Minimizing the Age of Information in Wireless Networks with Stochastic Arrivals

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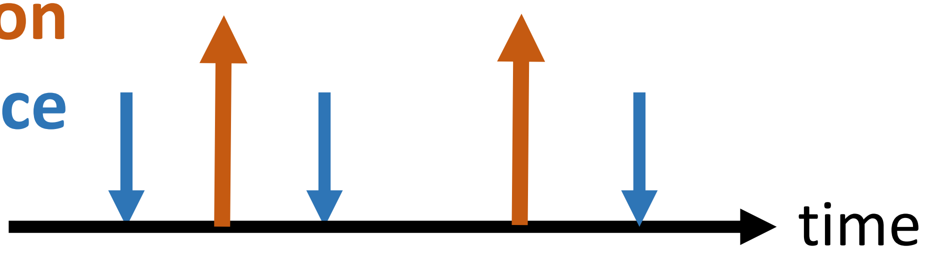
Outline

- Age of Information
- Network Model
- Scheduling Policies and Performance Guarantees
  - Stationary Randomized Policy
  - Max-Weight Policy
- Numerical Results
- Final Remarks & Current Work: from theory to practice (short video)

Age of Information

Delivery of packets to the destination

Packet generation at the source

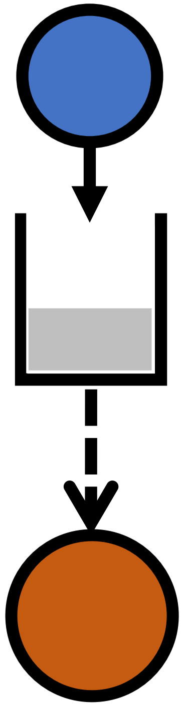


Source

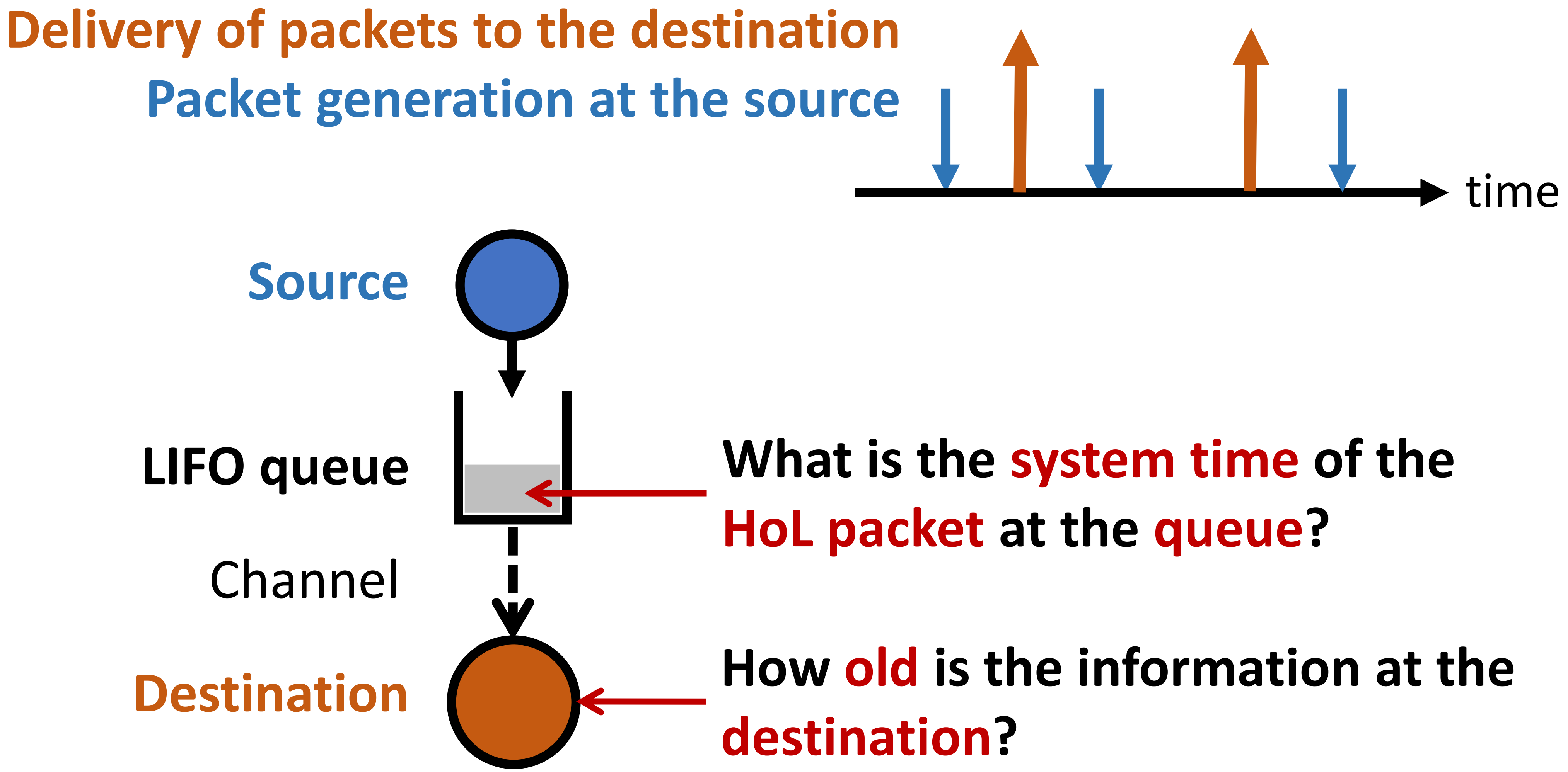
Single packet queue
≡ LIFO queue

Channel

Destination



Age of Information



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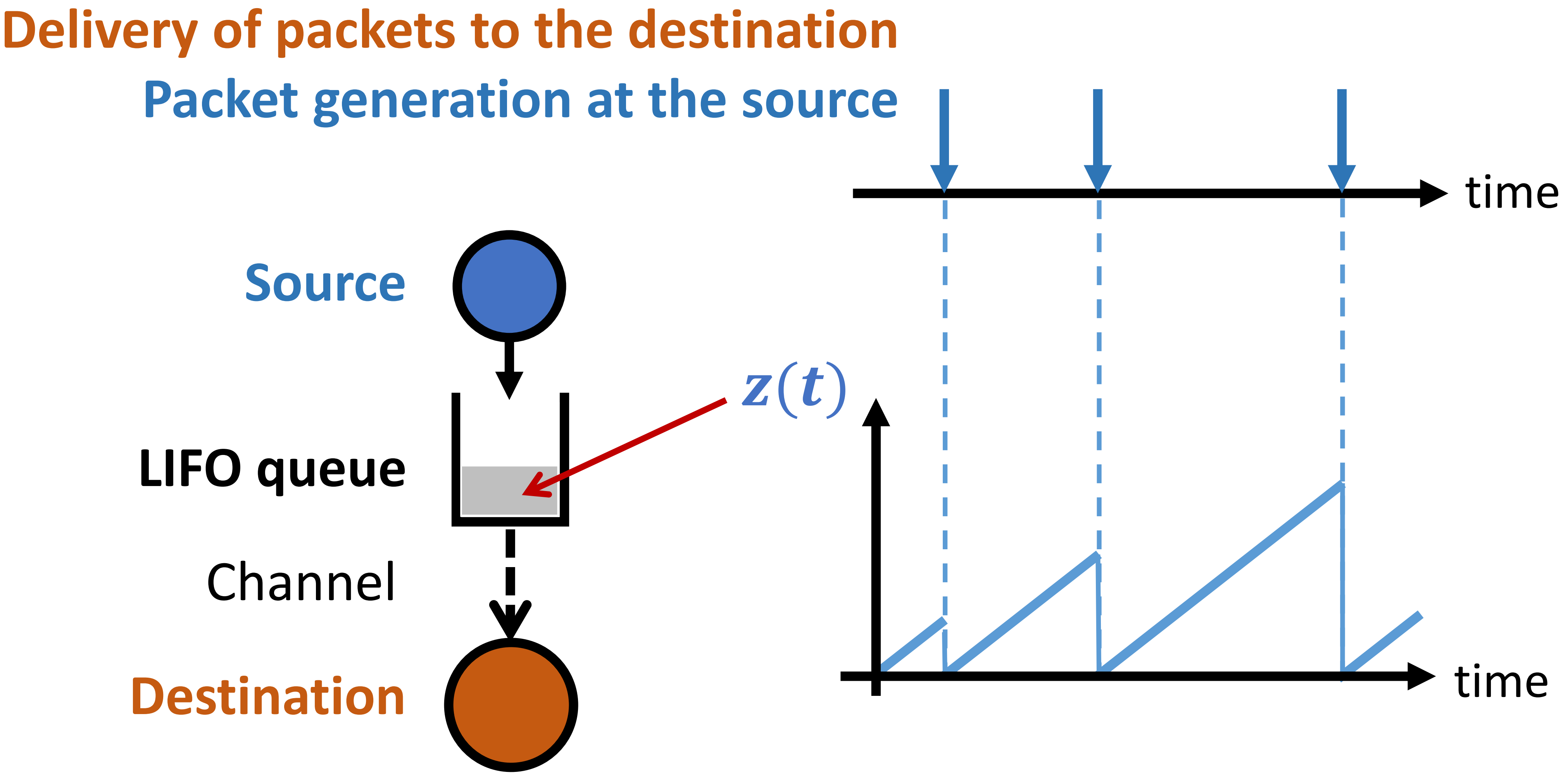

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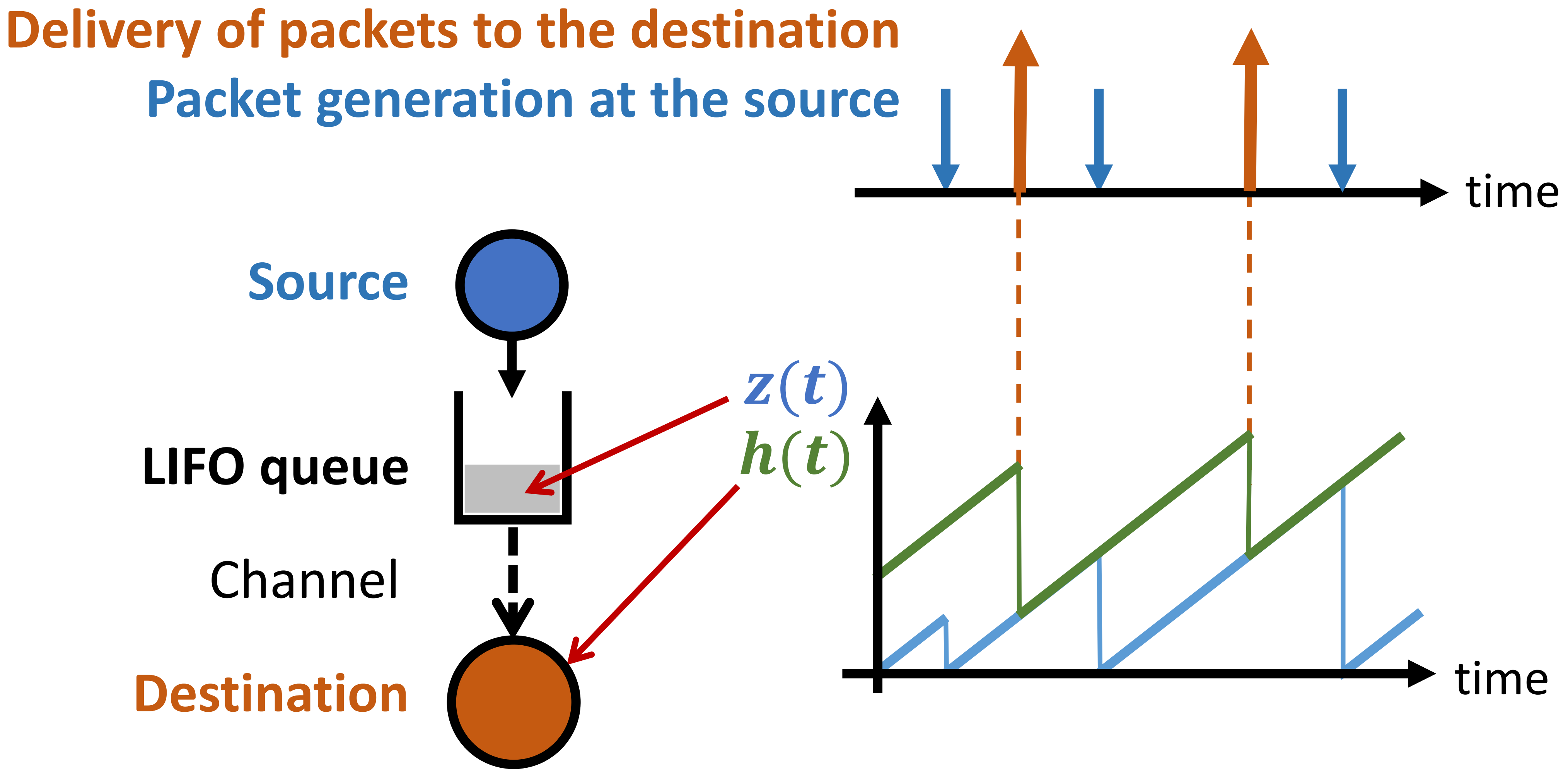

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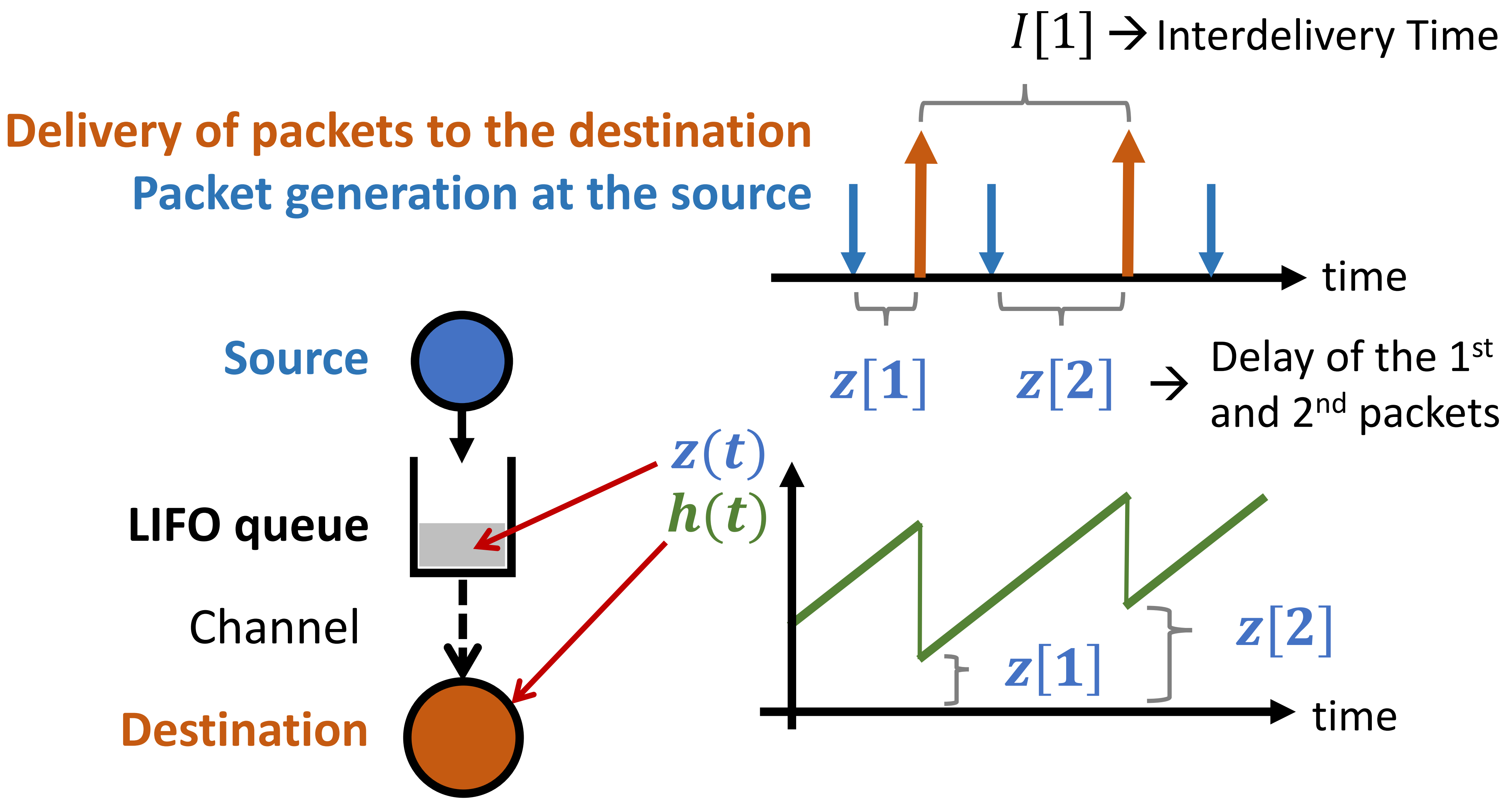
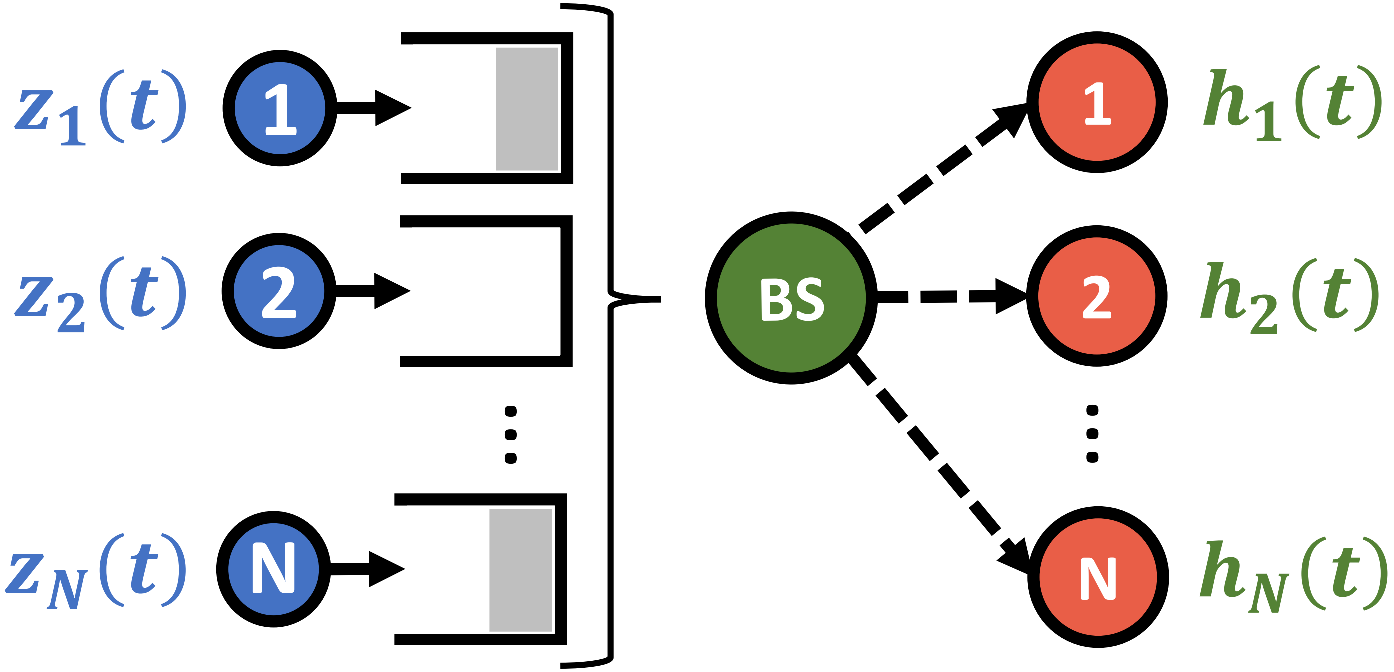


Network Model

Packet **arrivals** from each stream

Unreliable transmissions on a **shared** medium



Scheduling Policy at the Base Station attempts to **minimize the average AoI in the network.**

Literature

	Packet Arrivals			Channel Reliability			Queueing Discipline		
Papers	Active	Arbitr. Given	Stoch.	Reliable	Known	Unreliab.	FIFO	LIFO	Other
Bedewy 19		X		X					X
Sun 18		X		X					X
Hsu 18			X	X					X
Joo 17			X	X			X		
Lu 18			X		X			X	
Kaul 17	X					X		X	
Kadota INF 18	X					X		X	
Kadota ToN 18	X					X		X	
Talak 18	X					X		X	
Talak 18 sec. IV			X			X	X		
This work			**X**			**X**	**X**	**X**	**X**

Literature

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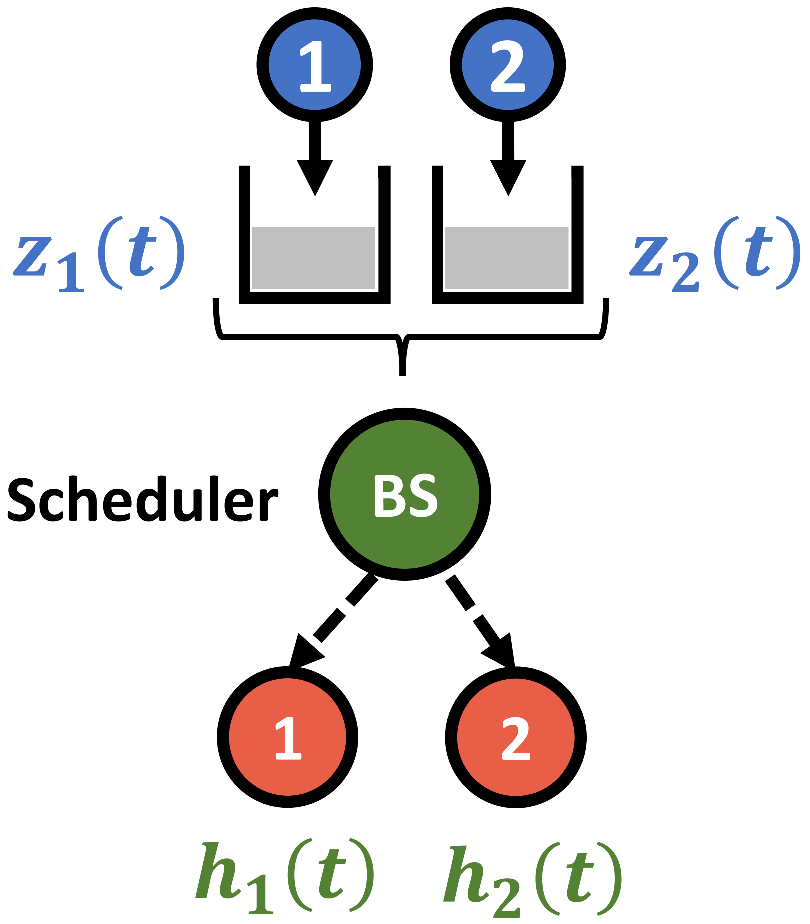
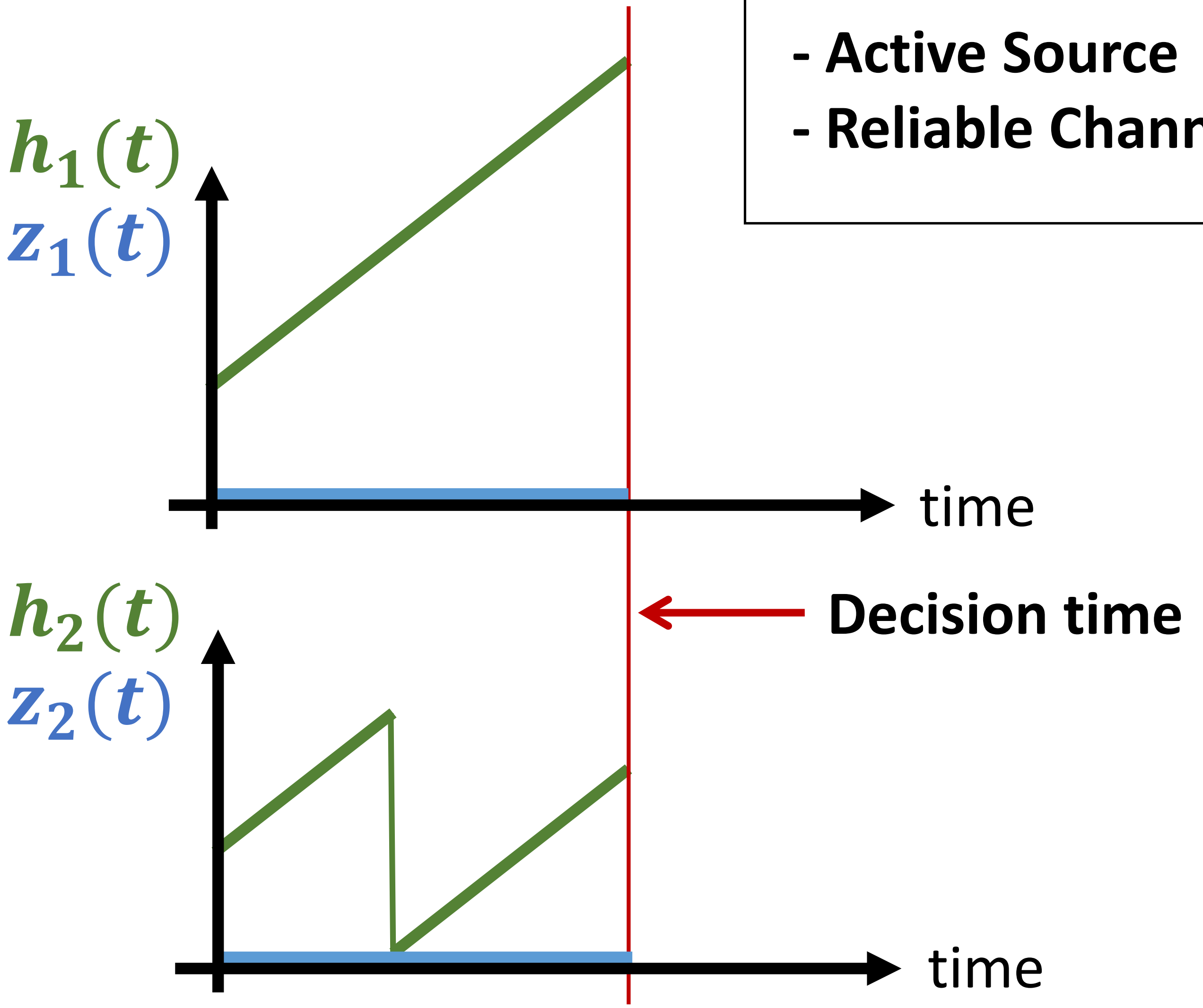
Hsu 18			X	X					X
Joo 17			X	X			X		
Lu 18			X		X			X	
Kaul 17	X					X		X	
Kadota INF 18	X					X		X	
Kadota ToN 18	X					X		X	
Talak 18	X					X		X	
Talak 18 sec. IV			X			X	X		
This work			**X**			**X**	**X**	**X**	**X**

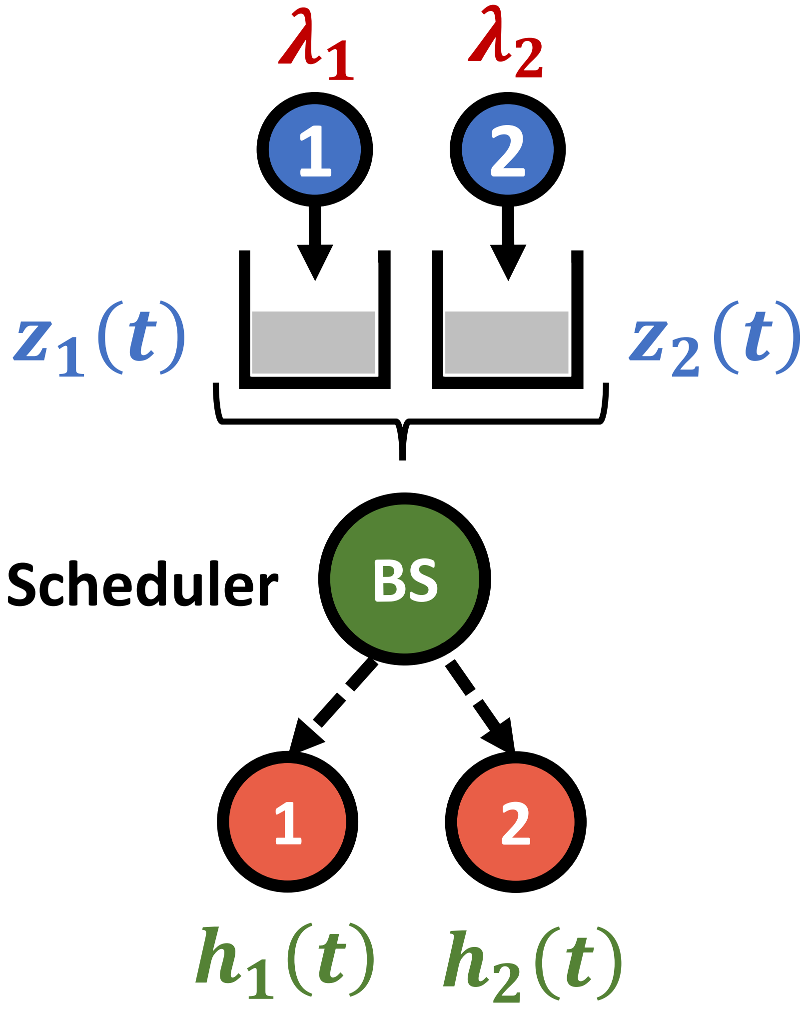
Network Model - Two Streams Example

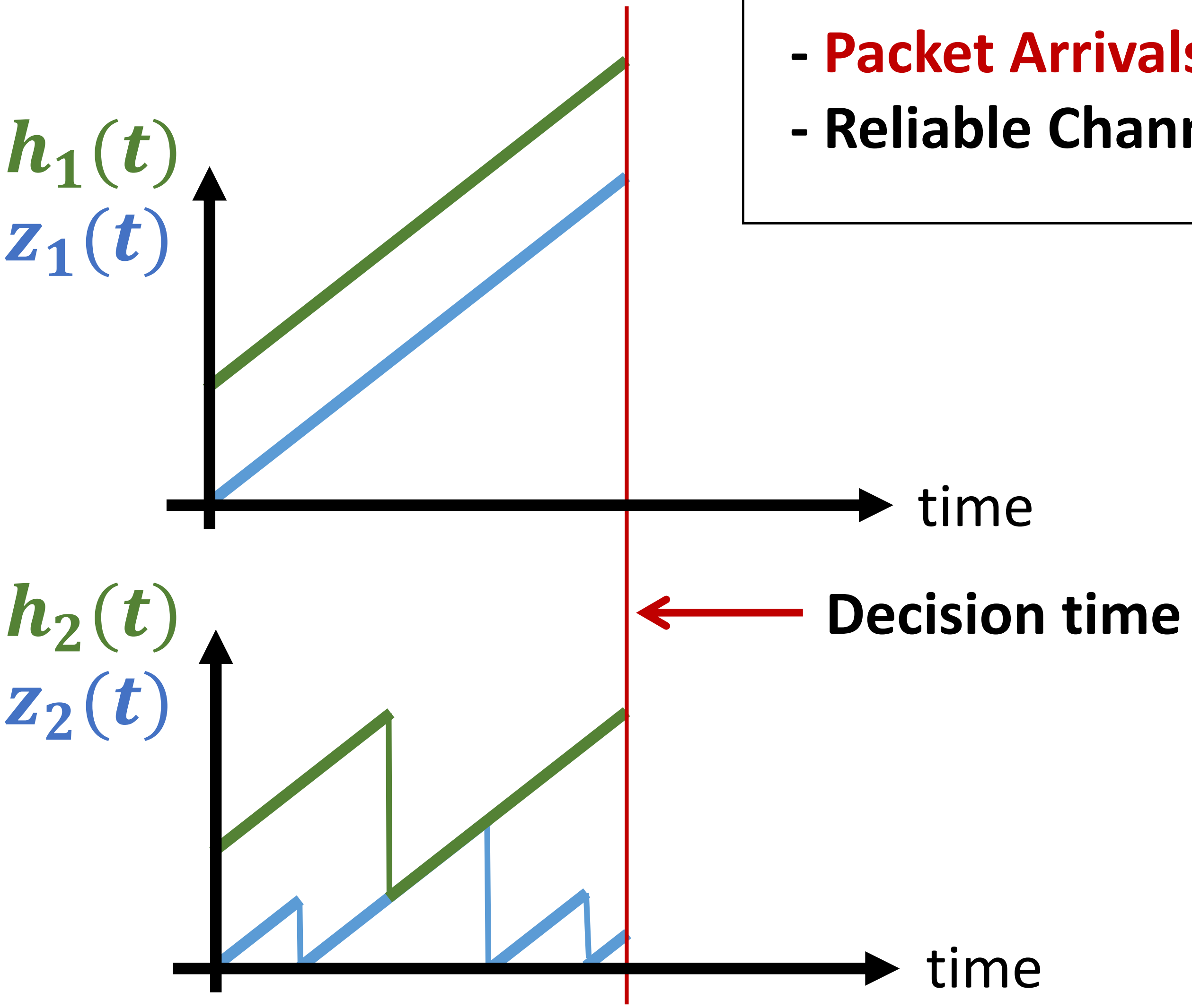




- **Active Source**
- **Reliable Channel**

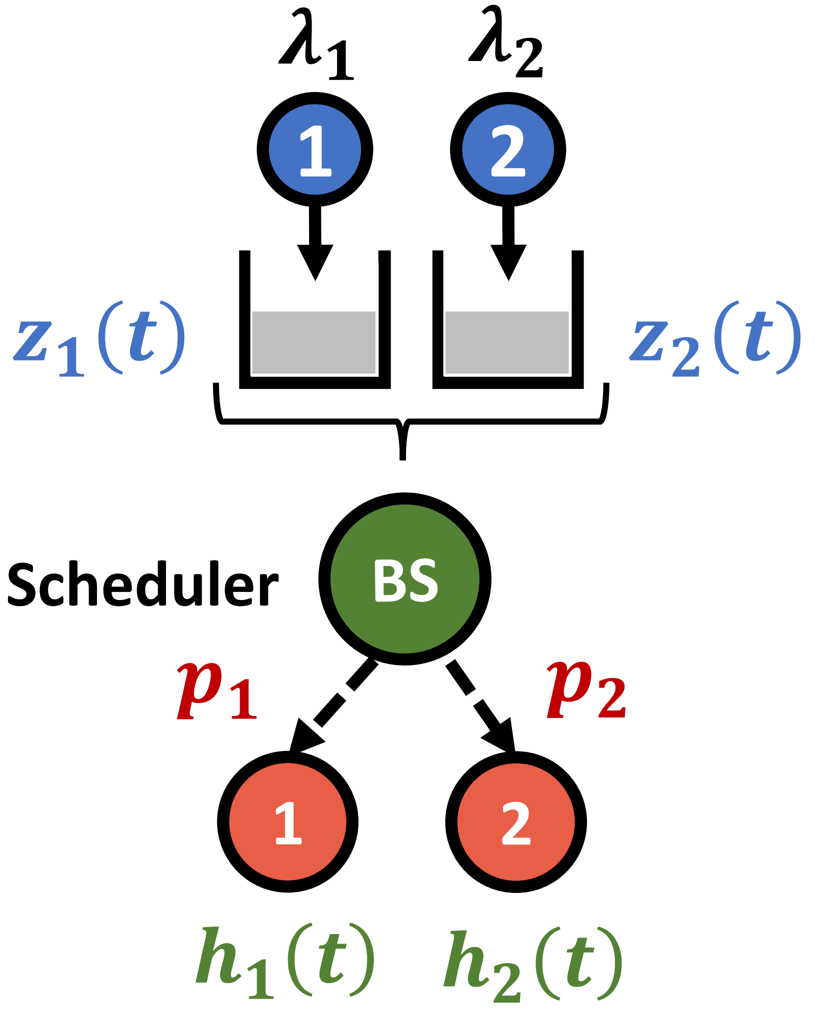
Network Model - Two Streams Example

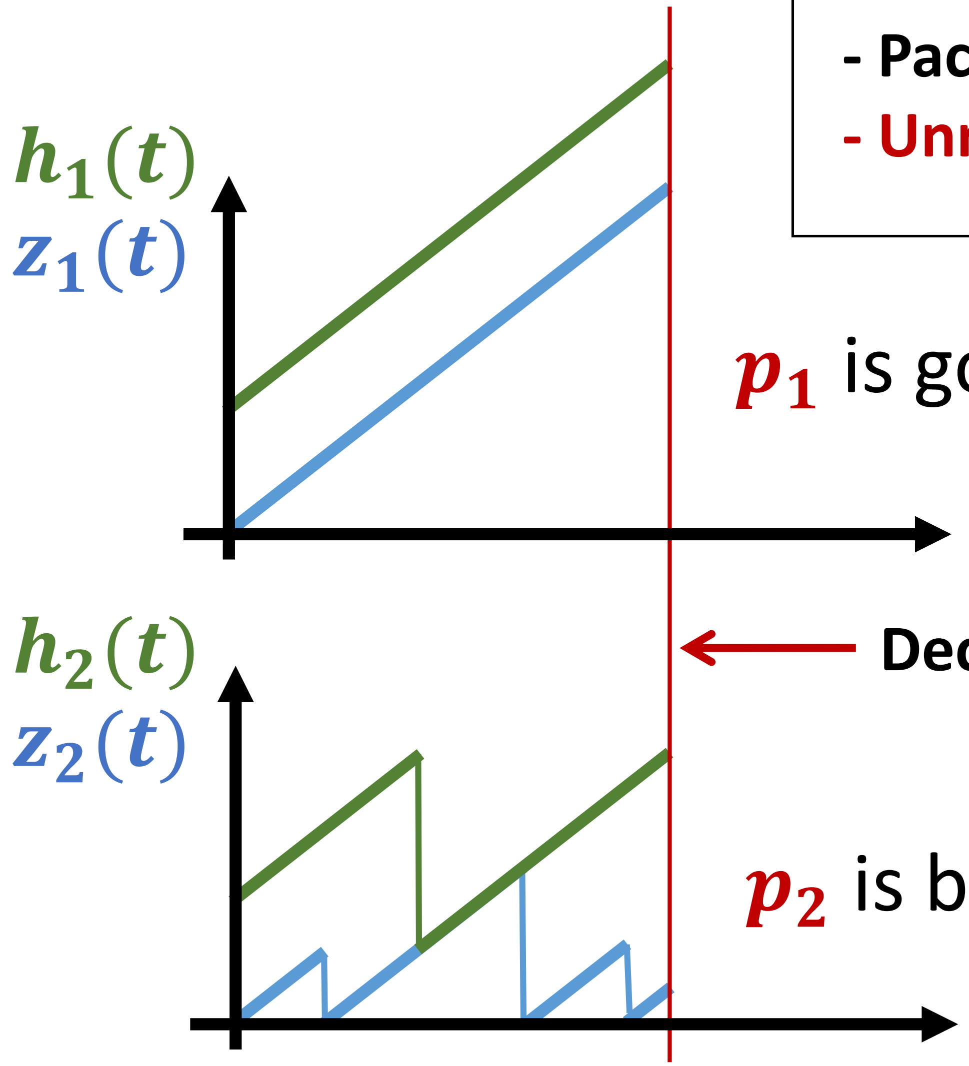




- **Packet Arrivals**
- **Reliable Channel**

Network Model - Two Streams Example





- **Packet Arrivals**
- **Unreliable Channel**

p_1 is good

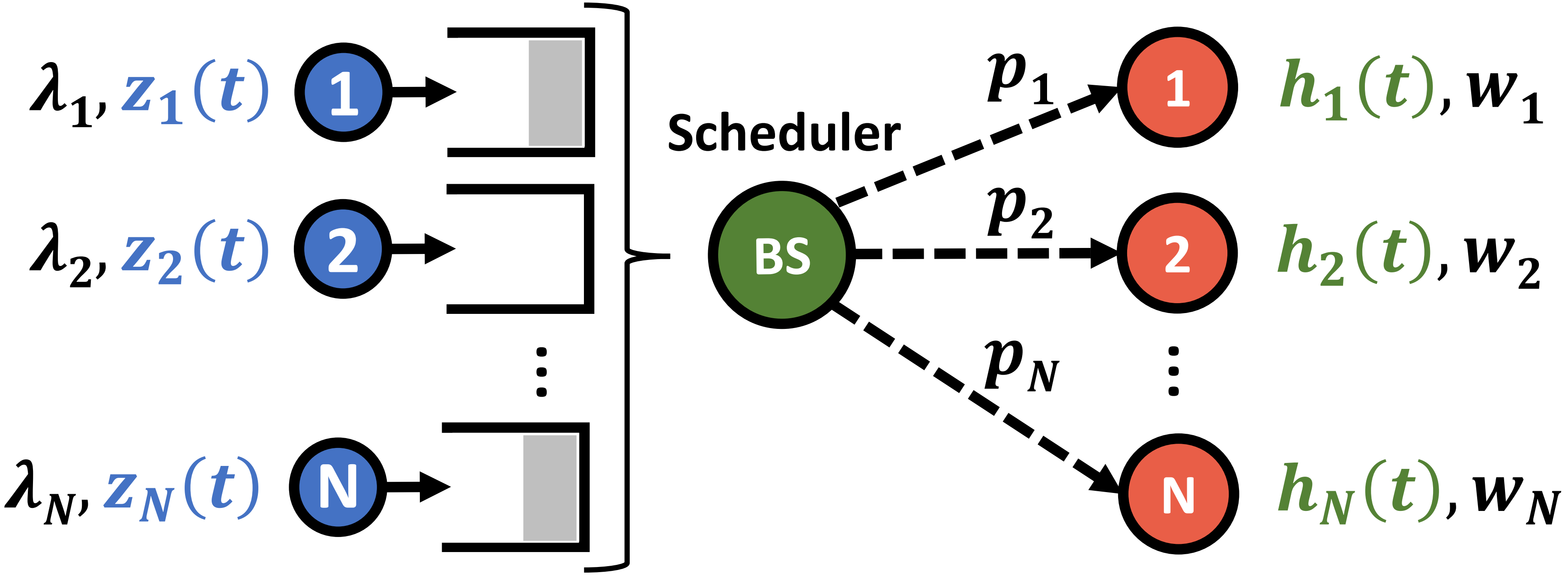
Decision time

p_2 is bad

Network Model

Bernoulli packet arrivals with rate, λ_i

Probability of a successful packet transmission, p_i



Weights w_i represent priority

Values of N, λ_i, w_i, p_i are **fixed and known**. Values of $h_i(t)$ and $z_i(t)$ are known by the BS.

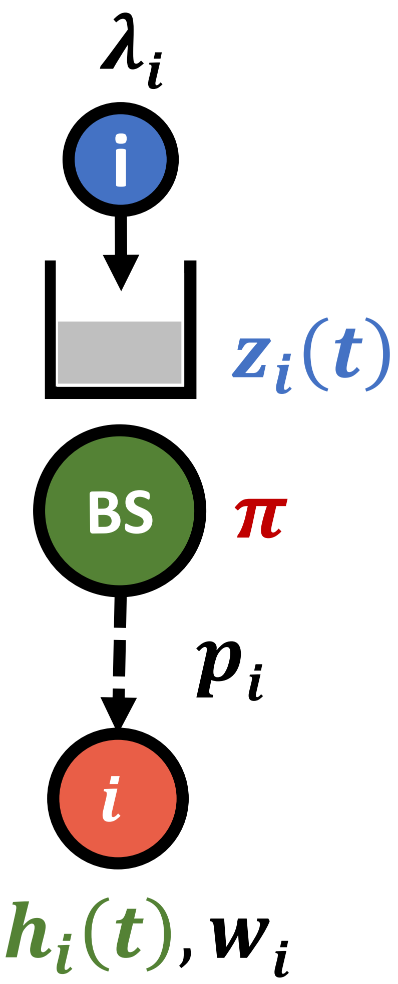
Network Model - Scheduling Policy π

During slot t:

1) A **new packet arrives** to the queue of stream i w.p. $\boldsymbol{\lambda_i}$, $\forall i \in \{1, \ldots, N\}$ $[a_i(t) = 1]$

2) BS runs the transmission scheduling policy π and **selects a single stream i** $[u_i(t) = 1]$

3) HoL packet of stream i is successfully **delivered** to destination i w.p. $\boldsymbol{p_i}$ $[d_i(t) = 1]$



Class of non-anticipative policies Π. **Arbitrary policy $\pi \in \Pi$.**

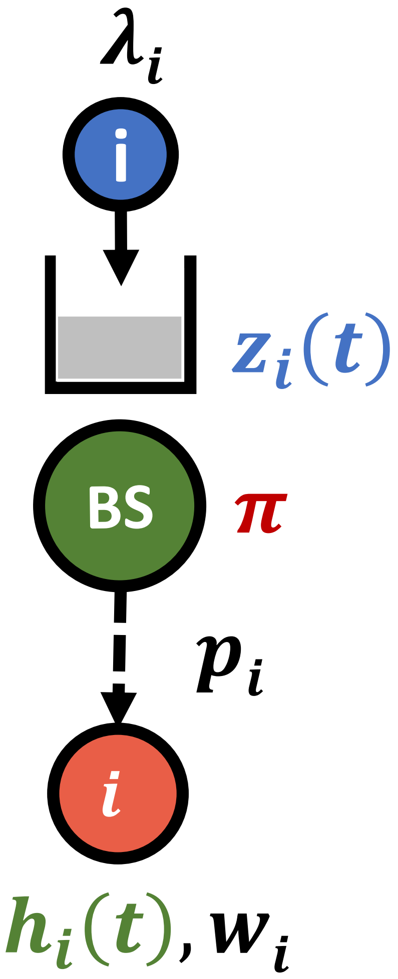
Network Model - Objective Function

- **Expected Weighted Sum AoI** when policy π is employed:

$$\mathbb{E}[J^{\pi}] = \lim_{T\to\infty} \frac{1}{TN} \sum_{t=1}^{T} \sum_{i=1}^{N} \boldsymbol{w_i} \mathbb{E}[\boldsymbol{h_i^{\pi}(t)}], \text{ where}$$

where $\boldsymbol{h_i^{\pi}(t)}$ is the AoI of stream i at the beginning of slot t and $\boldsymbol{w_i}$ is the positive weight.

- **AoI-optimal policy** achieves: $\mathbb{E}[J^{*}] = \min_{\pi\in\Pi} \mathbb{E}[J^{\pi}]$

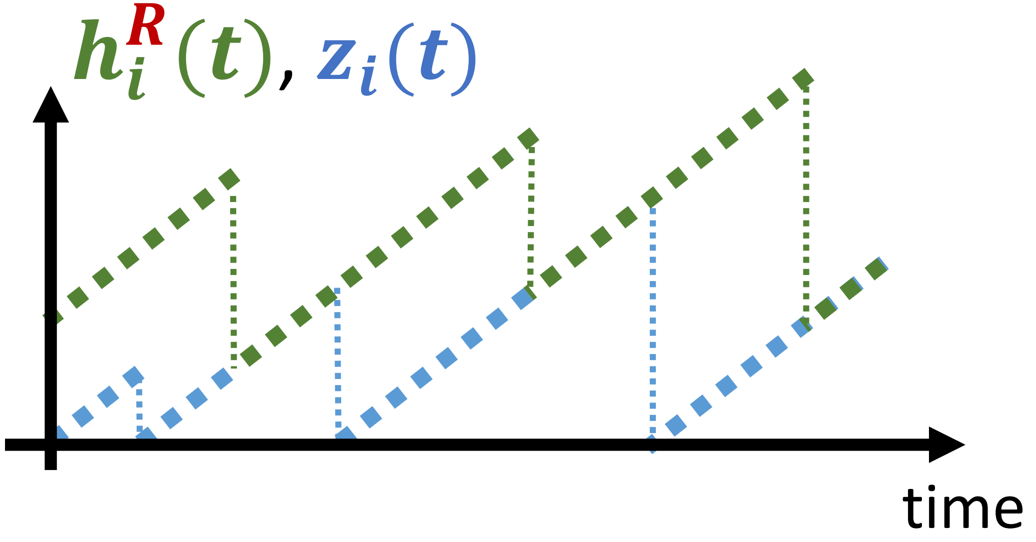


Stationary Randomized Policies

- **Policy R**: in each slot t, select stream i with probability $\boldsymbol{\mu_i} \in (0,1]$
- Sequence of transmission schedules from stream i is a renewal process

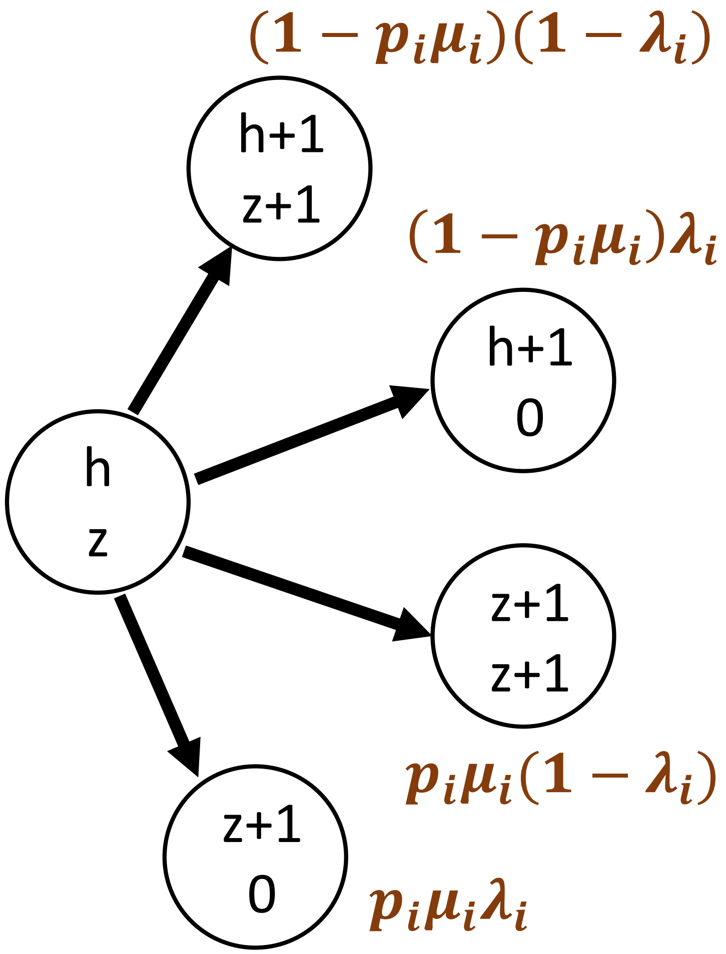
Stationary Randomized Policies

- **Policy R**: in each slot t, select stream i with probability $\mu_i \in (0,1]$
- Sequence of transmission schedules from stream i is a renewal process
- Evolution of AoI is NOT STOCHASTICALLY RENEWED after every packet delivery
- Expression for time-average $\mathbb{E}[h_i^R(t)]$?



Stationary Randomized Policies

- **Policy R**: in each slot t, select stream i with probability $\boldsymbol{\mu_i} \in (0,1]$
- Under policy **R**, $\left(h_i(t), z_i(t)\right)$ evolves as a 2-dim MC with countably-infinite state space

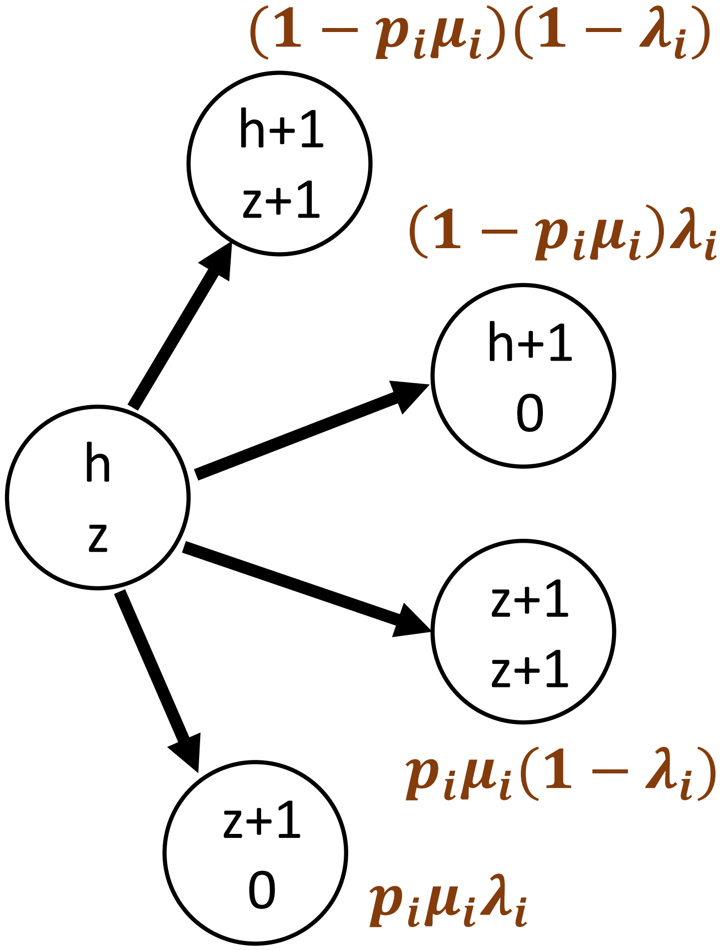


Stationary Randomized Policies

- **Policy R**: in each slot t, select stream i with probability $\boldsymbol{\mu_i} \in (0,1]$
- Under policy **R**, $\left(h_i(t), z_i(t)\right)$ evolves as a 2-dim MC with countably-infinite state space
- Analyzing this MC, we obtain:

$$\mathbb{P}(h) = \lambda_i p_i \mu_i \left[\sum_{n=0}^{h-1} (1-\lambda_i)^{h-1-n} (1-p_i\mu_i)^n \right]$$

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\left[\boldsymbol{h_i^R(t)}\right] = \frac{1}{p_i\mu_i} + \frac{1}{\lambda_i} - 1$$



Stationary Randomized Policies

- Network with parameters (N, p_i, λ_i, w_i) and **LIFO queues:**

Optimal Randomized policy for Single packet queues

$$\mathbb{E}\left[J^{R^S}\right] = \min_{R \in \Pi_R} \left\{ \frac{1}{N} \sum_{i=1}^{N} w_i \left(\frac{1}{\lambda_i} - 1 + \frac{1}{p_i \mu_i} \right) \right\}$$

$$s.t. \ \sum_{i=1}^{N} \mu_i \leq 1 \ ;$$

- **Theorem:** the optimal scheduling probabilities are: $\mu_i \sim \sqrt{w_i / p_i}, \forall i$ and the performance of the optimal policy is such that:

$$\mathbb{E}[J^*] \leq \mathbb{E}\left[J^{R^S}\right] \leq 4\,\mathbb{E}[J^*]$$

Stationary Randomized Policies

- Network with parameters (N, p_i, λ_i, w_i) and **FIFO queues**:

Optimal Randomized policy for FIFO queues

$$\mathbb{E}\left[J^{R^F}\right] = \min_{R \in \Pi_R} \left\{ \sum_{i=1}^{N} \frac{w_i}{N} \left[\frac{1}{p_i\mu_i} + \frac{1}{\lambda_i} + \left[\frac{\lambda_i}{p_i\mu_i}\right]^2 \frac{1-p_i\mu_i}{p_i\mu_i - \lambda_i} \right] \right\}$$

$$\text{s.t. } \sum_{i=1}^{N} \mu_i \leq 1\ ;$$

$$p_i\mu_i > \lambda_i, \forall i\ .$$

- **Theorem:** the optimal scheduling probabilities are given by Algorithm 2 which uses the *bisection method* to find the set of μ_i^*. [details are omitted]

Max-Weight Policy for any queueing discipline

- Lyapunov Function: $L(t)$ high value at undesirable states
- Lyapunov Drift: $\Delta(t) = \mathbb{E}\{L(t+1) - L(t)\}$
- Max-Weight policy attempts to minimize $\Delta(t)$ at every slot t

Max-Weight Policy for any queueing discipline

- Lyapunov Function: $L(t) = \frac{1}{N}\sum_{i=1}^{N} \beta_i \boldsymbol{h_i(t)}$, where $\beta_i > 0$ is a constant
- Lyapunov Drift: $\Delta(t) = \mathbb{E}\{\, L(t+1) - L(t) \,|\boldsymbol{h_i(t)}, \boldsymbol{z_i(t)}\}$
- Substituting the expression for the evolution of $\boldsymbol{h_i(t+1)}$ into the drift:

$$\Delta(t) = -\frac{1}{N}\sum_{i=1}^{N} \beta_i \boldsymbol{p_i}\,(\boldsymbol{h_i(t)} - \boldsymbol{z_i(t)})\mathbb{E}[\boldsymbol{u_i(t)}|\boldsymbol{h_i(t)}, \boldsymbol{z_i(t)}] + \frac{1}{N}\sum_{i=1}^{N} \beta_i$$

- **MW policy:** in slot t, schedule stream ($u_i(t) = 1$) with highest value of:

$$\beta_i \boldsymbol{p_i}\,(\boldsymbol{h_i(t)} - \boldsymbol{z_i(t)})$$

Max-Weight Policy for any queueing discipline

- **MW policy:** in slot t, schedule stream ($u_i(t) = 1$) with highest value of:

$$\beta_i \boldsymbol{p_i} \left(\boldsymbol{h_i(t)} - \boldsymbol{z_i(t)}\right)$$

- For different queueing disciplines substitute the corresponding $\mathbf{z_i(t)}$
- $\boldsymbol{p_i} \left(\boldsymbol{h_i(t)} - \boldsymbol{z_i(t)}\right)$ represents the expected AoI reduction from selecting i

- **Theorem:** consider a network employing LIFO queues. The performance of MW policy when $\beta_i = \sqrt{w_i/p_i}, \forall i$ is such that:

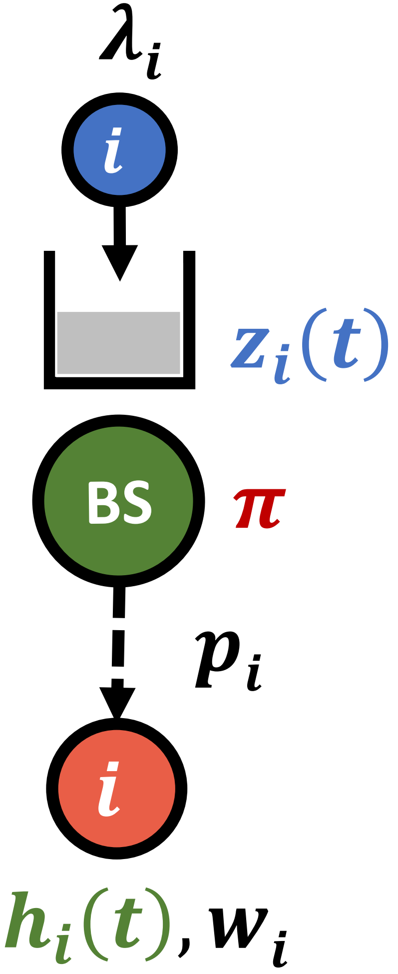
$$\mathbb{E}\left[J^{MW^S}\right] \leq \mathbb{E}\left[J^{R^S}\right]$$

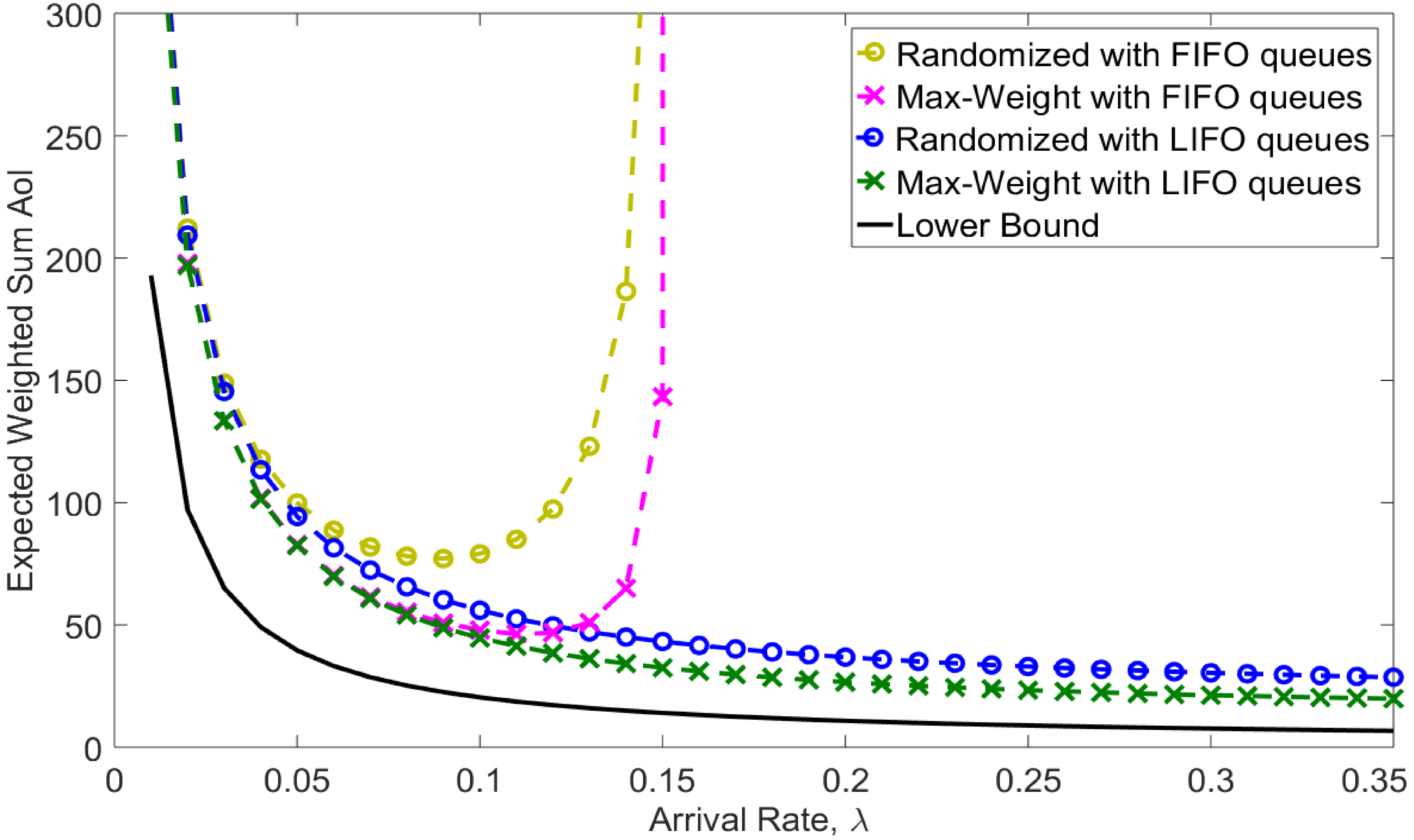
Numerical Results

- Metric:
  - Expected Weighted Sum AoI : $\mathbb{E}[\, J^{\pi} \,]$

- Network setup with $N = 4$ streams. Stream i has:
  - channel reliability $\boldsymbol{p_i} = i/N$
  - arrival rate $\boldsymbol{\lambda_i} = \lambda \times (N + 1 - i)/N$
  - weights: $w_1 = w_2 = 4$ and $w_3 = w_4 = 1$

- Arrival rate in the range $\lambda \in \{0.01, 0.02, \ldots, 0.35\}$
- Each simulation runs for $T = 2 \times 10^6$ slots
- Each data point is an average over 10 simulations



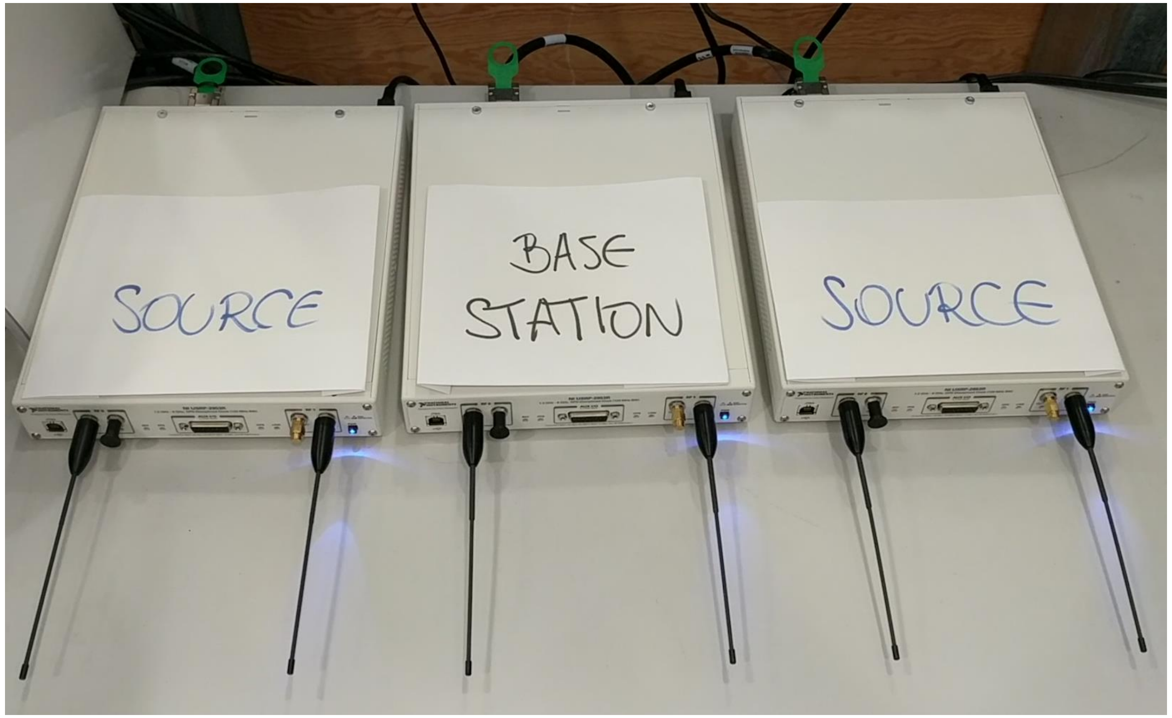

Randomized with FIFO queues
Max-Weight with FIFO queues
Randomized with LIFO queues
Max-Weight with LIFO queues
Lower Bound
Expected Weighted Sum AoI
Arrival Rate, λ
300
250
200
150
100
50
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

Video of Testbed

Current work on the AoI scheduling problem.

Testbed implementation is not complete. There are a few missing parts/tests.

Video shows a short demo.



Network Setup:

- Two sources generating packets
  - Bernoulli arrivals
  - Single packet queue discipline
- Sources send packets to Base station according to Greedy Policy on $h_i(t)$

Final Remarks

In this presentation:

- Developed scheduling policies for wireless networks with stochastic arrivals, unreliable channels and different queueing disciplines
- Described performance guarantees
- Conclusion: **Max-Weight** with **LIFO queues** has superior performance.

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Questions? Thank you!