is always finite.

CASIMIR: I agree entirely with Peierls' first remark but I think that one should first elaborate the non-relativistic theory somewhat further before looking for relativistic refinements.

As to the second remark there is perhaps no objection against speaking of subtraction if we only bear in mind that this subtraction has to be carried out in the Hamiltonian and not in the final results for the energy levels.

DIRAC: I would like to ask Casimir about the shape of his electron when it accelerates. There are two assumptions one might make: (I) the electron is always rigidly a sphere in the frame of reference in which it is instantaneously at rest (II) the surface of the electron is deformable and there are some forces which tend to keep it spherical.

CASIMIR: With respect to Dirac's questions I am sorry that I have not considered these problems in any detail.

PAIS: In regard to the stability of the model it may be recalled that the equilibrium due to Poincaré stresses is not a stable one for arbitrary deformations.

FIERZ: Ist es wahr, dass die Selbstenergie „skalarer Teilchen" (Pauli-Weisskopf) wie k^4 divergiert? Soll man nicht diese Divergenz als nur scheinbar ansehen, indem man einen Teil abspaltet, der zwar unbestimmt ist, aber als 0 definiert werden kann? Was übrig bleibt ist sodann ebenfalls nur logarithmisch divergent.

KÄLLEN: As a reply to Fierz it can be remarked that it is rather the logarithmic divergency in the self-energy of the Fermi particles which is a spurious effect because we here have a cancelling between two terms which separately diverge linearly.

HEISENBERG: What justification can be given for introducing an electron radius $\sim 10^{-13}$ cm? If one imagines quantum electrodynamics separated from other particles, then one can call the radius of the electron either zero or $\hbar/mc$; but e^2/mc^2 does *not* come into the formalism of quantum electrodynamics. The only justification for 10^{-13} cm seems to be the existence of the π-meson, and that means, that one does *not* separate quantum electrodynamics from nucleon-physics.

CASIMIR: Although the model I propose is admittedly very crazy I should like to point out that in the second version it is not necessary to assume any definite value for the radius of the electron since a compensation is obtained for an arbitrary value of R. Therefore I do not think that Heisenberg's remark is the most important objection against this model.

BELINFANTE: It is possible to interpret positon theory without using the hole concept. Instead of giving the state vector as a one-column matrix with the (4^n-component) n-electron function on the $(n + 1)$st row, we can write the state vector as an infinite square matrix having on the $(p + 1)$st place in the $(n + 1)$st row a situation function for p positons and n negatons. This function may be a simple product of a state vector $\eta\,(1, 2, \ldots, p)$ for the positons, and a state vector $\varphi\,(1, 2, \ldots, n)$ for the negatons, each factor antisymmetrized in the one kind of particles only.

Let, for some observable, Ω be the Hermitian operator that operates on the negaton function $\varphi\,(1, 2, \ldots, n)$, then the operator acting on the positon function $\eta\,(1, 2, \ldots, p)$ will be $(-\,\Omega^*)$.

The second-quantized wave function $\boldsymbol{\psi}$ may be split into two parts:

$$\boldsymbol{\psi} = \boldsymbol{\varphi} + \boldsymbol{\eta}^*; \qquad \boldsymbol{\psi}^* = \boldsymbol{\eta}^* + \boldsymbol{\varphi}^*.$$

Here, $\boldsymbol{\varphi}$ and $\boldsymbol{\eta}$ annihilate (and their conjugates create) negatons and positons respectively. Explicit formulas can be given showing how $\boldsymbol{\varphi}$, $\boldsymbol{\eta}$, $\boldsymbol{\varphi}^*$, and $\boldsymbol{\eta}^*$ operate on the state vector. In momentum space, they operate much in the usual way as in Fock's theory: $\boldsymbol{\varphi}$ and $\boldsymbol{\varphi}^*$ act on each separate column of the state-vector matrix, and $\boldsymbol{\eta}$ and $\boldsymbol{\eta}^*$ on each row. However, in order to make $\boldsymbol{\varphi}$ *anti*commutative with $\boldsymbol{\eta}$ and $\boldsymbol{\eta}^*$, one has to postulate an additional factor (minus one to the power total number of *negatons*) in the definition of the *positon* wave functions (operators), or conversely.

The main advantage of our formalism is that it makes the operation with infinite numbers of negative-energy electrons superfluous, and that the interpretation of the theory becomes clearer. Although only finite numbers of positive-energy particles need be considered, we can describe pair creations and annihilations and we can calculate such phenomena as the vacuum polarization and the Uehling effect.