Games

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A *deterministic game* is a two-player game that

- has no element of randomness, and
- has perfect information; that is, no information is ever kept a secret from any player.

The goal of problems involving combinatorial games is often to determine the winning player. Symmetry usually plays an integral role in the solution of such problems – oftentimes by mirroring an opponent's move. If symmetry cannot be used, it is helpful to make a table of the player with the winning strategy for each possible configuration.

Problem 1. There is a pile of 20 stones, and Ugyen and Tsundue alternate taking either 1, 2, or 3 stones from the pile at a time. Ugyen goes first, and a player wins if they take the last stone. Who has the winning strategy?

Problem 2. There is a pile of 20 stones, and Jigme and Phuntsho alternate taking either 1, 3, or 4 stones from the pile at a time. Jigme goes first, and a player wins if they take the last stone. Who has the winning strategy?

Problem 3. Jigme and Kesang play a game on a 4×6 grid. Jigme goes first, and they alternate placing non-overlapping dominoes on the grid such that each domino covers exactly two adjacent squares. The last player to make a move wins. Who has the winning strategy?

Problem 4. Jigme and Dorji are playing a game on a chocolate bar divided into a 3×5 grid of chocolate pieces. Jigme goes first, and they alternate breaking one of the chocolate chunks into exactly two smaller pieces along the grid of the chocolate bar (the break line can be zigzag-shaped as long as it follows the grid lines). The last player to make a move wins. Who has the winning strategy?

Problem 5. Jigme and Jetsun are playing a game on a chocolate bar divided into a 3×5 grid of chocolate pieces. Jigme goes first, and they alternate choosing a chocolate piece, eating it, and also eating the pieces that are below it and to its right. A player loses if they eat the top-left piece. Who has the winning strategy?

You can play the following games for fun. Determining the winning strategy for these games is difficult.

Problem 6. Yeshi and Dorji play a game using a pile of three stones, a pile of four stones, and a pile of five stones. Yeshi goes first, and they alternate taking a nonzero number of stones from a single pile. A player wins if they take the last stone. Which player has a winning strategy?

Problem 7. Pema and Choeden play a game by connecting dots on a piece of paper. Pema goes first, and they alternate connecting two dots with a path, then placing a dot in the middle of their path. Paths may not intersect, and no dot may have more than three routes emanating from it. A player wins if their opponent cannot draw a valid path. If there are initially four dots, which player has a winning strategy?