Equations

Ben Kang, Holden Mui, Mark Saengrungkongka

Name: ______________________________

Date: ______________________________

Two methods used to solve systems of equations are *elimination* and *substitution*. Elimination is done by manipulating equations and then adding or subtracting them from each other to get rid of variables. Substitution is done by writing a variable in terms of other variables and then plugging this expression into the other equations.

Problem 1.

(a) Let x and y be real numbers satisfying

$$\begin{aligned} x - y &= 18 \\ 2x + 3y &= 4. \end{aligned}$$

Find y.

(b) Let r and s be real numbers satisfying

$$\begin{aligned} 5r + 12s &= 13 \\ 16r - 12s &= 20. \end{aligned}$$

Find r.

(c) Let a, b, and c be real numbers satisfying

$$\begin{aligned} a+b+c&=1\\ a+2b+4c&=8\\ a+3b+9c&=27. \end{aligned}$$

Find $a+4b+16c$.

(d) Let a and b be real numbers satisfying

$$\begin{aligned} a+b+c&=6\\ a-b+c&=7\\ a-c&=8. \end{aligned}$$

Find b.

(e) Let a, b, c, and d be real numbers satisfying

$$\begin{aligned} a-3b+c&=1\\ b-3c+d&=1\\ c-3d+a&=1\\ d-3a+b&=2. \end{aligned}$$

Find a.

(f) Let a, b, c, and d be real numbers satisfying

$$\begin{aligned} a+b+c+d&=1\\ a-b+c-d&=2\\ a-b-c+d&=6\\ a+b-c-d&=24. \end{aligned}$$

Find $a-b-c-d$.

Problem 2.

(a) Let $P(x)$ be a linear polynomial such that $P(2) = 4$ and $P(3) = 9$. Find $P(4)$.

(b) Let $P(x)$ be a quadratic polynomial such that $P(1) = 1$, $P(2) = 8$, and $P(3) = 27$. Find $P(4)$.

(c) Let $P(x)$ be a cubic polynomial such that $P(1) = 1$, $P(2) = 16$, $P(3) = 81$, and $P(4) = 256$. Find $P(5)$.

Theorem 3. Let a, b, and c be real numbers such that $a \neq 0$. Then the two values of x satisfying $ax^2 + bx + c = 0$ are

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and

$$\frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Problem 4. Find all pairs of real numbers (x, y) satisfying:

(a) $x + y = 8$ and $xy = 15$

(b) $x - y = 9$ and $x^2 + y^2 = 125$

(c) $x - y = 5$ and $x^2 - y^2 = 49$

(d) $x + \frac{1}{y} = 2$ and $y + \frac{1}{x} = \frac{9}{4}$

MISTI

When solving more complicated systems, clever usage of algebraic identities is important. Here are a few things to keep in mind.

- Try to factor the given equations. Do they factor? If not, can you combine them to make the result factor?
- Make use of common identities. Do the given equations remind you of common identities?

Problem 5. Find all pairs of real numbers (x, y) satisfying:

(a) $x^2 + xy = 7$ and $y^2 + xy = 9$

(b) $x^2 + y^2 = 58$ and $xy = 21$

(c) $x^3 + 3xy^2 = 260$ and $y^3 + 3x^2y = 252$

(d) $x^3 + xy^2 = 170$ and $y^3 + x^2y = 102$

(e) $x^3 + y^3 = 400$ and $x^2y + xy^2 = 200$

MISTI

Problem 6. Let x be a solution to $x + \frac{1}{x} = 3$. Find:

(a) $x^2 + \frac{1}{x^2}$

(b) $x^3 + \frac{1}{x^3}$

(c) $x^4 + \frac{1}{x^4}$

(d) $x^5 + \frac{1}{x^5}$

Problem 7. Find all triples of real numbers (x, y, z) satisfying:

(a) $xy = 12$, $xz = 15$, and $yz = 20$

(b) $xy + z = 11$, $xz + y = 10$, and $x + y + z = 9$

(c) $x + y + z = 11$, $x^2 + xy + z = 16$, and $y^2 + xy + z = 21$

Problem 8. Find all pairs of real numbers (x, y) satisfying:

(a) $x^3 = 3x + y$ and $y^3 = 3y + x$

(b) $x^3 + 3xy^2 = 5x + 4y$ and $y^3 + 3x^2y = 5y + 4x$

MSTL