# Vieta's Formulas

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*Vieta's formulas* give a way of relating the roots of a polynomial with its coefficients. The formulas can be used to solve problems involving the roots of a polynomial, without determining what the roots actually are.

**Problem 1.** Let $a$ and $b$ be the two roots of the quadratic $2x^2 - 4x - 9$. Use the quadratic formula to find:

(a) $a + b$

(b) $ab$

(c) $a^2 + b^2$

**Problem 2 (Vieta's Formula for quadratic polynomials).** Let $a$, $b$, and $c$ be real numbers such that $a \neq 0$, and let the quadratic expression $ax^2 + bx + c$ have roots $r$ and $s$. Prove that $r + s = -\frac{b}{a}$ and $rs = \frac{c}{a}$.

**Problem 3.** Let $a$ and $b$ be the two roots of the quadratic $2x^2 - 4x - 9$. Use Vieta's formula to find:

(a) $a + b$

(b) $ab$

(c) $a^2 + b^2$

(d) $(a + 1)(b + 1)$

**Problem 4.** Let $r$ and $s$ be the two roots of the quadratic $x^2 - 7x + 5$. Find:

(a) $(r - s)^2$

(b) $(r^2 - 1)(s^2 - 1)$

(c) $\dfrac{1}{r} + \dfrac{1}{s}$

(d) $\dfrac{1}{r^2} + \dfrac{1}{s^2}$

**Problem 5.** Let $r$ and $s$ be the two roots of the quadratic $x^2 + 2x - 4$. Find:

(a) $r^2 + s^2$

(b) $r^3 + s^3$

(c) $r^4 + s^4$

(d) $\dfrac{1}{r} + \dfrac{1}{s}$

(e) $\dfrac{1}{r^2} + \dfrac{1}{s^2}$

(f) $\dfrac{1}{r^3} + \dfrac{1}{s^3}$

**Problem 6 (Vieta's formula for cubic polynomials).** Let $a$, $b$, $c$, and $d$ be real numbers for which $a \neq 0$, and suppose the cubic $ax^3 + bx^2 + cx + d$ has roots $r$, $s$, and $t$. Prove that:

$$r + s + t = -\frac{b}{a}$$

$$rs + rt + st = \frac{c}{a}$$

$$rst = -\frac{d}{a}.$$

**Problem 7.** Let $a$, $b$, and $c$ be the three roots of the cubic $8x^3 - 10x^2 - 25x + 15$. Find:

(a) $a + b + c$

(b) $ab + ac + bc$

(c) $abc$

(d) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

(e) $a(1 + b + c) + b(1 + c + a) + c(1 + a + b)$

(f) $(a + 1)(b + 1)(c + 1)$

**Problem 8.** Let $r$, $s$, and $t$ be the three roots of the cubic $x^3 - 2x^2 - 3x + 4$. Find:

(a) $r^2 + s^2 + t^2$

(b) $r^3 + s^3 + t^3$

(c) $\dfrac{1}{r^2} + \dfrac{1}{s^2} + \dfrac{1}{t^2}$

(d) $\dfrac{r+s}{t} + \dfrac{r+t}{s} + \dfrac{s+t}{r}$

(e) $(r+s)(r+t)(s+t)$

(f) $(r^2 - 1)(s^2 - 1)(t^2 - 1)$

**Problem 9.** Let $r$, $s$, and $t$ be the three roots of the cubic $x^3 - 2x - 1$. Find:

(a) $r^2 + s^2 + t^2$

(b) $r^3 + s^3 + t^3$

(c) $\frac{1}{r^2} + \frac{1}{s^2} + \frac{1}{t^2}$

(d) $(r + s - t)(r + t - s)(s + t - r)$

(e) $(r^2 - 4)(s^2 - 4)(t^2 - 4)$