

Verify Green's Theorem in Normal Form

Verify that $\oint_C M dy - N dx = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dA$ when $\mathbf{F} = x\hat{\mathbf{i}} + x\hat{\mathbf{j}}$ and C is the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$.

Answer:

Right hand side: Here $M = N = x$, so $\iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dA = \iint_R 1 dA = 1$.

Left hand side: $\oint_C M dy - N dx = \oint_C x dy - x dx$. We evaluate this line integral in four parts.

- $(0, 0)$ to $(1, 0)$.

$$\int_{x=0}^{x=1} x \cdot 0 - x dx = \left. \frac{x^2}{2} \right|_0^1 = \frac{1}{2}.$$

- $(1, 0)$ to $(1, 1)$.

$$\int_{y=0}^{y=1} 1 dy - 1 \cdot 0 = 1.$$

- $(1, 1)$ to $(0, 1)$.

$$\int_{x=1}^{x=0} x \cdot 0 - x dx = -\frac{1}{2}.$$

- $(0, 1)$ to $(0, 0)$.

$$\int_{y=1}^{y=0} 0 dy - 0 \cdot 0 = 0.$$

Since the sum of the line integrals along the components of C is 1, $\oint_C x dy - x dx = 1$. This confirms that the normal form of Green's Theorem is true in this example.

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