Another Reduction Formula: $\int x^n e^x \, dx$

To compute $\int x^n e^x \, dx$ we derive another reduction formula. We could replace e^x by $\cos x$ or $\sin x$ in this integral and the process would be very similar.

Again we'll use integration by parts to find a reduction formula. Here we choose

$$u = x^n$$

because

$$u' = nx^{n-1}$$

is a simpler (lower degree) function. If $u = x^n$ then we'll have to have

$$v' = e^x, \quad v = e^x.$$

(Note that the antiderivative of v is no more complicated than v' was — another indication that we've chosen correctly.)

On the other hand, if we used $u = e^x$, then $u' = e^x$ would not be any simpler.

Performing the integration by parts we get:

$$\int \underbrace{x^n e^x}_{uv'} \, dx = \underbrace{x^n e^x}_{uv} - \int \underbrace{x^{n-1} e^x}_{u'v} \, dx$$

So, if:

$$G_n(x) = \int x^n e^x \, dx$$

then we get the reduction formula:

$$G_n(x) = x^n e^x - nG_{n-1}(x).$$

Let's illustrate this by computing a few integrals. First we directly compute:

$$G_0(x) = \int x^0 e^x \, dx = e^x + c.$$

Now we can use the reduction formula to conclude that:

$$\begin{aligned} G_1(x) &= xe^x - G_0(x) \\ &= xe^x - e^x + c \end{aligned}$$

So $\int xe^x \, dx = xe^x - e^x + c.$

Question: How do you know when this method will work?

Answer: Good question! The answer is "only through experience and practice". To use this method on an integrand, we need one factor u of the integrand to get simpler when we differentiate and the other factor v not to get more complicated when we integrate.

We've seen how to use integration by parts to derive reduction formulas. We could also find these formulas by advanced guessing — guess what the formula should be and then check it. Either method is valid.