Basic Physics

Holden Mui

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Background: high school physics

- High school physics is pattern matching on formulas
- Wanted to understand "axioms" of physics
- Physics contest friends would be confused about my questions
  - What is a force?
  - What is energy? Is conservation of energy a law?
  - Is Newton's first law a special case of the second law?

My journey

- Goals:
  - Understand the definition tree
  - Derive every formula from scratch
- Result: *Physics Napkin*
- The journey did not go as expected...

Some questions

1. What is a force?
2. Can Newton's first law be derived from his second law?
3. What kinds of configurations do Newton's laws apply to? For example, is $\mathbf{F} = m\mathbf{a}$ still true when the force is applied at an angle?
4. Is conservation of momentum derivable? How about conservation of angular momentum?
5. What is the definition of energy? Is conservation of energy derivable?
6. Can the rotational motion formulas (e.g. $\tau = I\alpha$) be derived?
7. How does a moving particle affect the electric and magnetic fields?

Force

Question

What is a force?

A *force* is one of:

- Gravity
- Normal force
- Friction force
- Spring force

Each type of force is governed by some law:

- Newton's law of universal gravitation
- "Objects do not pass through each other"
- Amontons' laws of friction
- Hooke's law

Newton's laws

Can Newton's first law be derived from his second law? More generally...

Question

What are Newton's three laws?

1. There exists a frame of reference where every point particle moves with constant velocity if and only if there is no net force acting on it.
2. In an inertial reference frame, the instantaneous change in momentum of a point particle is equal to the net force acting on the point particle; that is,
$$\mathbf{F} = \frac{d\mathbf{p}}{dt}.$$
3. If point particle A exerts a force $\mathbf{F}$ on point particle B, then B must exert a force $-\mathbf{F}$ on A. In other words, the forces are equal in magnitude and opposite in direction. Additionally, the force must in the direction of the line connecting A and B.

Newton's law for rigid bodies?

Question

Is $\mathbf{F} = m\mathbf{a}$ true when the force does not point towards the center of mass?

In fact:

Question

What is a rigid body?

Definition

A *rigid body* is a system of point particles where the point particles do not move relative to each other.

Question

Do Newton's laws hold for rigid bodies?

Conservation of momentum

Question

Is conservation of momentum derivable?

Theorem (Law of conservation of momentum)

In a closed system, the total momentum is constant.

Question

Is conservation of angular momentum derivable?

Conservation of angular momentum

Definition

The *orbital angular momentum* of a point particle relative to $\mathbf{x}_0$ is

$$\mathbf{L} = (\mathbf{x} - \mathbf{x}_0) \times (m\,(\mathbf{v} - \mathbf{v}_0)) = \mathbf{x}' \times \mathbf{p}',$$

where $\mathbf{p}'$ is the momentum relative to $\mathbf{x}_0$. The *angular momentum* of a system, such as a rigid body, relative to $\mathbf{x}_0$ is a vector equal to the sum of the orbital angular momenta of the point particles relative to $\mathbf{x}_0$ that comprise the system.

Theorem (Law of conservation of angular momentum)

In a system not acted upon by external torques relative to a fixed point $\mathbf{x}_0$*, the total angular momenta relative to* $\mathbf{x}_0$ *is constant. Additionally, when not acted upon by external torques relative to the center of mass of the system, the spin angular momentum of a system is constant.*

Rotational dynamics

Question

Can $\tau = I\alpha$ be derived (for some suitable definition of torque, moment of inertia, and angular acceleration)?

Answer: It cannot be derived since it is not true!

Theorem (Euler's rotation equation)

The net torque exerted on a rigid body is

$$\boldsymbol{\tau} = \mathbf{I}_{cm}\boldsymbol{\alpha} + \boldsymbol{\omega} \times \mathbf{L}_s.$$

$\tau = I\alpha$ is only true for fixed-axis rotation. In fact; fixed-axis rotation is special; general objects do not rotate about fixed axes (including Earth).

Energy

Question

What is energy?

Answer: one of

- Kinetic energy
- Gravitational potential energy
- Spring potential energy

They are defined using work.

Definition

The *work* done by a force on a particle over a path C is

$$W = \int_C \mathbf{F} \cdot d\mathbf{x} = \int_{t_1}^{t_2} \mathbf{F} \cdot \mathbf{v}\, dt.$$

Conservation of energy is not guaranteed by Newton's three laws!

Electricity and Magnetism

Question

What are the assumptions and what can be derived?

Assumptions:

- Maxwell's four laws
- Symmetry

Derivable facts:

- Superposition principle
- Conservation of charge
- Kirchhoff's circuit laws

Electrodynamics

What is the electric and magnetic field of a moving point charge?

Theorem (Liénard–Wiechert)

The electric and magnetic fields are

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q(\mathbf{n}_s - \boldsymbol{\beta}_s)}{\gamma^2(1-\mathbf{n}_s\cdot\boldsymbol{\beta}_s)^3|\mathbf{r}-\mathbf{r}_s|^2} + \frac{q\mathbf{n}_s\times\left((\mathbf{n}_s-\boldsymbol{\beta}_s)\times\dot{\boldsymbol{\beta}}_s\right)}{c(1-\mathbf{n}_s\cdot\boldsymbol{\beta}_s)^3|\mathbf{r}-\mathbf{r}_s|}\right)_{t_r}$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mu_0}{4\pi}\left(\frac{qc(\boldsymbol{\beta}_s\times\mathbf{n}_s)}{\gamma^2(1-\mathbf{n}_s\cdot\boldsymbol{\beta}_s)^3|\mathbf{r}-\mathbf{r}_s|^2} + \frac{q\mathbf{n}_s\times\left(\mathbf{n}_s\times\left((\mathbf{n}_s-\boldsymbol{\beta}_s)\times\dot{\boldsymbol{\beta}}_s\right)\right)}{(1-\mathbf{n}_s\cdot\boldsymbol{\beta}_s)^3|\mathbf{r}-\mathbf{r}_s|}\right)_{t_r}$$

where $t_r(\mathbf{r}, \mathbf{r_s}, t) = t - \frac{1}{c}|\mathbf{r}-\mathbf{r}_s|$, $\mathbf{n}_s = \frac{\mathbf{r}-\mathbf{r}_s}{|\mathbf{r}-\mathbf{r}_s|}$, *and*

$$\varphi(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{(1-\mathbf{n}_s\cdot\boldsymbol{\beta}_s)|\mathbf{r}-\mathbf{r}_s|}\right)_{t_r}$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0 c}{4\pi}\left(\frac{q\boldsymbol{\beta}_s}{(1-\mathbf{n}_s\cdot\boldsymbol{\beta}_s)|\mathbf{r}-\mathbf{r}_s|}\right)_{t_r} = \frac{\boldsymbol{\beta}_s(t_r)}{c}\varphi(\mathbf{r}, t).$$

A hint of something more...

- Deriving Liénard-Wiechert requires special relativity
- Laws of physics were not Galilean invariant
- The correct way to the electric and magnetic fields was through the electromagnetic field; through the lens of special relativity, electricity and magnetism are the same phenomenon

Thank you!

Questions?