Tangent Circles

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Name: ______________________________

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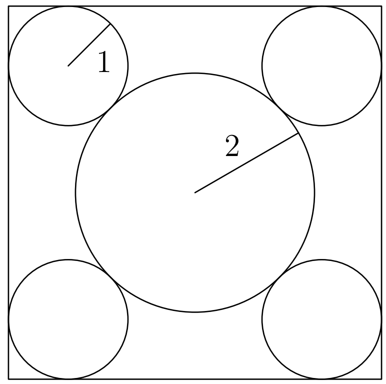
Geometry problems involving tangent circles can usually be solved by encoding each tangency condition as a length one by connecting the two centers with the tangency point. Oftentimes, the Pythagorean Theorem is used to obtain an equation involving the desired quantity, which then can be used to solve the problem.

Example 1. Two circles with centers B and C are externally tangent, and both are internally tangent to a circle with center A. If $AB = 6$, $AC = 5$, and $BC = 9$, find the largest circle's radius.

Example 2. Four circles of radius 1 are each tangent to two sides of a square and externally tangent to a circle of radius 2, as shown. Find the area of the square.



Example 3. A flat board has a circular hole with radius 1 and a circular hole with radius 2 such that the distance between the centers of the two holes is 7. Two spheres with equal radii sit in the two holes such that the spheres are tangent to each other. Find the radius of the spheres.

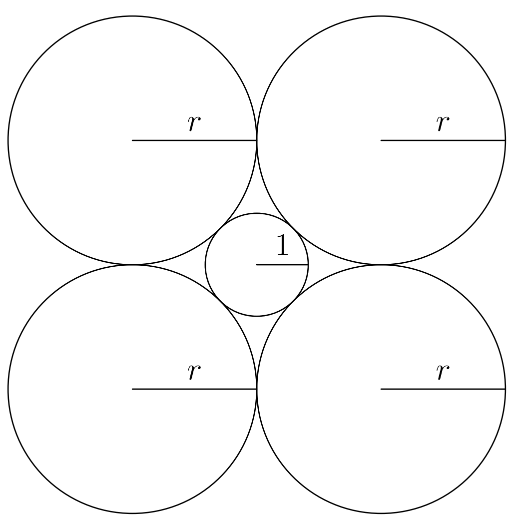
Example 4. Circles A, B and C are externally tangent to each other, and internally tangent to circle D. Circles B and C are congruent. Circle A has radius 1 and passes through the center of D. What is the radius of circle B?

Problems

Problem 1. Two externally tangent unit circles are tangent to a line. Find the radius of the circle tangent to both circles and the line.

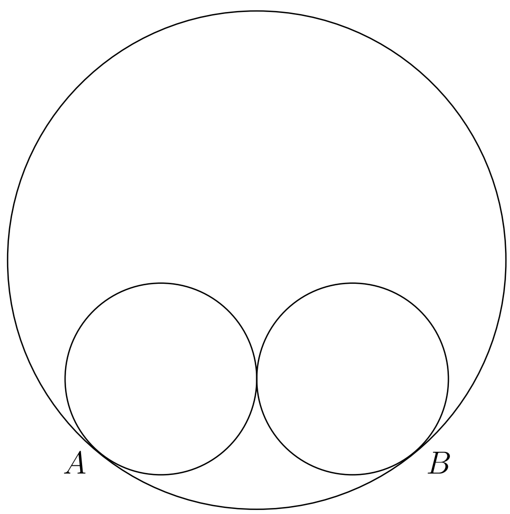
Problem 2. Points A and B lie on a circle centered at O, and $\angle AOB = 60^\circ$. A second circle is internally tangent to the first and tangent to both $\overline{OA}$ and $\overline{OB}$. What is the ratio of the area of the smaller circle to that of the larger circle?

Problem 3. A circle of radius 1 is surrounded by 4 circles of radius r as shown. Find r.



Problem 4. Three circles of radius 1 are externally tangent to each other and internally tangent to a larger circle. What is the radius of the large circle?

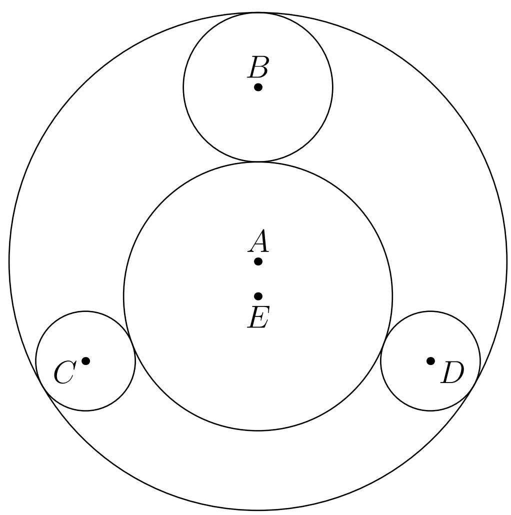
Problem 5. Two circles of radius 5 are externally tangent to each other and are internally tangent to a circle of radius 13 at points A and B, as shown in the diagram. Find AB.



Problem 6. Circle C with radius 2 has diameter $\overline{AB}$. Circle D is internally tangent to circle C at A. Circle E is internally tangent to circle C, externally tangent to circle D, and tangent to $\overline{AB}$. Find the radius of circle D, given that it is three times the radius of circle E.

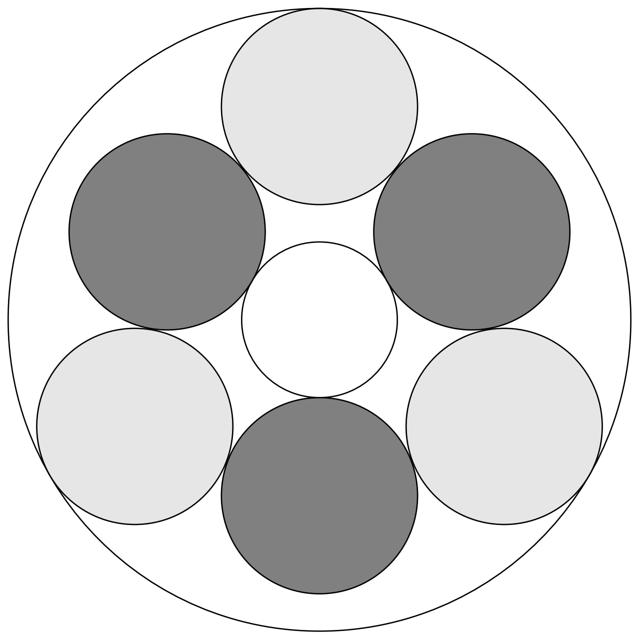
Problem 7. A circle with radius 3 and a circle with radius 4 are externally tangent, and their common external tangent is line ℓ. Find the largest possible radius of a circle tangent to both circles and ℓ.

Problem 8. Equilateral triangle T is inscribed in circle A, which has radius 10. Circle B with radius 3 is internally tangent to circle A at one vertex of T. Circles C and D, both with radius 2, are internally tangent to circle A at the other two vertices of T. Circles B, C, and D are all externally tangent to circle E. Find circle E's radius.



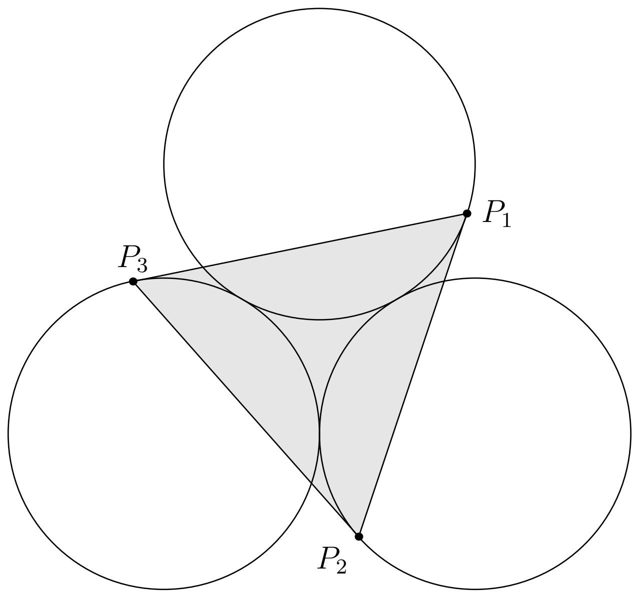
Problem 9. Three pairwise-tangent spheres of radius 1 rest on a horizontal plane. A sphere of radius 2 rests on them. What is the distance from the plane to the top of the larger sphere?

Problem 10. Two concentric circles have radii 1 and 4. Six congruent circles form a ring where each of the six circles is tangent to the two circles adjacent to it as shown. The three lightly shaded circles are internally tangent to the circle with radius 4 while the three darkly shaded circles are externally tangent to the circle with radius 1. Find the radius of the six congruent circles.



Problem 11. Circle ω_1 with radius 3 is inscribed in a strip S having border lines a and b. Circle ω_2 within S with radius 2 is tangent externally to circle ω_1 and is also tangent to line a. Circle ω_3 within S is tangent externally to both circles ω_1 and ω_2, and is also tangent to line b. Compute the radius of circle ω_3.

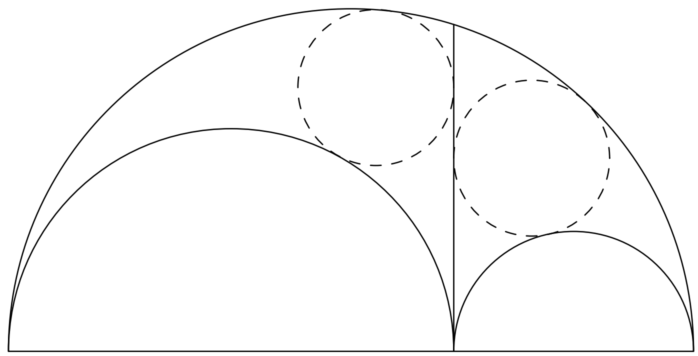
Problem 12. Circles ω_1, ω_2, and ω_3 each have radius 4 and are placed in the plane so that each circle is externally tangent to the other two. Points P_1, P_2, and P_3 lie on ω_1, ω_2, and ω_3 respectively such that $P_1P_2 = P_2P_3 = P_3P_1$ and line P_iP_{i+1} is tangent to ω_i for each $i = 1, 2, 3$, where $P_4 = P_1$. Find the area of $\triangle P_1P_2P_3$.



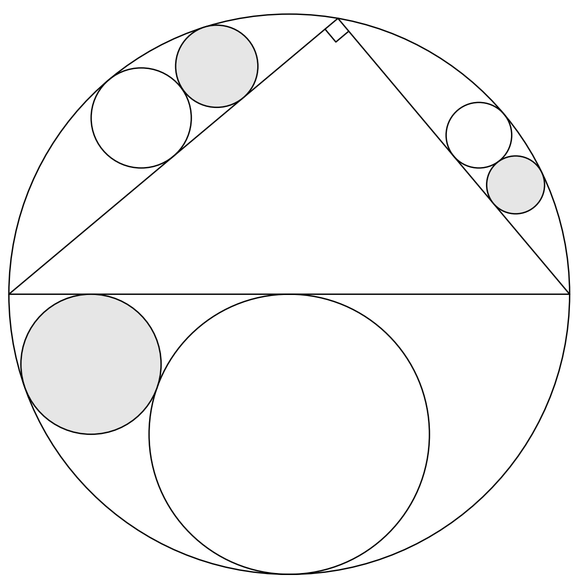
Problem 13. A semicircle is drawn on $\overline{AB}$, a line segment of length 2. Let C_1 be the circle of radius $\frac{1}{2}$ that is tangent to $\overline{AB}$ and the semicircle. For $n \geq 2$, let C_n be the smallest circle that is tangent to $\overline{AB}$, the semicircle, and circle C_{n-1}. Find the radius of C_5.

Challenge Problems

Challenge 1. Two semicircles ω_1 and ω_2 are inscribed inside a semicircle Γ, as shown. Let ℓ be the line tangent to both ω_1 and ω_2. Prove that the circle tangent to ℓ, Γ, and ω_1 has the same radius as the circle tangent to ℓ, Γ, and ω_2.



Challenge 2. Right triangle ABC has circumcircle Γ. Three circles are constructed, each tangent to Γ and tangent to a side of the triangle at a midpoint, and three more circles are constructed, as shown. Prove that the radius of the largest shaded circle equals the sum of the radii of the other two shaded circles.



Challenge 3. Let circles ω_1 and ω_2 intersect at X and Y. A circle ω is internally tangent to ω_1 and ω_2 at P and Q, respectively. $\overline{XY}$ intersects ω at points M and N. Rays $\overline{PM}$ and $\overline{PN}$ intersect ω_1 at points A and D, and rays $\overline{QM}$ and $\overline{QN}$ intersect ω_2 at points B and C respectively. Prove that $AB = CD$.

Challenge 4. A circle is internally tangent to $\triangle ABC$'s circumcircle and is tangent to $\overline{AB}$ and $\overline{AC}$ at M and N. Prove that the midpoint of $\overline{MN}$ is $\triangle ABC$'s incenter.

Challenge 5. A circle through B and a circle through C are both externally tangent to the incircle and A-excircle of $\triangle ABC$. Prove $\overline{BC}$ cuts the circles into congruent chords.