Optimizing Age of Information in Wireless Networks with Throughput Constraints

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Outline

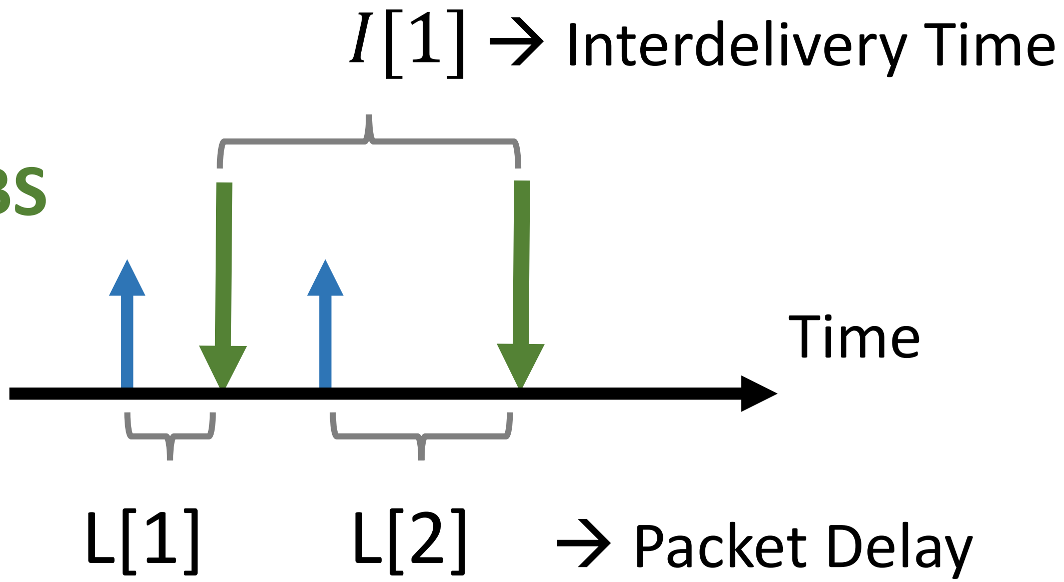
- Age of Information and Motivation
- Network Model
- **Scheduling Policies and Performance Guarantees**
  - **Stationary Randomized Policy**
  - **Max-Weight Policy**
  - **Whittle’s Index Policy**
- Numerical Results

Age of Information (AoI)

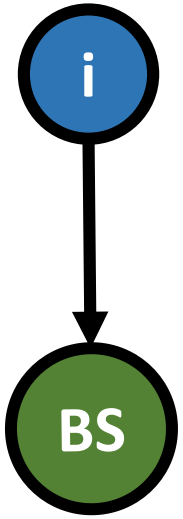
Example:

Delivery of Packets to BS

Packet Generation at i



Single Source

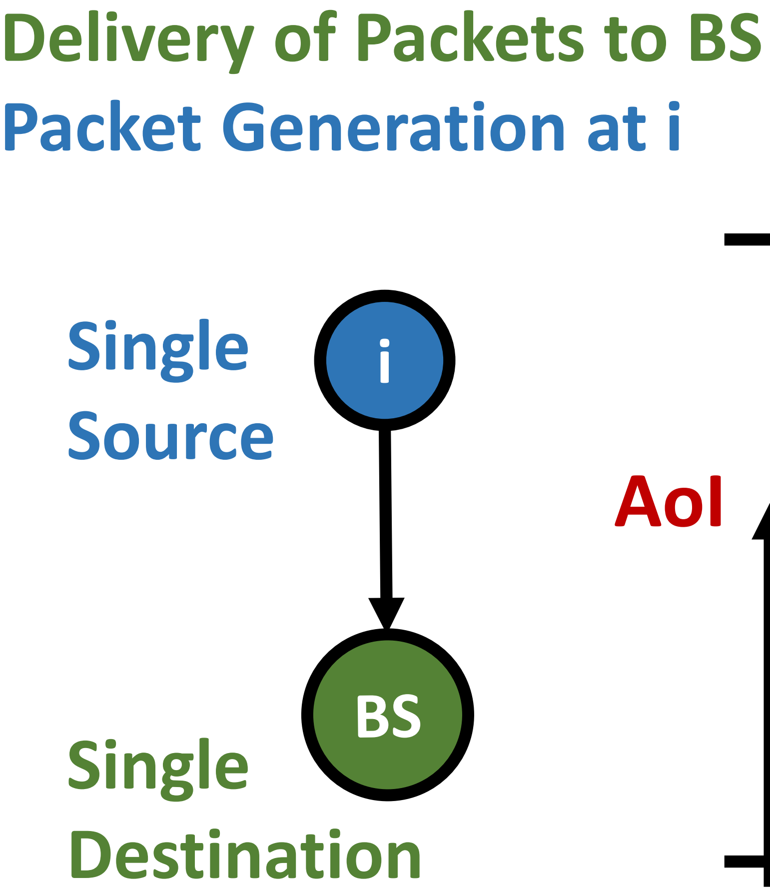


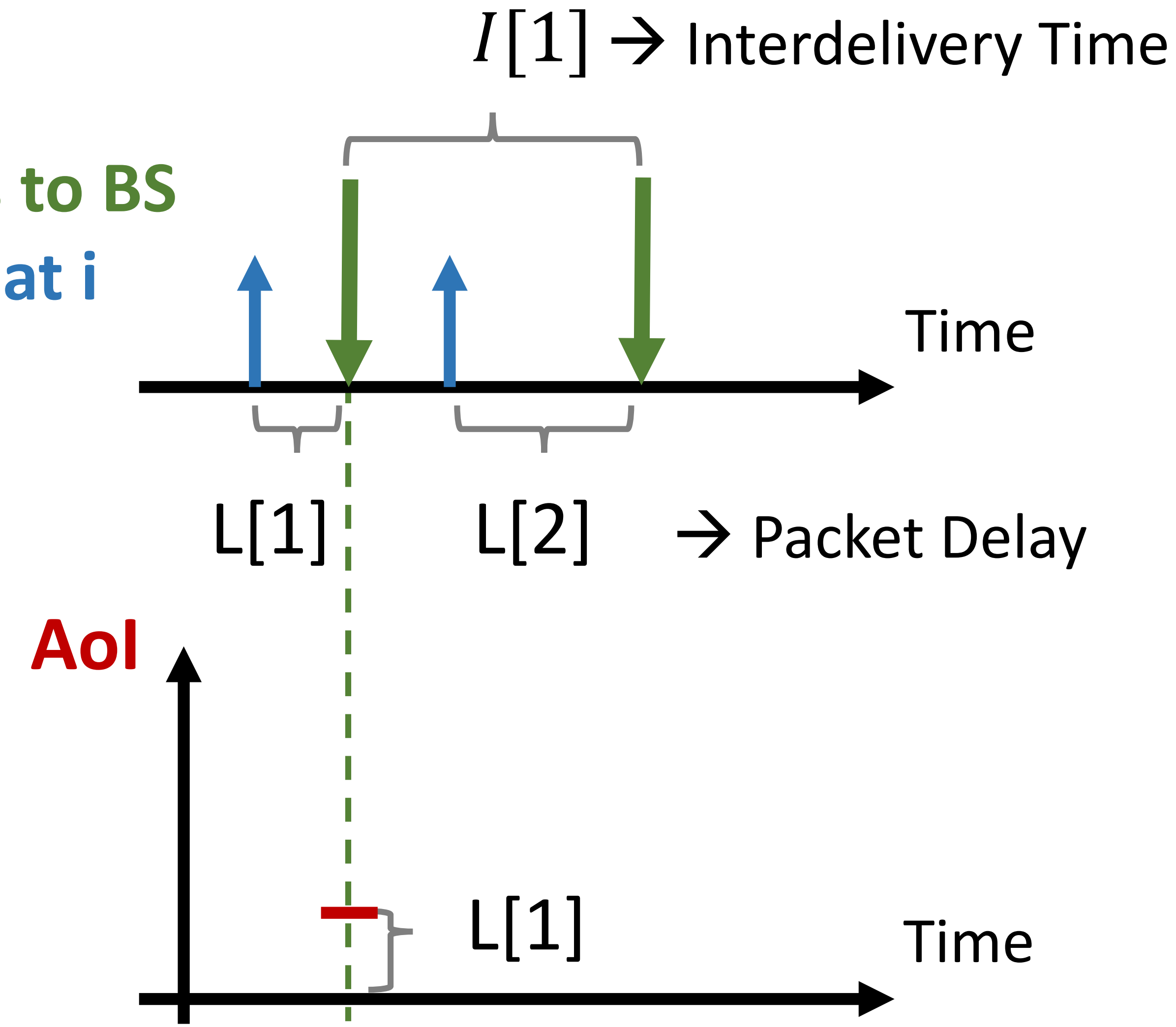
Single Destination

How fresh is the information at the destination?

Age of Information (AoI)

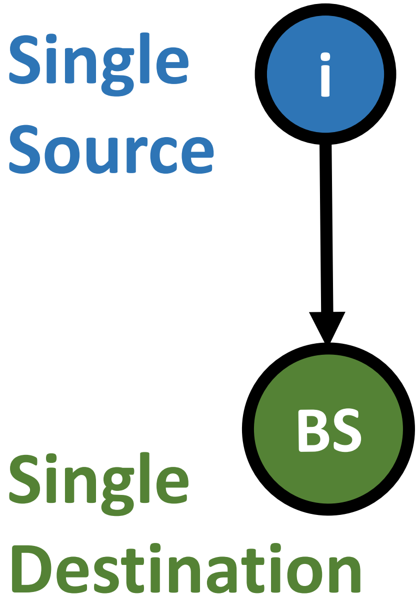
Example:

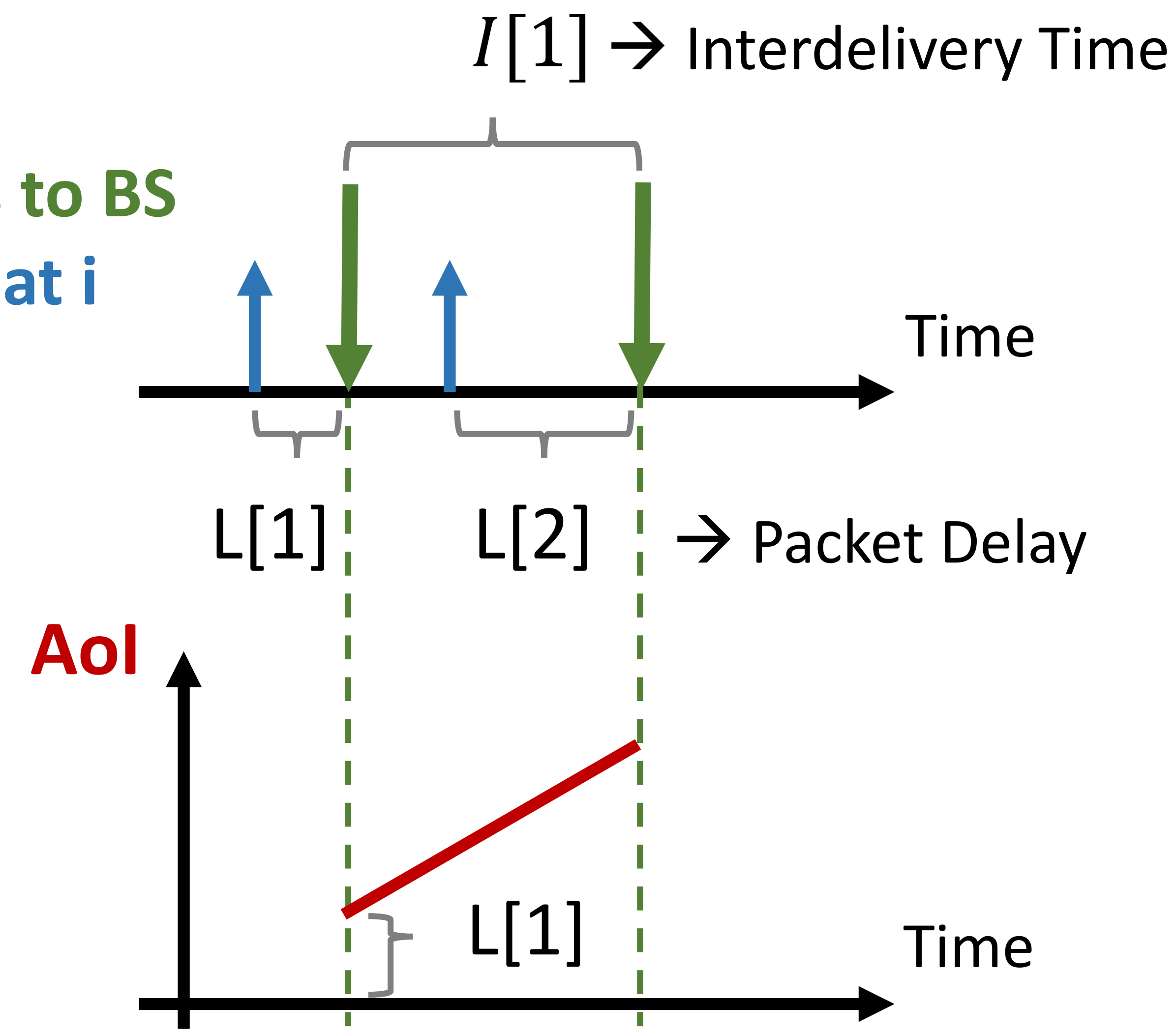




Age of Information (AoI)

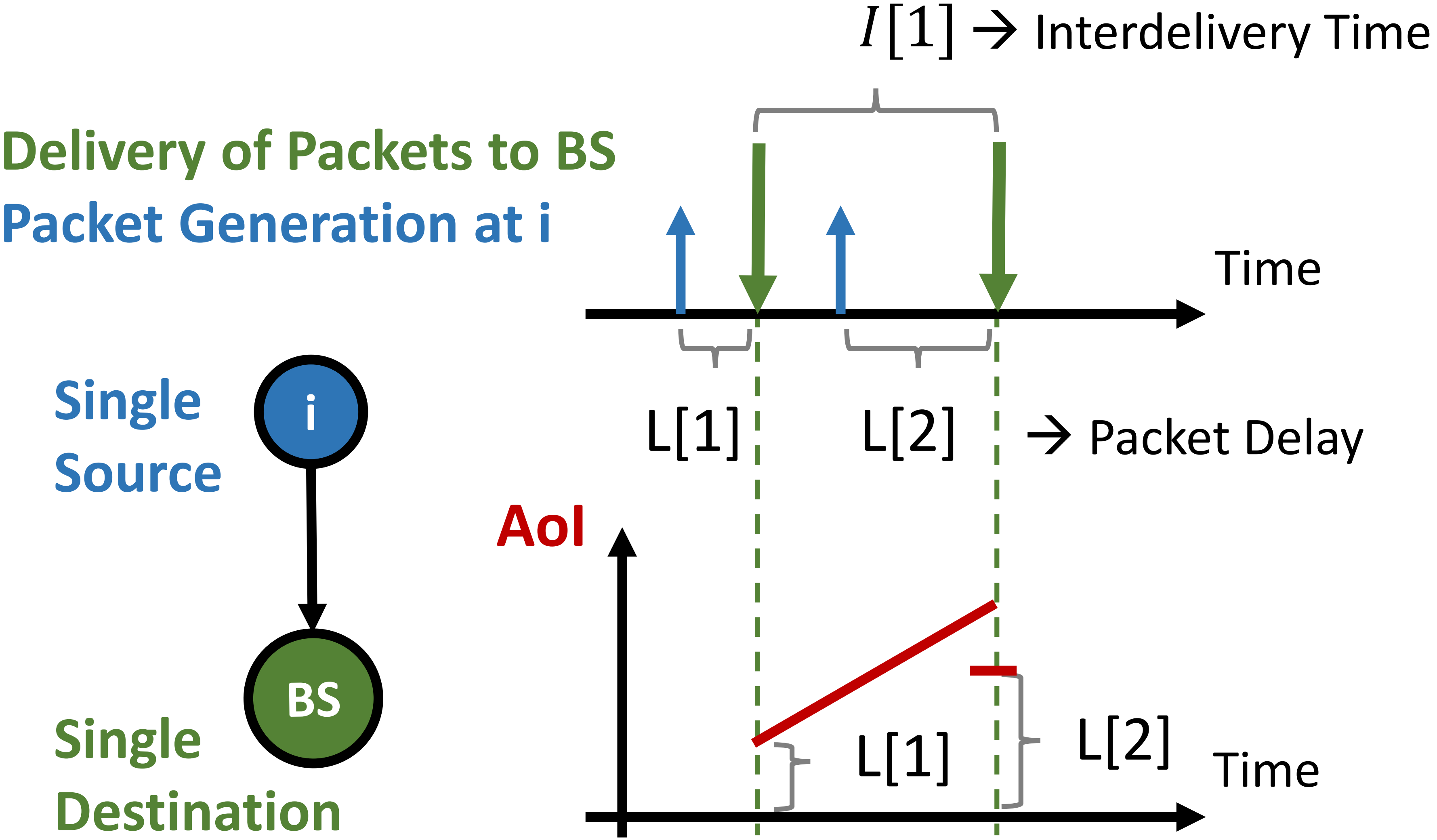
<u>Example</u>:





Age of Information (AoI)

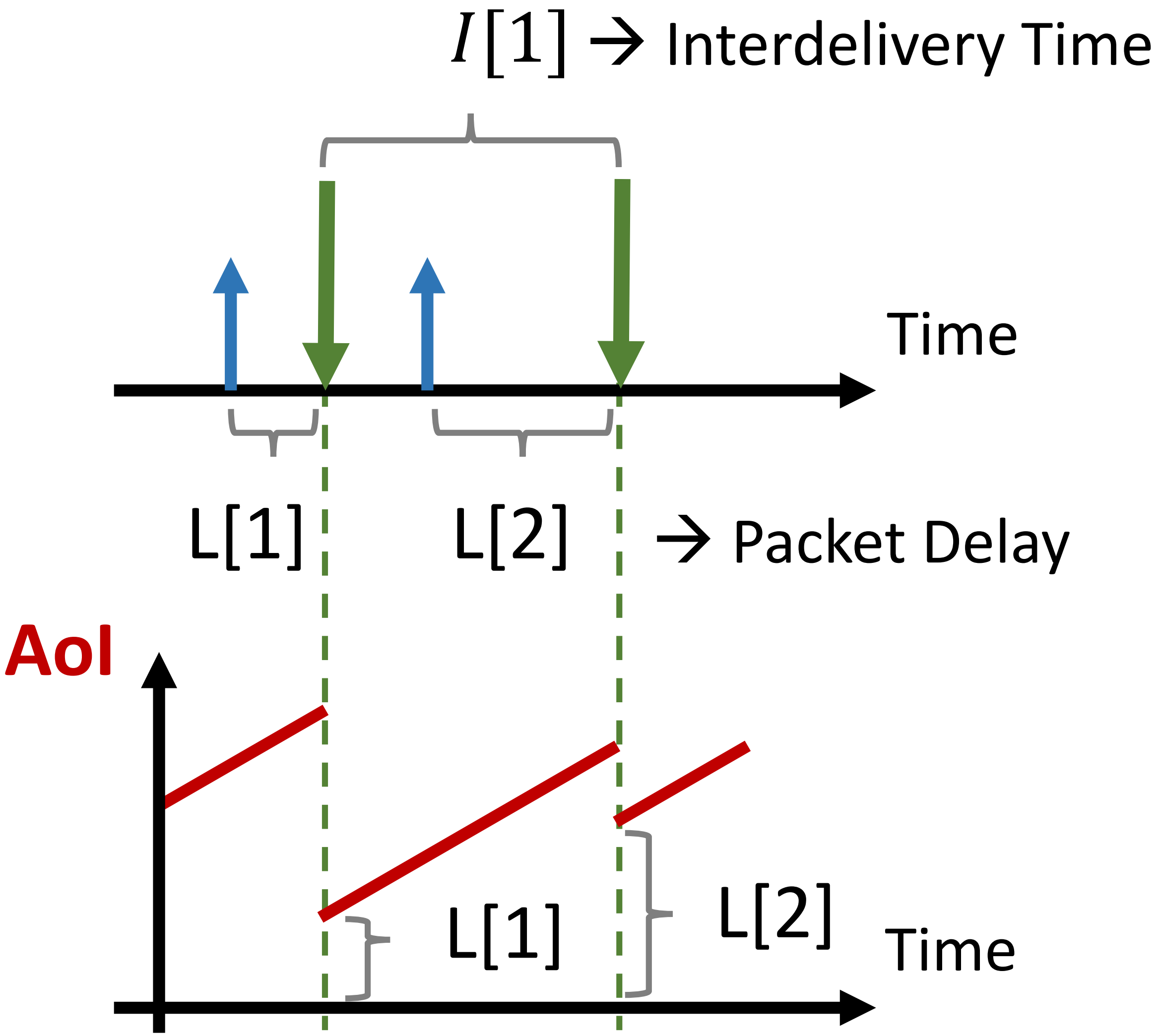
Example:



Age of Information (AoI)

AoI: time elapsed since the most recently delivered packet was generated.

Relation between AoI, delay and interdelivery time?

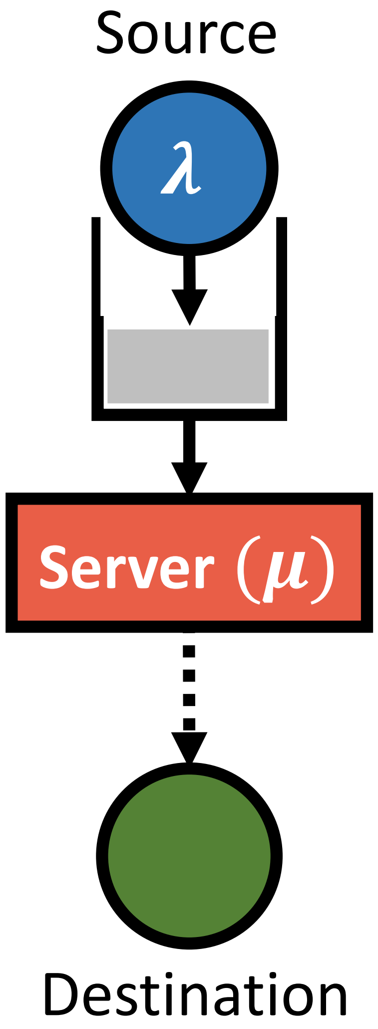


AoI, Delay and Interdelivery time

- Example: (M/M/1): (∞/FIFO) system

  Controllable arrival rate λ and fixed service rate $\mu = 1$ packet per second.

λ	$\mathbb{E}[delay]$	$\mathbb{E}[interdel.]$	Average AoI
0.01	1.01	**100.00**	
0.53	2.13	1.89	
0.99	**100.00**	1.01	



[1] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?", 2012.

AoI, Delay and Interdelivery time

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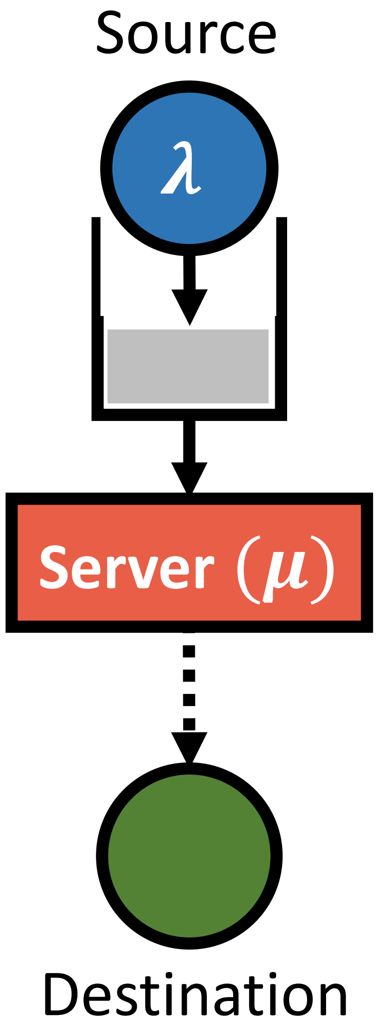
λ	$\mathbb{E}[delay]$	$\mathbb{E}[interdel.]$	Average AoI
0.01	1.01	**100.00**	**101.00**
0.53	2.13	1.89	**3.48**
0.99	**100.00**	1.01	**100.02**



Low time-average AoI when packets with **<u>low delay</u>** are **<u>delivered regularly</u>**.

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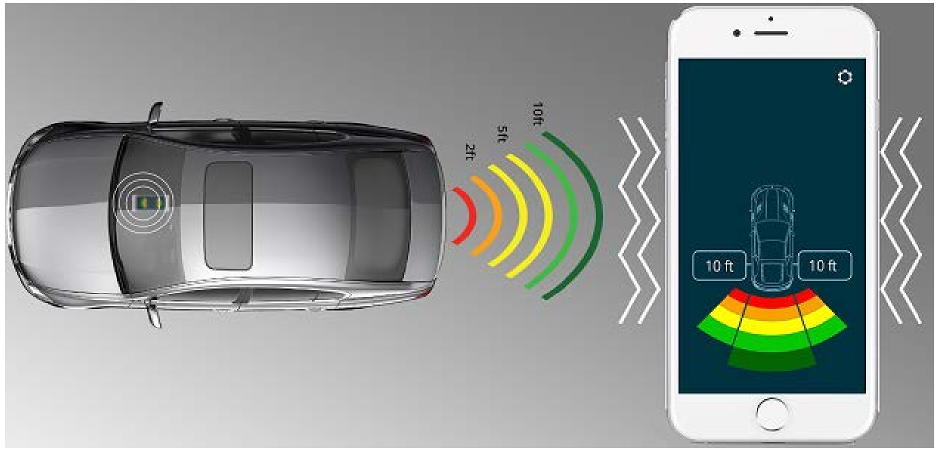
Network - Example

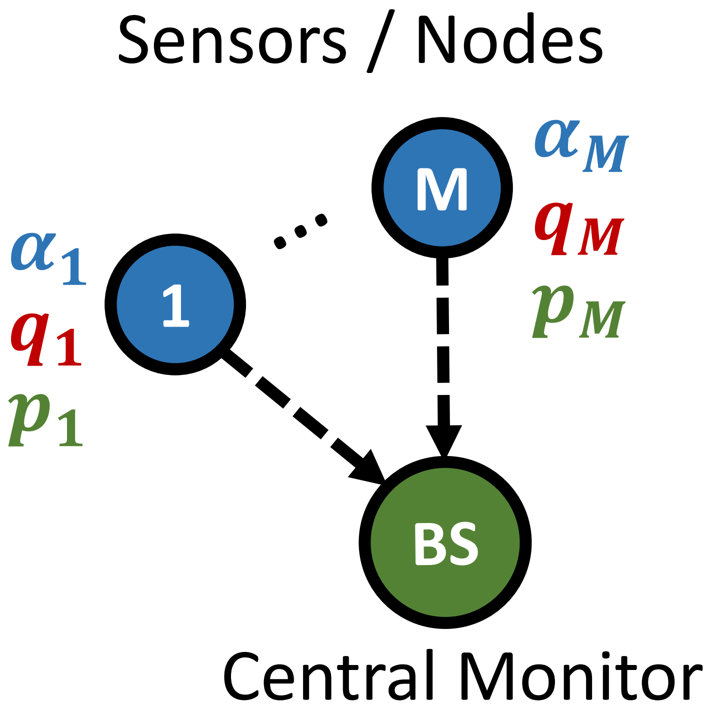
Wireless Parking Sensor



Wireless Rearview Camera

Wireless Tire-Pressure Monitoring System

Network - Description

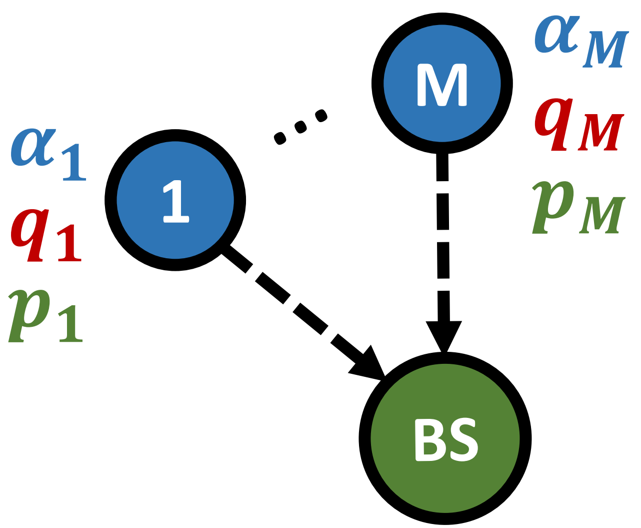


1) Low network-wide AoI
2) Weights α_i represent priority
3) Minimum throughput requirement, q_i
4) Channel is **shared** and **unreliable,** p_i

Values of M, α_i, q_i, p_i are fixed and known

Network - Scheduling Policy π

Sensors / Nodes



Central Monitor
Running policy $\boldsymbol{\pi}$

During slot k:

1) **BS selects** a single node i $[u_i(k) = 1]$
2) Selected node **samples** new data and then **transmits**
3) Packet is successfully **delivered** to the BS with probability p_i $[d_i(k) = 1]$
4) Packet Delay = 1 slot

Class of non-anticipative policies Π. **Arbitrary policy $\boldsymbol{\pi} \in \boldsymbol{\Pi}$.**

Network - Age of Information

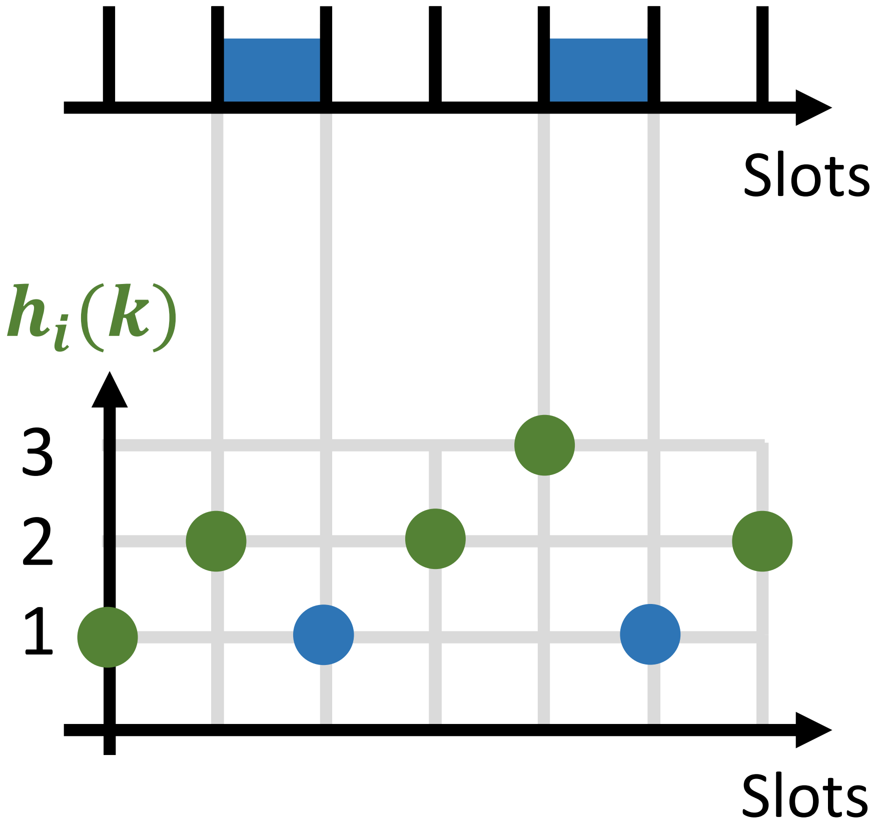
- Age of Information associated with node i at the beginning of slot k is given by $h_i(k)$.

- **Recall:** Packet Delay = 1 slot

- Evolution of AoI:

$$\mathbf{h_i}(k+1) = \begin{cases} 1, & \text{if } d_i(k) = 1 \\ h_i(k) + 1, & \text{otherwise} \end{cases}$$

Delivery of packets from sensor i to the BS



Network - Objective Function

- **Expected Weighted Sum AoI** when policy π is employed:

$$\mathbb{E}[\, J_K^{\pi} \,] = \frac{1}{KM}\mathbb{E}\left[\sum_{k=1}^{K}\sum_{i=1}^{M}\boldsymbol{\alpha_i h_i^{\pi}(k)}\right]$$

, where $\boldsymbol{h_i^{\pi}(k)}$ is the AoI of node i and $\boldsymbol{\alpha_i}$ is the positive weight

- **Minimum Throughput Requirements**:

$$\hat{q}_i^{\pi} := \lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\mathbb{E}[\boldsymbol{d_i(k)}] \geq \boldsymbol{q_i}\,, \forall i \in \{1,2,\dots,M\},$$

where set $\{\boldsymbol{q_i}\}_{i=1}^{M}$ is feasible.

- **Channel Interference:** $\sum_{i=1}^{M}\boldsymbol{u_i(k)} \leq 1\,, \forall k \in \{1,2,\dots,K\}$

Scheduling Policies

AoI Optimization

$$\mathrm{OPT}^* = \min_{\pi\in\Pi}\left\{\lim_{K\to\infty}\frac{1}{KM}\mathbb{E}\left[\sum_{k=1}^{K}\sum_{i=1}^{M}\alpha_i h_i(k)\right]\right\} \quad (8a)$$

(Age of Information)

$$\text{s.t. } \hat{q}_i^{\pi} \geq q_i \;, \forall i \;; \quad (8b)$$

(Minimum Throughput)

$$\textstyle\sum_{i=1}^{M} u_i(k) \leq 1 \;, \forall k \;. \quad (8c)$$

(Channel Interference)

- Policy π^* that solves (8) is **AoI-optimal** and achieves OPT^*

- Policy $\eta \in \Pi$ that attains OPT_η is ψ-optimal when $OPT^* \leq OPT_\eta \leq \psi OPT^*$

Scheduling Policies

Summary of Results:

Scheduling Policy	**Technique**	**Optimality Ratio**	**Simulation Result**
Optimal Stationary Randomized Policy	Renewal Theory	2-optimal	~ 2-optimal
Max-Weight Policy	Lyapunov Optimization	4-optimal	close to optimal
Whittle's Index Policy	RMAB Framework	8-optimal	close to optimal

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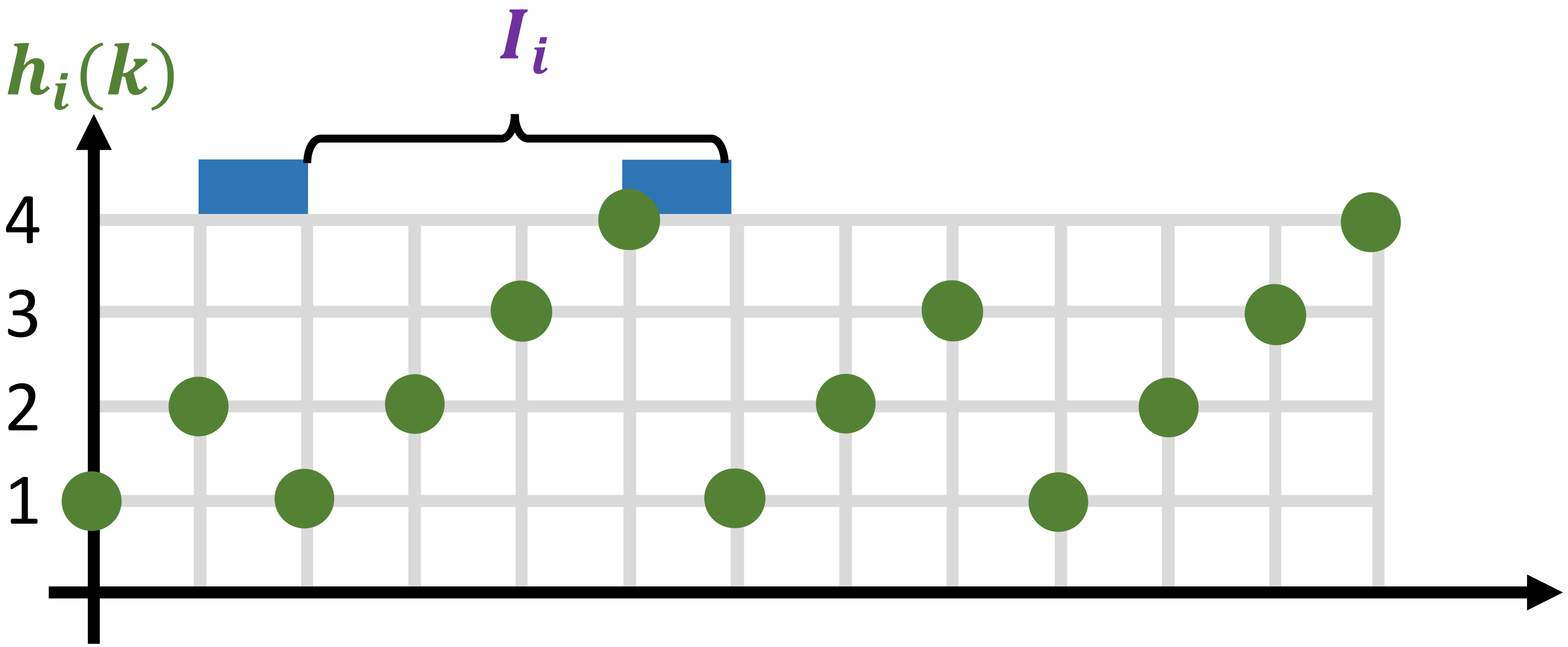
Stationary Randomized Policies

- **Policy R**: in each slot k, select node i with probability $\mu_i \in (0,1]$.

- Packet deliveries from node i is a renewal process with $I_i \sim Geo(\mu_i p_i)$
- Sum of $h_i(k)$ over time is a **renewal-reward process.**



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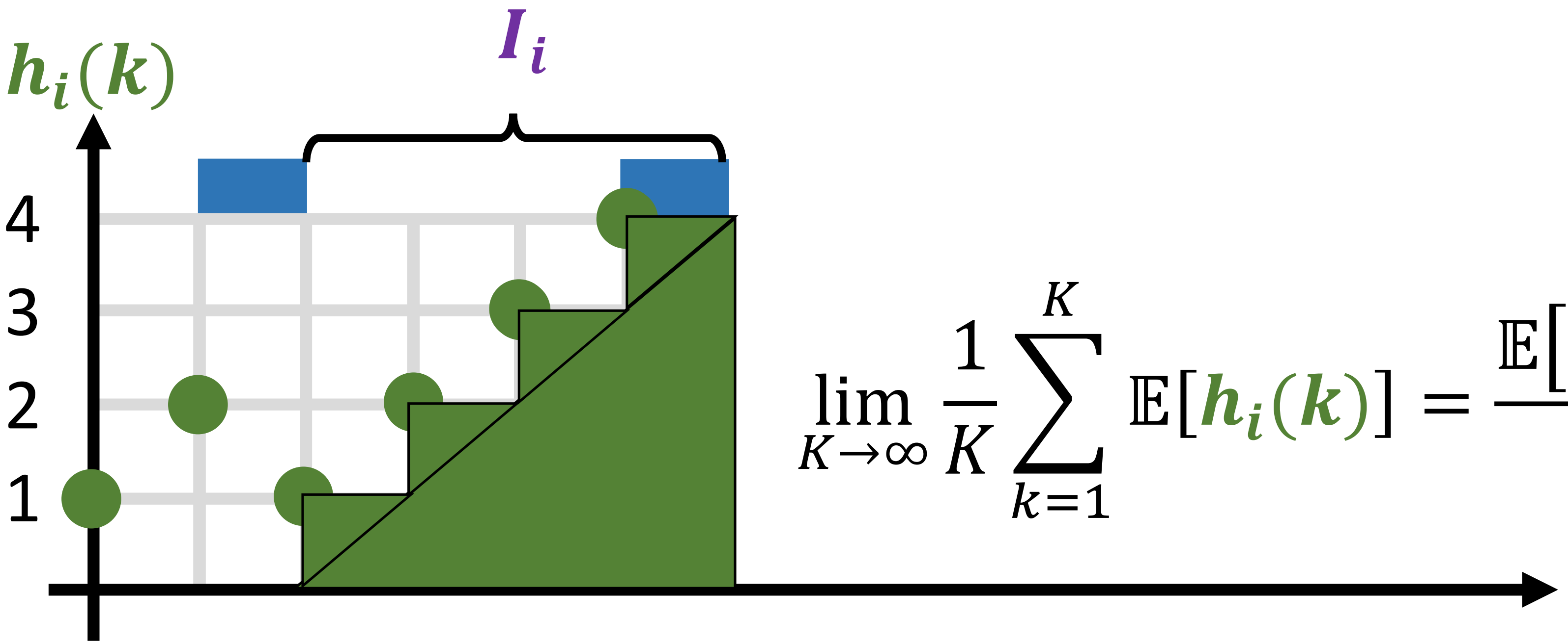

$$\lim_{K\to\infty} \frac{1}{K}\sum_{k=1}^{K} \mathbb{E}[h_i(k)] = \frac{\mathbb{E}\left[I_i^2/2 + I_i/2\right]}{\mathbb{E}[I_i]} = \frac{1}{\mu_i p_i}$$

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Optimization over Randomized policies

$$\mathrm{OPT}_{R^*} = \min_{R \in \Pi_R} \left\{ \frac{1}{M} \sum_{i=1}^{M} \frac{\alpha_i}{p_i \mu_i} \right\} \tag{25a}$$

(Age of Information)

$$\text{s.t. } p_i \mu_i \geq q_i \, , \forall i \, ; \tag{25b}$$

(Minimum Throughput)

$$\textstyle\sum_{i=1}^{M} \mu_i \leq 1 \, , \forall k \, . \tag{25c}$$

(Probability)

Stationary Randomized Policies

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- Optimal Stationary Randomized Policy **R*** uses probabilities $\{\boldsymbol{\mu_i^*}\}_{\mathrm{i}=1}^{\mathrm{M}}$ that can be obtained offline with Algorithm 1 (omitted in this presentation).

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Theorem: for **any** network configuration, policy **R*** is **2-optimal**.

Scheduling Policies

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Max-Weight Policy

- Minimum Throughput Requirements:

$$\hat{q}_i^{\pi} := \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} \mathbb{E}[\boldsymbol{d_i(k)}] \geq \boldsymbol{q_i}, \forall i \in \{1,2,\dots,M\}, \text{ where set } \{\boldsymbol{q_i}\}_{i=1}^{M} \text{ is feasible.}$$

- Throughput debt: $\boldsymbol{x_i(k+1)} = k\boldsymbol{q_i} - \sum_{t=1}^{k} \boldsymbol{d_i(t)}$

- Lyapunov Function: $L(k) \coloneqq \frac{1}{2} \sum_{i=1}^{M} \left(\boldsymbol{\alpha_i h_i^2(k)} + V[x_i^+(k)]^2 \right)$, where V is a constant

and $x_i^+(k) = \max\{0, x_i(k)\}$

Max-Weight Policy

- Max-Weight is designed to reduce the Lyapunov Drift.
- Lyapunov Function: $L(k) := \frac{1}{2}\sum_{i=1}^{M}\left(\boldsymbol{\alpha_i h_i^2(k)} + V[x_i^+(k)]^2\right)$
- Lyapunov Drift: $\Delta(k) := \mathbb{E}\{\, L(k+1) - L(k) \mid (\boldsymbol{h_i(k)}, x_i(k))_{i=1}^{M}\}$

$$\Delta(k) \le -\sum_{i=1}^{M} \mathbb{E}\left\{\boldsymbol{u_i(k)}\middle|\left(\boldsymbol{h_i(k)}, x_i(k)\right)_{i=1}^{M}\right\} W_i(k) + B_i(k)$$

- **Policy MW**: in each slot k, select the node with highest value of $W_i(k)$, where:

$$W_i(k) = \frac{\boldsymbol{\alpha_i p_i}}{2}\boldsymbol{h_i(k)}[\boldsymbol{h_i(k)} + 2] + V\boldsymbol{p_i}x_i^+(k)$$

Max-Weight Policy

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Theorem: MW satisfies ANY feasible set of throughput requirements $\{q_i\}_{i=1}^M$

Theorem: for every network with $V \leq 2\sum_{i=1}^M \alpha_i / M$, the **MW** is **4-optimal**.

Max-Weight Policy vs Whittle's Index Policy

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Theorem: MW satisfies ANY feasible set of throughput requirements $\{q_i\}_{i=1}^{M}$

Theorem: for every network with $V \leq 2\sum_{i=1}^{M} \alpha_i / M$, the **MW** is **4-optimal**.

- **Policy Whittle**: in each slot k, select the node with highest value of $C_i(k)$, where:

$$C_i(k) = \frac{\alpha_i p_i}{2} h_i(k) \left[h_i(k) + \frac{2}{p_i} - 1 \right] + \theta_i$$

Scheduling Policies

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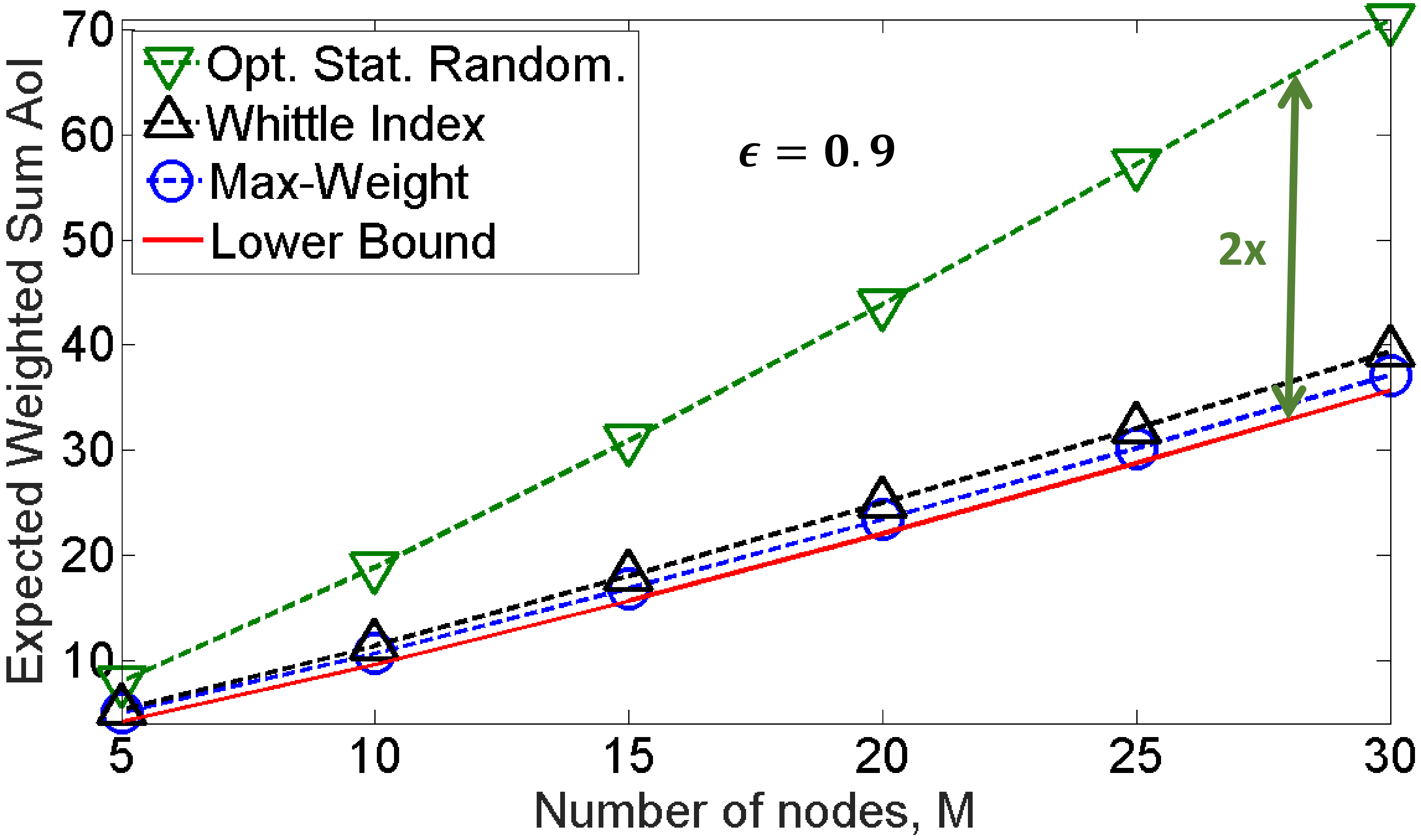
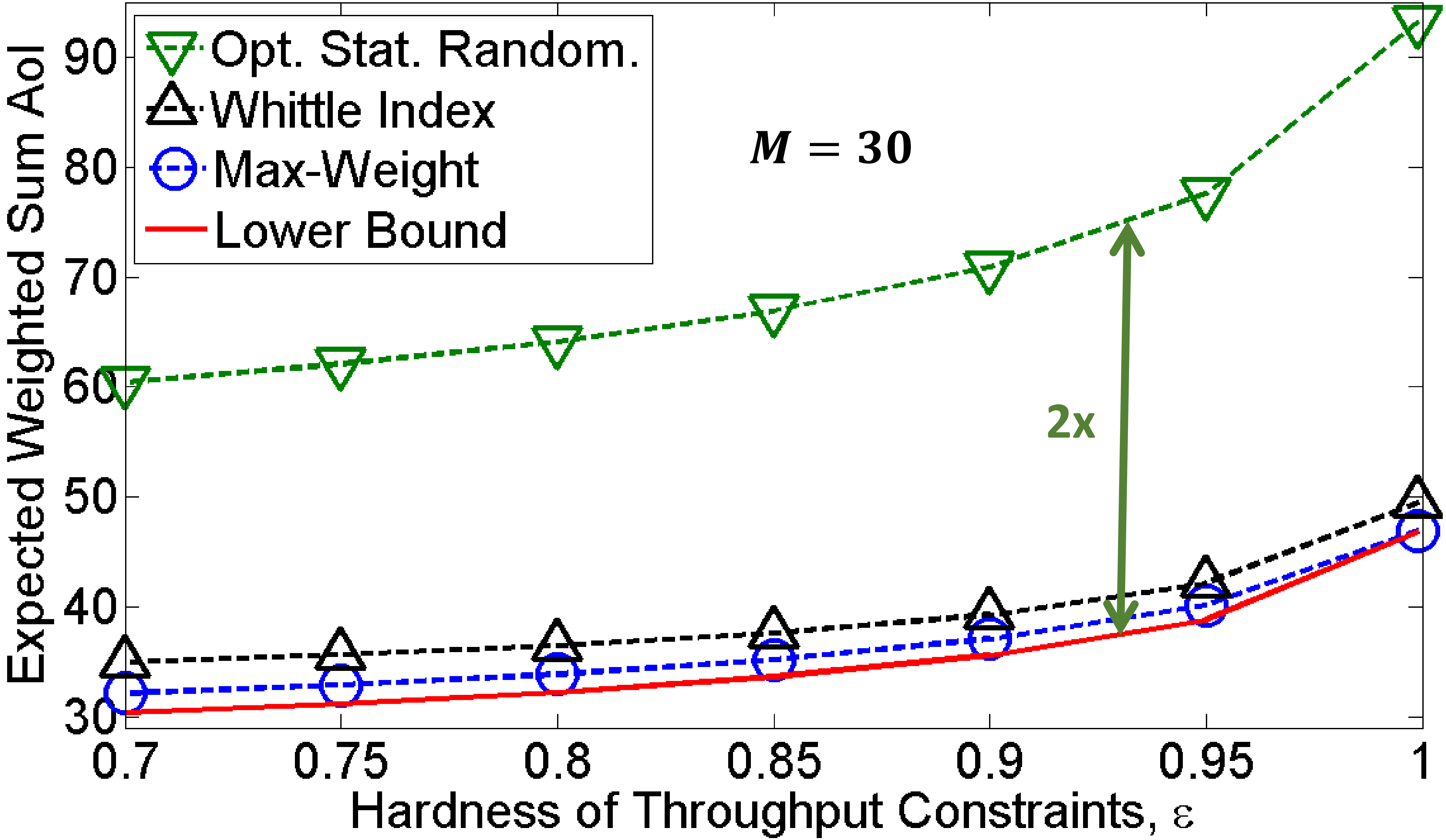
Numerical Results

- Metric:
  - Expected Weighted Sum AoI : $\mathbb{E}[J_K^\pi]$

- Network Setup with M nodes. Node i has:
  - channel reliability $\boldsymbol{p_i} = i/M$ [increasing]
  - weight $\boldsymbol{\alpha_i} = (M + 1 - i)/M$ [decreasing]
  - throughput requirement $\boldsymbol{q_i} = \epsilon \boldsymbol{p_i}/\mathrm{M}$, where ϵ in $[0;\ 1)$

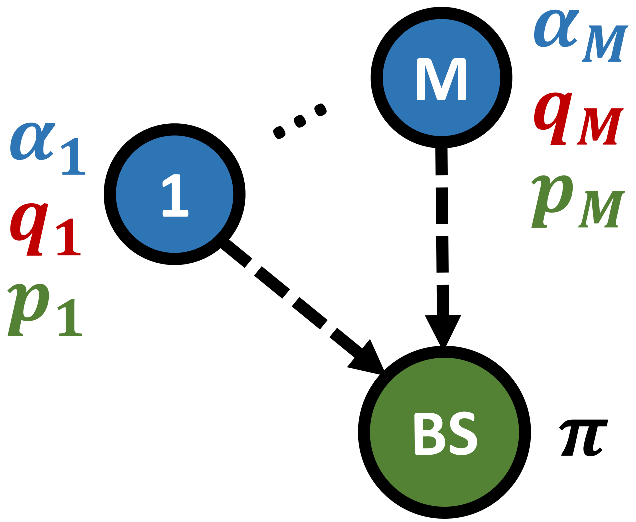
- Each simulation runs for $K = M \times 10^6$ slots
- Each data point is an average over 10 simulations


Opt. Stat. Random.
Whittle Index
Max-Weight
Lower Bound
$\epsilon = 0.9$
2x
Expected Weighted Sum AoI
Number of nodes, M
70
60
50
40
30
20
10
5
10
15
20
25
30



Opt. Stat. Random.
Whittle Index
Max-Weight
Lower Bound
M = 30
2x
90
80
70
60
50
40
30
0.7
0.75
0.8
0.85
0.9
0.95
1
Expected Weighted Sum AoI
Hardness of Throughput Constraints, ε

Final Remarks



- In this presentation:
  - Age of Information and Network Model
  - Three low-complexity scheduling policies
  - Performance guarantees
  - Numerical Results: **Max-Weight** has superior performance

- In the paper:
  - Derive Universal Lower Bound on Age of Information
  - Discuss Indexability and Whittle's Index Policy
  - Additional simulation results

- Recent result not in the paper: Drift-Plus Penalty Policy is 2-optimal