



Note on Life Cycle Diffusion Models

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Imagine that you have been offered an opportunity to be the brand manager of a new durable product, say a color television based on quantum mechanics. This television is so good that it will, eventually, replace other forms of television. However, it is sufficiently expensive that few families will purchase more than one quantum television. You've done your homework and sales look promising. In fact, the sales have been growing for the last thirty months (Figure 1). The total sales to date have been 43 million units. Impressive! You decide to accept the job. Your future looks bright.

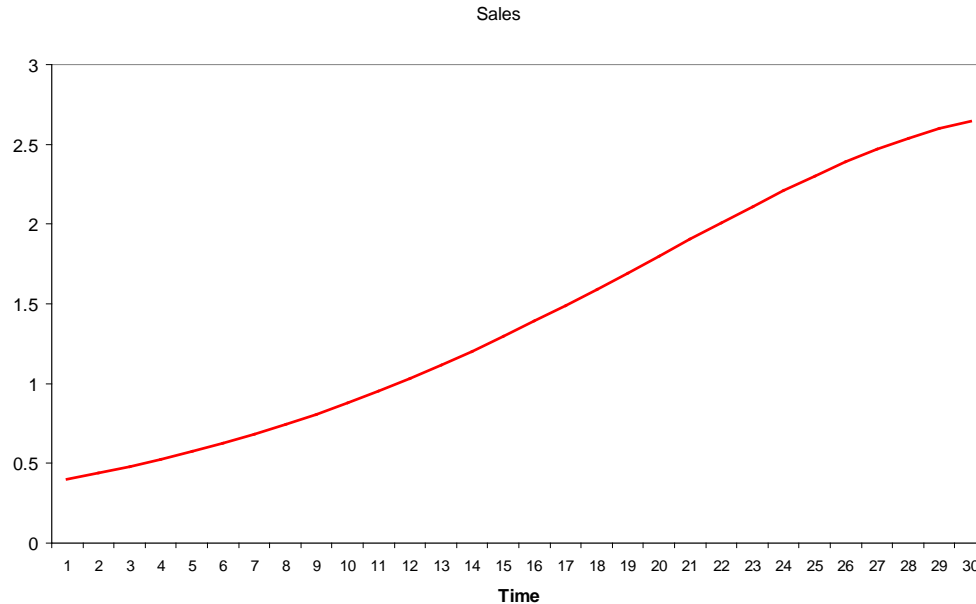


Figure 1. Sales of Quantum Television in the First 30 Months

A year later, you are worried. Although total sales are now over 73 million units, the monthly figures are not so optimistic (Figure 2). The CEO is blaming you and wants you to resign. You wonder whether this decline is due to your decisions or is the decline due to other reasons.

After studying the situation, you argue that there are only 100 million households that could afford a quantum television. If 73 million already have a quantum television and there are no upgrades available, this means that only 27 million could possibly purchase a quantum television in the coming months. The product development division was so optimistic based on the first thirty months that they saw no need for a replacement or upgrade and it is likely that none will be available for at least a year. To save your job you would like to present data and a theory that the decline is not due to your marketing. Perhaps you can argue that, to save the product, the firm must create a new sales cycle, a recycle, by accelerating product development. Intuitively, you might argue that it is much easier to sell quantum televisions when the potential market is 100 million (73 million replacement, 27 million new) than when you are trying to sell only to the 27 million who have so far not adopted this new technology.

However, you would like to have a more-quantitative model to support your argument.

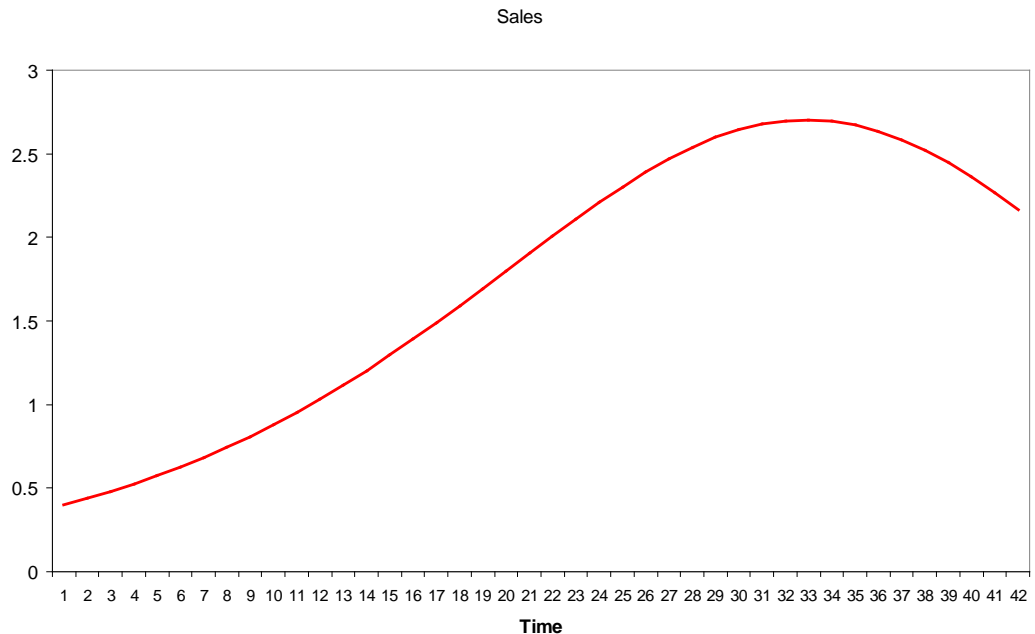


Figure 2. Sales of Quantum Television in the First 42 Months

This scenario is fictitious, but not atypical. From CB radios, to Hummer 2's, to plasma televisions, to PDAs, markets saturate. Sales decline, crises develop, and there is a need for innovation. (The same applies to more mundane products such as pitcher or faucet-mounted water filters.)

In this note we explore the classic diffusion analysis that was developed by Frank Bass.¹ This analysis is known as the “Bass model.” It applies to the first adoption of a new technology and has been applied to all types of technologies from hybrid corn, to color televisions, to personal computers.

For marketing management we want to understand the basic phenomena. Advanced marketing courses, e.g., New Product Development (15.828), explore how the model can be used to forecast the sales of new durable goods. For ex-

¹ Frank M. Bass (1969), “A New Product Growth Model for Consumer Durables,” *Management Science*, 15, 5, (January), 215-227. See also <http://www.basseconomics.com/BE/Modules/About/About.aspx>.

ample, had you used the Bass model based on the first 30 months of sales, you would have been able to predict months 31 through 42. Not only could you have used the analysis to set expectations, but you could have convinced product development to invest in the next generation of technology to restart the lifecycle. (More importantly, you could have convinced the product development team to develop a replacement or upgrade while they still had time to do so.) Such a restart, or upgrade, overlays a “recycle” on top of the basic Bass life cycle.

The remainder of this note provides the mathematics behind the Bass model. This note tries to balance mathematical notation with intuitive explanations. The later are, in many cases, more important for strategic thinking about product life cycles.

Market Size, Innovation, and Contagion

Market Size, Sales and Cumulative Sales

Marketing actions, such as reduced price, improved positioning, or advertising can increase the market size. However, for illustration, we will assume that there is a fixed market size. We will call this market size, m .

When the product is first launched, all m consumers are potential adopters. As more products are sold, there are fewer consumers who could adopt the product. (Here we are assuming each household adopts at most once.) To model this effect, we define sales as S , and recognize that sales will change as a function of time, t . We also define cumulative sales as Y and recognize that it, too, is a function of time. That is:

m = total number of households who could ever adopt the new product

$S(t)$ = the number of households who adopt the new product in the t^{th} month

$Y(t)$ = the total number of households who have adopted the new product up to and including the t^{th} month

Notice that we have the following simple structural definition. Cumulative sales by month t are the sum of all sales in the months up to and including month t .

$$Y(t) = \sum_{\tau=1}^t S(\tau)$$

We also recognize that we can calculate the number of consumers, who have not yet adopted, with the following equation. Those who have not yet adopted are simply those in the market, m , minus those who have already adopted, $Y(t)$.

$$(1) \quad m - Y(t) = \text{potential adopters at month } t$$

Innovation and Contagion

Due to our marketing actions, some consumers will adopt the product on their own, without talking to other consumers. We call these consumers innovators. Mathematically, we say that there is some innovation rate, p . We can influence p with our marketing mix. For example, more advertising makes more innovators aware of the products and informs them of the products differential benefits. Greater distribution makes the product available to more innovators. Thus we have:

p = the rate of innovative adoption as a percent of those that have not yet adopted

However, active marketing is not the only way that consumers find out about our new product. Potential adopters talk to other consumers, look for reviews (e.g., ePinions, Facebook, YouTube, Amazon), and hear about the prod-

uct in the media. The more products that are in circulation, the stronger these “word-of-mouth” forces. In an analogy to the propagation of epidemics, we call this effect contagion.

To model contagion we define a contagion effect, q . In particular, q times the number of current adopters determines the rate at which new consumers adopt the product via contagion. In particular, if we multiply q times the number of consumers who have already adopted the product, we get the contagion effect:

$$qY(t) = \text{the rate of adoption as a percent of those that have not yet adopted (contagion effect)}$$

We simply add up innovators and imitators to get the total rate of adoption as:

$$(2) \quad p + qY(t) = \text{total rate of adoption.}$$

Bass Model

The basic Bass model is very simple. The number of new adopters is just the rate of adoption times the size of the potential market. This gives us a simple equation for new sales. We simply combine Equations (1) and (2).

$$(3) \quad \text{Sales} = \text{rate of adoption} \times \text{potential market}$$

$$S(t) = [p + qY(t)][m - Y(t)]$$

We can now see that

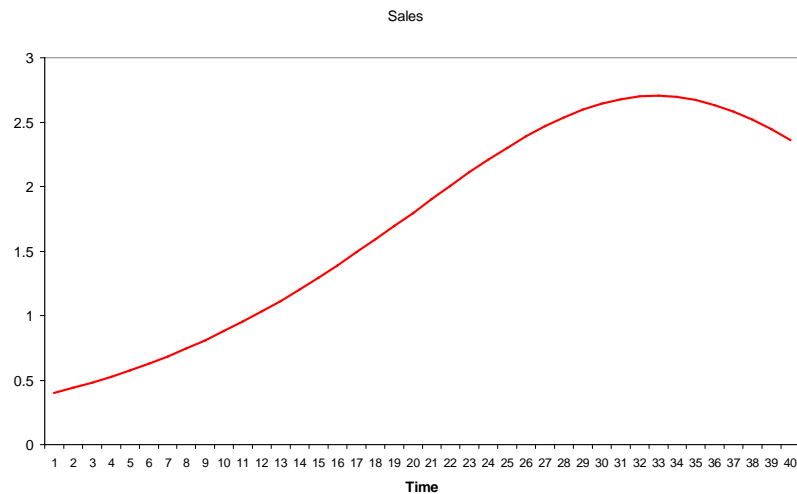
- in the beginning, when cumulative sales, $Y(t)$, is small compared to the total market, m , there is an acceleration in sales. More adopters mean more contagion and, hence, more sales. The market takes off and everyone is overly optimistic.

- however, as cumulative sales, $Y(t)$, approaches the total number of customers who might ever purchase, m , the market saturates and sales slow to a trickle. Managers are surprised and careers are ruined, which is sad because it is so predictable with only a little analysis and logic.

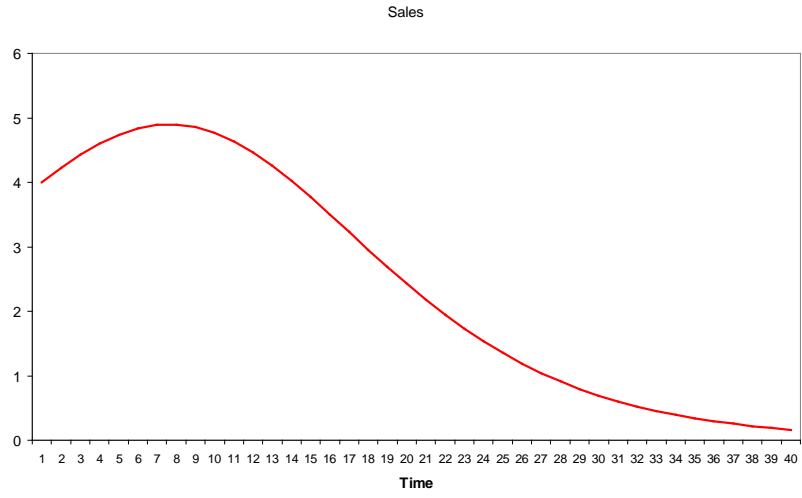
Illustrative Examples

I created a simple Excel spreadsheet that enables you to play with the Bass model. Just change the three parameters to see their effect on sales. You can download this spreadsheet from Stellar. For example, consider the following four diagrams.

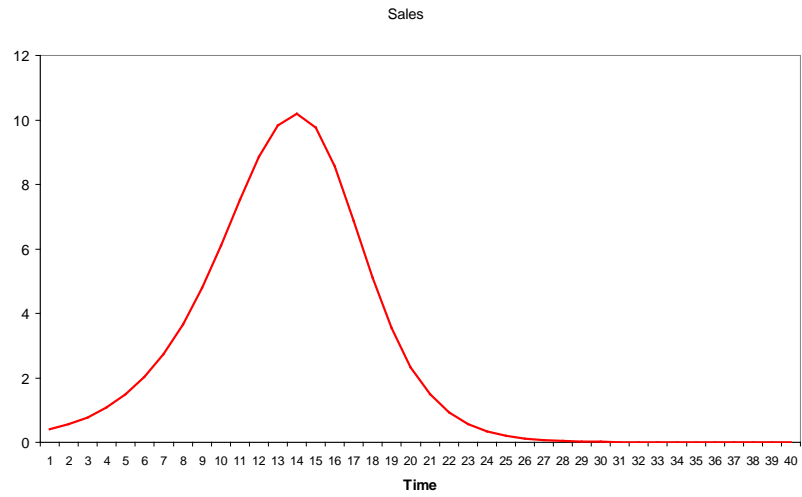
Low innovation (0.004) and low contagion (0.001)

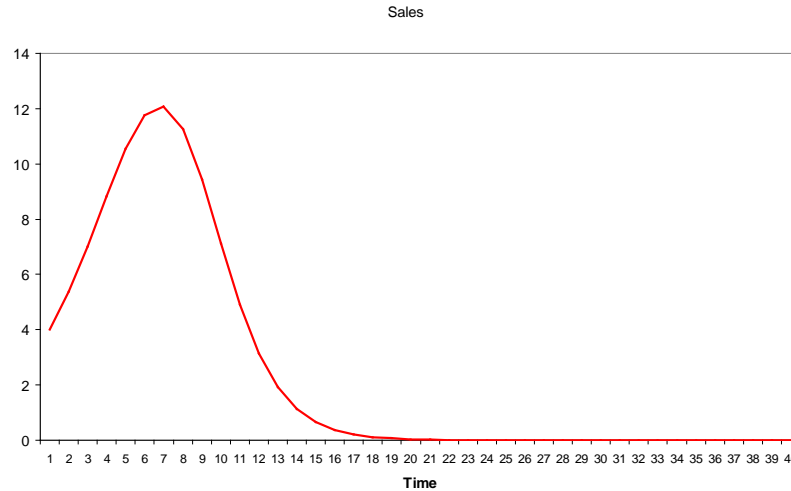


High innovation (0.04), but low contagion (0.001)



Low innovation (0.004), but high contagion (0.004)



High innovation (0.01) and high contagion (0.004)**Summary**

The Bass Model illustrates the effects of market saturation, innovation, and contagion. It has been used quantitatively to forecast the sales of new products. However, if you are thinking about being an entrepreneur or getting involved in product development or becoming a financial analyst who must forecast the future of a firm, you should understand the basic forces that are represented in the Bass model.

To use the Bass model quantitatively, you must estimate the three unknown parameters in the model, m , p , and q , from market research data. The specific methods are discussed in advanced marketing courses at MIT Sloan. (For example, purchase intention questions provide basic input to estimate p and m in the Bass model. If sufficient observations on sales are already available, either regression or maximum-likelihood models provide estimates of the parameters.)

To understand the basic implications of the Bass model for evaluating a new product or a firm, I encourage you to try different parameters in the spreadsheet until you understand how each parameter affects the sales forecasts. In doing this, you should think carefully about what these parameters mean and

you should think carefully about how marketing tactics affect each of these parameters.

The Bass Model is accurate, but it only applies to each cycle. For example, color television has gone through many recycles as new technologies supplant old technologies and/or as new uses (secondary, small televisions) restart the innovation cycle. These cycles are the customer side of the equation and are related to S-curves that you will see in strategy to describe the supply side of the equation. The Bass Model is a start, but practical application requires the recognition of the potential for recycle. There are literally hundreds of academic articles written about the Bass model and its extensions.² For further information you can use the Web of Science, which is available through the MIT library system.

Finally, there has been some interesting research on how various networks of consumers affect diffusion. The Bass model assumes a particularly simple network called a random network – the contagion effect is based only on the total number of customers who have already adopted the new product. However, there are other networks such as “scale free” networks and “small world” networks. In a scale-free network, some consumers are connected to lots of other consumers. These consumers are called “nodes” and have a major impact on diffusion. They might be reached by directed advertising as explored in 15.810. A small-world network recognizes that there are many “cross-over” links between consumers. You may have heard about this in terms of six degrees of freedom, that is anyone is only six connections away from anyone else. This is only an approximation, but it is an interesting effect and suggests that viral marketing might be effective. For an example in academic marketing, see <http://www.mconnectivity.com/>.

² In fact, the 1969 paper is one of the ten most cited articles ever published in *Management Science*.