# Midterm 1

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Name: ____________________________

Date: ____________________________

This midterm is an exam designed to measure both your progress learning the material and our ability to teach the material. Five of the problems are problems you have already seen in class or on your homework, and five of the problems will be problems you have not seen before.

You will have unlimited time for this exam. Please write up your solutions clearly and concisely. Even for problems that you cannot solve, please write down any partial progress or ideas that you may have. Enjoy the problems!

**Problem 0.**

(a) How much time did you spend on homework every day outside of classroom time, on average?

(b) Write any comments you have about this mathematics camp (including lecture material, homework, and teaching style) so far.

(c) Are there any mathematical topics you want to learn about next week?

**Problem 1.** List all the ways to answer a four-question true-false test. How many ways are there?

**Problem 2.** Find, with proof, all real numbers $x$ satisfying

$$2x - \sqrt{3x^2 + 1} = 1.$$

**Problem 3.** Find, with proof, the remainder when

$$1! + 2! + 3! + 4! + 5! + 6! + 7! + 8! + 9! + 10!$$

is divided by 9.

**Problem 4.** Let $\Omega$ be a circle with diameter $\overline{PQ}$, and let $X$ be a point on $\Omega$ different from $P$ and $Q$. Prove that $\overline{PX}$ is perpendicular to $\overline{QX}$.

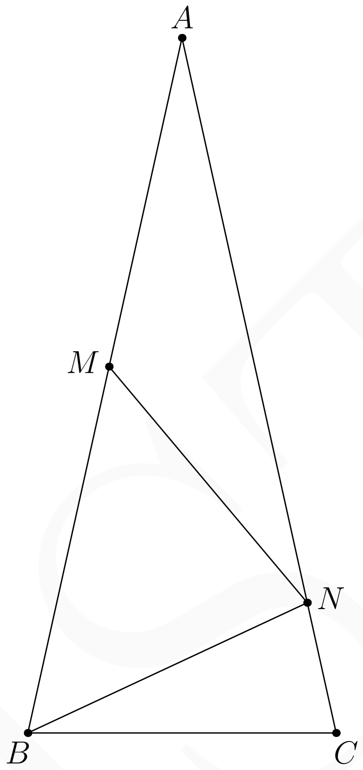
**Problem 5.** Jigme and Dorji are playing a game on a chocolate bar divided into a $3 \times 5$ grid of chocolate pieces. Jigme goes first, and they alternate breaking one of the chocolate chunks into exactly two smaller pieces along the grid of the chocolate bar (the break line can be zigzag-shaped as long as it follows the grid lines). The last player to make a move wins. Find, with proof, the player with the winning strategy.

**Problem 6.** Call a number *threeven* if it is of the form $3k - 1$ for some integer $k$, and call a number *throdd* if it is of the form $3k - 2$ for some integer $k$.

(a) Is the product of two threeven numbers always threeven?

(b) Is the product of two throdd numbers always throdd?

**Problem 7.** Let $ABC$ be a triangle with $AB = AC$. Point $M$ lies on side $\overline{AB}$ and point $N$ lies on side $\overline{AC}$ such that $AM = MN = NB = BC$. Find, with proof, the measure of $\angle BAC$.



**Problem 8.** How many ways are there to arrange the letters in "BHUTAN" such that A and B are adjacent, and T and U are adjacent?

**Problem 9.** Let $ABC$ be an acute triangle with circumcenter $O$, and let $AD$ be an altitude of triangle $ABC$. Prove that $\angle BAD = \angle OAC$.

**Problem 10.** Let $x$ be a real number satisfying $2^x = 3$. Prove that $x$ is irrational.