Summation Notation

You'll have noticed working with sums like $1^2 + 2^2 + 3^2 + \cdots + (n-1)^2 + n^2$ is extremely cumbersome; it's really too large for us to deal with. Mathematicians have a shorthand for calculations like this which doesn't make the arithmetic any easier, but does make it easier to write down these sums.

The general notation is:

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n.$$

The summation symbol Σ is a capital sigma. So, for instance,

$$\frac{1^2 + 2^2 + 3^2 + \cdots + (n-1)^2 + n^2}{n^3} = \frac{1}{n^3} \sum_{i=1}^{n} i^2.$$

We just showed that:

$$\lim_{n \to \infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2 = \frac{1}{3}.$$

When using the summation notation, we'll have a formula describing each summand a_i in terms of i; for example, $a_i = i^2$. The expression $\displaystyle\sum_{i=1}^{n} a_i$ is just an abbreviation for the sum of the terms a_i.

Another difficult sum we encountered was:

$$\left(\frac{b}{n}\right)\left(\frac{b}{n}\right)^2 + \left(\frac{b}{n}\right)\left(\frac{2b}{n}\right)^2 + \left(\frac{b}{n}\right)\left(\frac{3b}{n}\right)^2 + \cdots + \left(\frac{b}{n}\right)\left(\frac{nb}{n}\right)^2$$

Using summation notation, we can rewrite this as:

$$\sum_{i-1}^{n} \left(\frac{b}{n}\right)\left(\frac{ib}{n}\right)^2.$$

We factored $\left(\frac{b}{n}\right)^3$ out of this sum earlier; we can also do this using our new notation:

$$\sum_{i-1}^{n} \left(\frac{b}{n}\right)\left(\frac{ib}{n}\right)^2 = \frac{b^3}{n^3} \sum_{i=1}^{n} i^2.$$

These notations just make our notes a little bit more compact. The concepts are still the same and the mess is still there hiding under the rug, but the notation at least fits on the page.