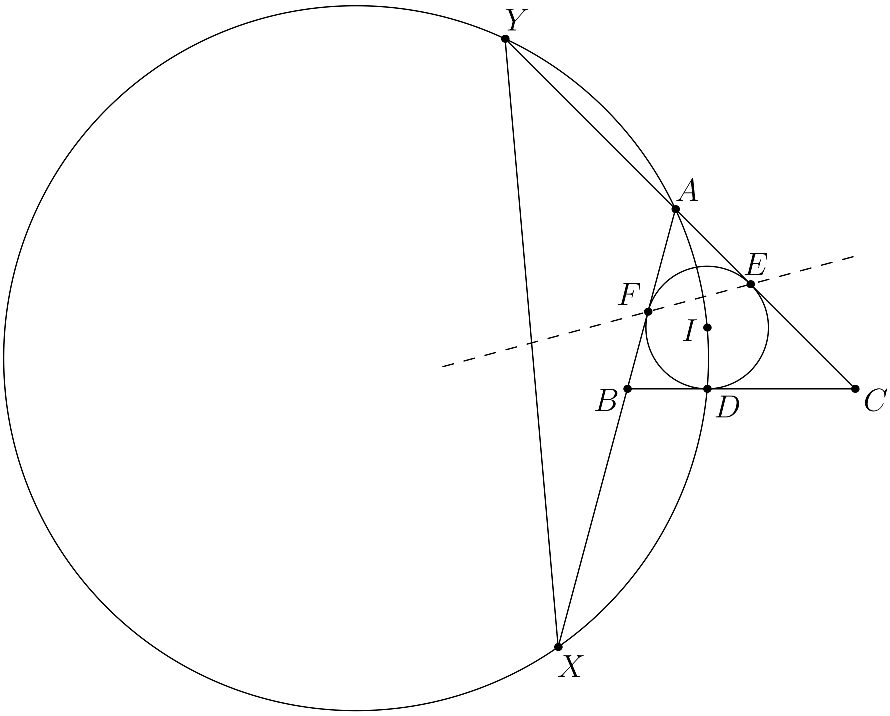
# § Problem Statement

$ABC$'s incircle has center $I$ and is tangent to $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$ at $D$, $E$, and $F$. If $(ADI)$ intersects $\overline{AB}$ and $\overline{AC}$ again at $X$ and $Y$, prove $\overline{EF}$ bisects $\overline{XY}$.

# § Diagram



# § Solutions

## Solution A

Since $\measuredangle IXY = \measuredangle IAY = \measuredangle XAI = \measuredangle XYI$, the midpoint of $\overline{XY}$ is the foot from $I$ to $\overline{XY}$. Then the desired collinearity is $I$'s Simson line with respect to $AXY$.

## Solution B

Since
$$\measuredangle IXY = \measuredangle IAY = \measuredangle XAI = \measuredangle XYI,$$
$IX = IY$, so $FX = \sqrt{IX^2 - FI^2} = \sqrt{IY^2 - EI^2} = EY$. Now,

$$\begin{aligned}
\mathrm{Pow}_{(AEF)}X - \mathrm{Pow}_{(DEF)}X &= FX \cdot AX - FX^2 \\
&= FX \cdot AF \\
&= EY \cdot AY \\
&= EY^2 - EY \cdot AY \\
&= \mathrm{Pow}_{(DEF)}(Y) - \mathrm{Pow}_{(AEF)}(Y)
\end{aligned}$$

so the midpoint of $\overline{XY}$ lies on $\overline{EF}$, by linearity of power.

## § Metadata

This problem was selected as Problem 4 of the 2023 USAMTS Round 2.

- Title: $\overline{EF}$ bisects $\overline{XY}$
- Author: Holden Mui
- Subject: geometry
- Description: $\overline{EF}$ bisects $\overline{XY}$ in incircle configuration
- Keywords: bisection, incircle, Simson line
- Difficulty: USAJMO 1/4
- Collaborators: Serena An, Ankit Bisain, Maxim Li, Carl Schildkraut
- Date written: November 2020
- Submission history: 2023 USAMO, 2023 Math Prize for Girls Olympiad, 2023 USAMTS