Introduction to Angle Chasing

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Name: ______________________________

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An *angle* is a figure formed by two rays; the size of an angle is measured in *degrees*. Many problems in geometry ask for the degree measure of a particular angle. *Angle chasing* is a method of solving such problems using properties of angles by finding the degree measures of other angles in the diagram. While angle chasing may appear to a simple method, clever applications of angle chasing can be used to solve to solve difficult problems.

Problem 1. Know the following facts and understand why they are true.

(a) Straight angles measure $180°$, and right angles measure $90°$.

(b) Vertical angles are equal.

(c) A line intersects two parallel lines at the same angle.

(d) The angles in a triangle sum to $180°$.

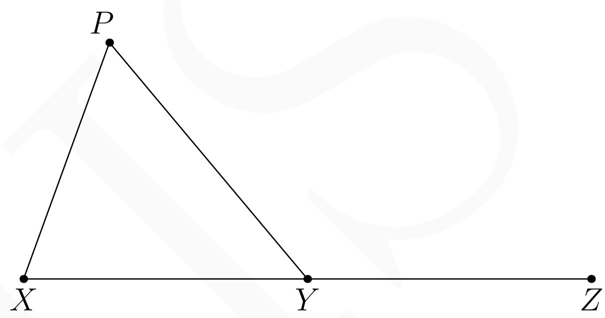
(e) The angles opposite the equal sides in an isosceles triangle are equal.

Problem 2.

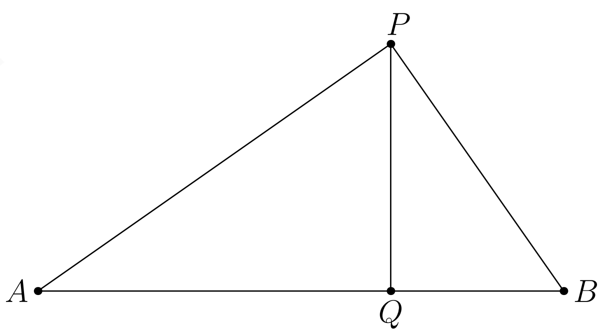
(a) Prove that the sum of the four angles in any quadrilateral is 360°.

(b) Prove that the sum of the five angles in any pentagon is 540°.

Problem 3. In the figure below, Y is the midpoint of $\overline{XZ}$. If $\angle XPY = 60^\circ$ and $\angle PXY = 70^\circ$, find $\angle PYZ$.



Problem 4. In the figure below, $\overline{AP} \perp \overline{BP}$ and $\overline{AB} \perp \overline{PQ}$. Given that $\angle BAP = 35^\circ$, find $\angle BPQ$.

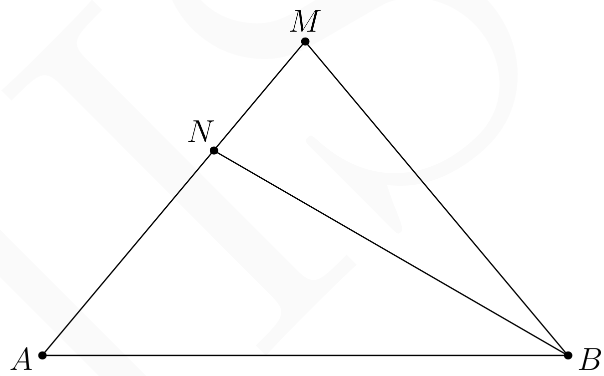


Problem 5.

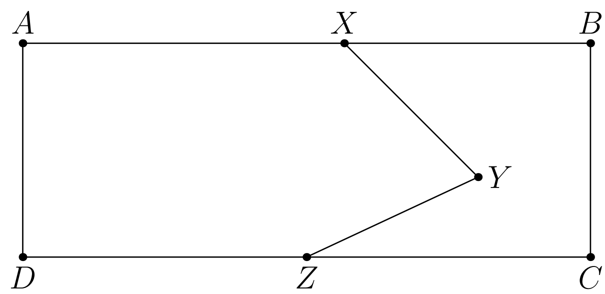
(a) An equilateral triangle ABC is located in the interior of square $BCDE$. Determine $\angle ADC$.

(b) A square $ABCD$ is located in the interior of a regular hexagon $CDEFGH$. Determine $\angle AEF$.

Problem 6. In the diagram below, $AM = MB = BN$ and $\angle BAM = 50°$. Find $\angle ANB$.

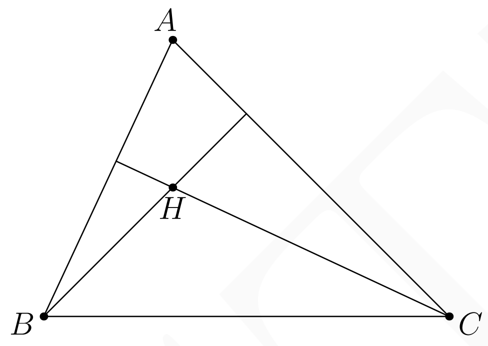


Problem 7. In the figure below, $ABCD$ is a rectangle, $\angle BXY = 45°$, $\angle CZY = 25°$, and $XY = YZ$. What is $\angle YXZ$?



Problem 8.

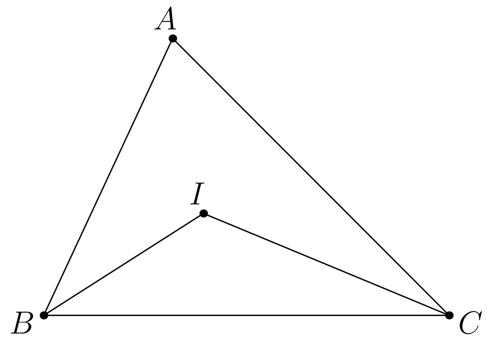
(a) Let ABC be a triangle with $\angle ABC = 65^\circ$ and $\angle ACB = 45^\circ$. The altitudes from B and C to $\overline{AC}$ and $\overline{AB}$ meet at point H. Find $\angle BHC$.



(b) Let ABC be an acute triangle. The altitudes from B and C to $\overline{AC}$ and $\overline{AB}$ meet at point H. Prove that $\angle BHC = 180^\circ - \angle BAC$.

Problem 9.

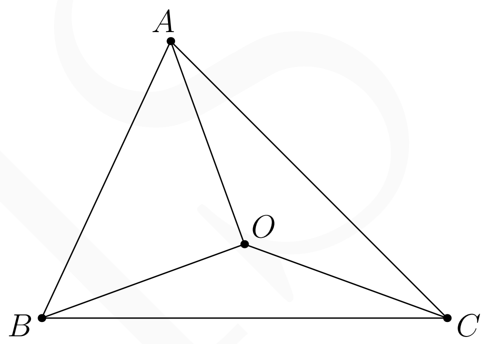
(a) Let ABC be a triangle with $\angle ABC = 65^\circ$ and $\angle ACB = 45^\circ$. The angle bisectors of angles B and C meet at point I. Find $\angle BIC$.



(b) Let ABC be a triangle. The angle bisectors of angles B and C meet at point I. Prove that $\angle BIC = 90^\circ + \frac{1}{2}\angle BAC$.

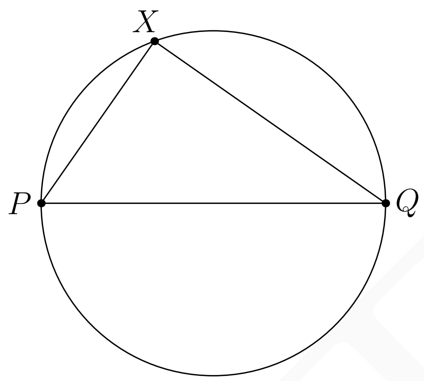
Problem 10.

(a) Let ABC be a triangle with $\angle ABC = 65^\circ$ and $\angle ACB = 45^\circ$. Let O be the circumcenter of triangle ABC. Find $\angle BOC$.



(b) Let ABC be an acute triangle. Let O be the circumcenter of triangle ABC. Prove that $\angle BOC = 2\angle BAC$.

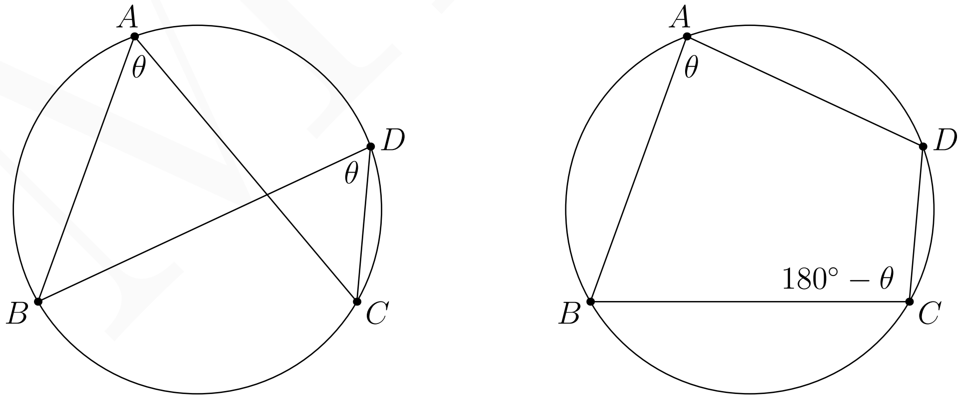
Problem 11. Let Ω be a circle with diameter $\overline{PQ}$, and let X be a point on Ω. Prove that $\overline{PX} \perp \overline{QX}$.



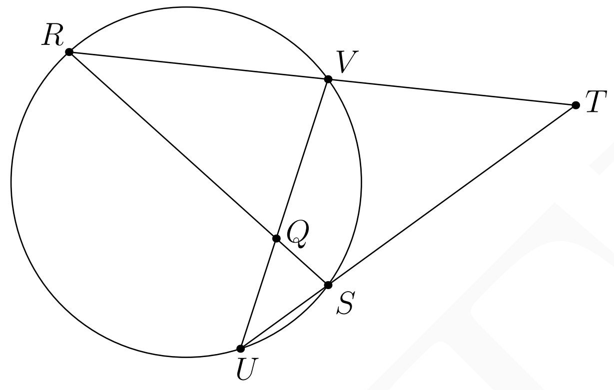
Problem 12. Let $ABCD$ be a cyclic quadrilateral. Prove that

(a) $\angle BAC = \angle BDC$.

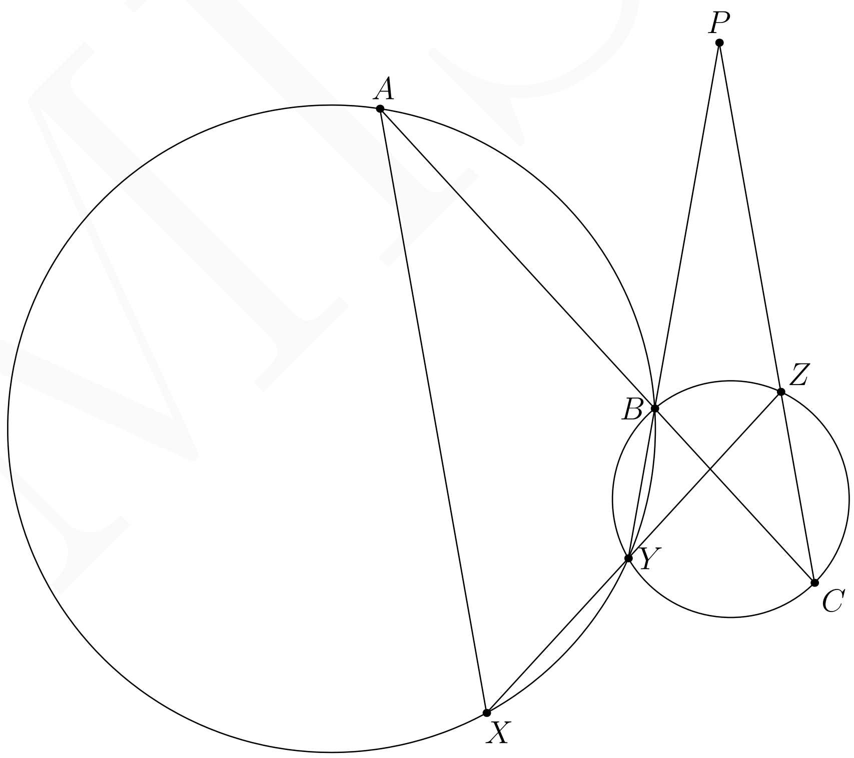
(b) $\angle A + \angle C = 180°$.



Problem 13. Consider the diagram below, where $\angle R = 36^\circ$ and $\angle T = 42^\circ$. Find $\angle RQV$.

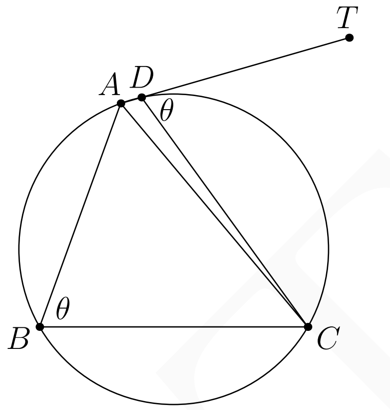


Problem 14. Consider the diagram below, where $\angle BAX = 50^\circ$ and $\angle AXY = 70^\circ$. Find $\angle BPZ$.



Problem 15.

(a) In the diagram below, explain why $\angle ABC = \angle CDT$.



(b) What happens when point D approaches point A?

(c) In the diagram below, point O is the center of the circle, and $\overline{TA}$ is tangent to the circle at point A. Prove that $\angle OAT = 90°$.