# Quantum creation of a universe with nontrivial topology

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Quantum creation of a universe with flat comoving 3-space is possible in nontrivial topology. For 3-torus topology, the relative probability of quantum birth of a spatially flat, isotropic world having finite 3-volume during the de Sitter (inflationary) stage is calculated, allowing for the vacuum energy-momentum tensor of massless quantum fields that will result from the nontriviality of the topology.

No sooner had the general theory of relativity been formulated than the question arose as to the topology of the universe. The first instance of a topology differing from the trivial $E^3$ topology of unbounded three-dimensional Euclidean space is a world closed upon itself with an $S^3$ topology for the section t = const a nontrivial topology. The simplest example is a 3-torus, that is, three-dimensional Euclidean space with the identification $x \equiv x + L_1$, $y \equiv y + L_2$, $z \equiv z + L_3$. The universe would then form a rectangular parallelepiped with the finite volume $V = L_1L_2L_3$. Such an identification would leave space homogeneous, but the spatial isotropy corresponding to the O(3) rotation group would be violated. Unless spatially inhomogeneous perturbations are present, however, this violation of isotropy would not induce a temperature anisotropy in the cosmic microwave background. Incidentally, in the cubically symmetric case $L_1 = L_2 = L_3$, the quadrupole asymmetry would vanish.

In several papers[1–3] (among others) the authors and colleagues have investigated Friedmann models with various identifications of points, the characteristic identification scales (the minimum and maximum "gluing" parameters) being assumed shorter than the current horizon distance of the Friedmann model. We have considered the prospects of observing one and the same astronomical object concurrently in different parts of the sky, and we have inquired into what effect a nontrivial topology might have on the adiabatic fluctuation spectrum and the apparent distribution of galaxies. Searches for such effects have invariably yielded negative results, setting a lower limit on the minimum identification scale – a few hundred megaparsecs.

The question of the spatial topology of the universe is now taking on new interest in light of the hypothesis that the universe may have been born spontaneously as a quantum fluctuation,[4–9] an idea which in turn was only put on the agenda in earnest after the theory of an inflationary universe had been broached.[10–12]. In a narrow technical sense one should regard the quantum birth of the universe as the stage when the classical equations of general relativity theory were invalid. In particular, it is well established that the classical solution for the scale factor of a closed Friedmann universe with zero Higgs scalar field and potential $V(0) = 3H^2/8\pi G$ is $a(t) = H^{-1}\cdot \mathrm{ch}(Ht)$ (throughout this letter we see $\hbar = c = 1$; $\mathrm{ch}\, x \equiv \cosh x$). For a closed world no other classical solution exists wherein $a$ would change from 0 to its minimum classical value $a_{\min} = H^{-1}$. Like a particle in ordinary quantum mechanics, the universe can tunnel through this interval. The tunneling of the universe through a potential barrier is described by the Wheeler–DeWitt quantum geometrodynamics equation,[13] an analog of the Schrödinger equation for quantum gravitation theory. In the case of a closed Friedmann world, the role of the barrier is played by the 3-space curvature.

On scales greater than the Planck scale ($a^2 \gg G$) one can solve the Wheeler–DeWitt equation in the WKB approximation both in the super-barrier regime (the classical case) and beneath the barrier. It is well recognized that in the sub-barrier regime the WKB approximation corresponds to motion in "imaginary time" $\tau = -it$. The probability of traversing the barrier may be reckoned as $e^{-|S|}$, where S represents the effect on the solution of the classical gravitation equations in Euclideanized geometry (with the t in the metric replaced by $i\tau$). In particular, the probability of a quantum birth of an empty closed de Sitter world of curvature H turns out to be proportional to $\exp(-\pi/GH^2)$ (assuming that $GH^2 \ll 1$). The exponent here differs in sign from Vilenkin's result[8,9]; this point is being discussed by one of us elsewhere.[14]

An essential feature in all these calculations is that we are dealing with a closed universe. The action S is proportional to the volume of the three-dimensional section $\tau$ = const. At the moment it is not clear just what the meaning of the "closed-world birth probability" is and how that probability is to be normalized. Moreover the universe might also have been "born classically," from a singularity. In that event the exponentially small factor in the probability describing the tunneling will not be present at all. One cannot infer, however, that classical creation is the more probable, because the absolute values of the probability are unknown.

Nevertheless, it is plain that if a flat or open Friedmann world has noncompact 3-space and hence infinite action S, then its quantum creation will have zero probability, and cannot occur. It is in this situation that a significant possibility would exist for the quantum birth of a flat but topologically nontrivial universe.

If the de Sitter (inflationary) stage of exponential expansion was long enough ($H\Delta t \gtrsim 70$), the periodicity cell size of the universe would today be larger than the horizon. Such a universe could not be distinguished observationally from an ordinary flat world. With a cosmological constant $\Lambda = 0$ today, the universe would expand indefinitely, apart from individual pieces that may collapse locally due to initial perturbations.

Let us consider the general form of a spatially homo-

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geneous universe having the topology of a 3-torus:

$$ds^2 = dt^2 - a^2(t)\,dx^2 - b^2(t)\,dy^2 - c^2(t)\,dz^2,$$
$$x + L \equiv x,\ y + L \equiv y,\ z + L \equiv z. \qquad (1)$$

Its 3-volume will be $abcL^3$. In the isotropic case $a \propto b \propto c$. Suppose that the universe is empty and that the effective cosmological constant produced by the scalar field or by gravitational vacuum polarization is $\Lambda = 3H^2 > 0$. Then in the isotropic case the classical solution would be $a \propto b \propto c \propto \exp(Ht)$. Unlike the trivial topology, for which $L = \infty$, the resultant metric will be geodesically complete and have no particle horizon as $t \to -\infty$. The surface $t = -\infty$ will be nonsingular. In this event no barrier will exist, but one may nevertheless speak of quantum creation in a certain sense, because the evolution of the metric will cease to be quasiclassical (that is, the WKB approximation cannot be used in the Wheeler–Dewitt equation) if $(aL)^3 < GH^{-1}$; the classical solution then will no longer be applicable.

If anisotropy is present, a true Kasner-type singularity will develop. Its general solution is

$$a = a_0 (\mathrm{sh}\, 3Ht)^{1/3} \left(\mathrm{th}\, \frac{3}{2} Ht\right)^{p_1 - 1/3},$$
$$b = b_0 (\mathrm{sh}\, 3Ht)^{1/3} \left(\mathrm{th}\, \frac{3}{2} Ht\right)^{p_2 - 1/3},$$
$$c = c_0 (\mathrm{sh}\, 3Ht)^{1/3} \left(\mathrm{th}\, \frac{3}{2} Ht\right)^{p_3 - 1/3}, \qquad (2)$$

where the $p_i$ are the Kasner indices ($p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1$). Here too there is no barrier. The metric will evolve quasiclassically near the singularity if $a_0 b_0 c_0 L^3 \gg GH^{-1}$ (for power-law regimes the criterion $abcL^3 \gg Gt$ is sufficient for the evolution to be quasiclassical). As $t \to \infty$ the metric will proceed to isotropize ($\dot{a}/a \approx \dot{b}/b \approx \dot{c}/c$), but the scale ratios $a/b$, $a/c$, $b/c$ will remain arbitrary. This is a special case of the general result that one of us has obtained.[15]

In the next approximation one should recognize that the presence of a nontrivial topology will induce a vacuum polarization of all quantum fields – the gravitational analog of the Casimir effect. Intuitively speaking, the continuous null-oscillation spectrum of the quantum fields will change over to a discrete spectrum. The effect corresponds to the difference between a sum and an integral in momentum space.[1)]

In the cubically symmetric case ($a = b = c$) the energy-momentum tensor of this topological vacuum polarization of massless fields will, by symmetry and dimensionality arguments, be proportional to the energy-momentum tensor of a perfect ultrarelativistic gas:

$$T_0^0 \propto a^{-4},\quad T_1^1 = T_2^2 = T_3^3 = -\frac{1}{3} T_0^0. \qquad (3)$$

On performing a calculation for a scalar field with the natural periodic identification conditions

$$\varphi(x + L) = \varphi(x),\quad \varphi(y + L) = \varphi(y),\quad \varphi(z + L) = \varphi(z) \qquad (4)$$

one finds[16–20] that

$$T_0^0 \simeq -0.8375\,(aL)^{-4}. \qquad (5)$$

For an electromagnetic field the quantity $T_0^0$ will be twice as large.

For fermion fields the result depends on the form of the identification conditions. If we suppose that a fermion field will obey the same periodicity (4) as boson fields, then for each fermion degree of freedom we will obtain the result (5), but with opposite sign. In supersymmetric theories, where the number of fermion and boson degrees of freedom is the same, the combined energy-momentum tensor of the topological vacuum polarization will then vanish, and we will revert to the situation discussed above, wherein no barrier exists.

But if, as is more natural for fermion fields, we adopt the antiperiodic identification conditions

$$\psi(x + L) = -\psi(x),\ \psi(y + L) = -\psi(y),\ \psi(z + L) = -\psi(z) \qquad (6)$$

(the advantages of such a choice are explained by Ford[21]), then for a two-component neutrino field, say, we will obtain

$$T_0^0 \simeq -0.3914\,(aL)^{-4}. \qquad (7)$$

The total value of $T_0^0$, summed over the boson and fermion fields, turns out to be negative: $T_0^0 = -A/(aL)^4$, with $A > 0$.

The solution of the classical equations of evolution for the metric (1) with $a = b = c$, an effective cosmological constant, and the energy-momentum tensor (3) of the topological vacuum polarization takes the form

$$a(t) = L^{-1}\left(\frac{8\pi GA}{3H^2}\right)^{1/4} (\mathrm{ch}\, 2Ht)^{1/2} \qquad (8)$$

and is nonsingular, with $a(t)$ not vanishing. Accordingly a barrier does exist and can be tunneled through so as to permit quantum creation of the universe. In fact there is a general relationship between the existence of t-symmetric nonsingular solutions of the classical equations and the possibility of a quantum birth of the universe by tunneling toward such a solution at the instant $t = 0$.

Using the WKB approximation to the Wheeler–DeWitt equation, one can calculate the probability that the universe will tunnel from $a = 0$ to $a = a_{\min} = L^{-1}(8\pi GA/3H^2)^{1/4}$ in the same manner as for the creation of a closed universe (Ref. 14). The creation probability proves to be proportional to the quantity

$$w \propto \exp\left(-\frac{(6(\pi)^{1/4}\Gamma(1/4)}{3\Gamma(3/4)}\left(\frac{A^3}{GH^2}\right)^{1/4}\right) = \exp\left(-2.055\left(\frac{A^3}{GH^2}\right)^{1/4}\right). \qquad (9)$$

This result remains qualitatively correct even if there is a slight departure from cubic symmetry ($a \approx b \approx c$). But if $c \ll a, b$, then the energy tensor of the topological vacuum polarization will have a very different structure. In that event we will have, for scalar and two-component neutrino fields,[20]

$$T_0^0(s=0) = -\frac{\pi^2}{90c^4L^4},\quad T_0^0\left(s=\frac{1}{2}\right) = -\frac{7\pi^2}{360c^4L^4},$$
$$T_1^1 = T_2^2 = -\frac{1}{3}T_3^3 = T_0^0. \qquad (10)$$

With an effective cosmological constant and the topological vacuum polarization (10), the metric (1) will begin to evolve from the classical singularity, where

$$a \propto b \propto t^{-2/9}, \quad c \propto t^{1/3}. \tag{11}$$

Again there is no barrier, just as when the quasiclassical condition is violated, so we will have a classical "birth" from a singularity. As $t \to \infty$ the solution has the same asymptotic behavior as the solution (2), the inequalities $c \ll a$, b continuing to hold. For completeness' sake we would point out that if $c \ll a$, b the solution can also have another asymptote as $t \to \infty$: $a \propto b \propto \exp(Ht\sqrt{2})$, with c = const, but it will be exponentially unstable. The instability has a time scale of order $H^{-1}$.

The examples we have given show that an empty, flat world with 3-torus topology can begin to evolve either from a classical singularity or by quantum creation with or even without tunneling. The quantum birth would correspond to the situation where the identification parameters $a$L, bL, cL are of the same order along all three axes; for emergence from a classical singularity, the parameters would differ sharply. If, however, the de Sitter (inflationary) stage in the cosmological expansion was protracted enough, then the observations cannot tell us which of the two situations prevailed. Nor, as we have said, is there presently any way to compare the probabilities of a classical and a quantum creation of the universe.

1)This topological vacuum polarization is in addition to the vacuum polarization of all quantum fields due to the curvature of spacetime. The relative amplitude of these two forms of gravitational polarization will depend on the ratios between $(aL)^{-1}$, $(bL)^{-1}$, $(cL)^{-1}$, and $\dot{a}/a$, $\dot{b}/b$, $\dot{c}/c$; it may be arbitrarily small or large.


1Ya. B. Zel'dovich, Comments Astrophys. Space. Phys. 5, 169 (1973).
2D. D. Sokolov and V. F. Shvartsman, Zh. Eksp. Teor. Fiz. 66, 412 (1974) [Sov. Phys. JETP 39, 196 (1975)].
3D. D. Sokolov and A. A. Starobinskii, Astron. Zh. 52, 1041 (1975) [Sov. Astron. 19, 629 (1975)].
4E. P. Tryon, Nature 246, 396 (1973).
5P. I. Fomin, "Gravitational vacuum instability and the cosmological prob- [in Ukrainian], Dokl. Akad. Nauk Ukrain. SSR (1975), 831.
6Ya. B. Zel'dovich, Pis'ma Astron. Zh. 7, 579 (1981) [Sov. Astron. Lett. 7, 322 (1981)].
7L. P. Grishchuk and Ya. B. Zel'dovich, in: Quantum Gravitation [in Russian] (Proc. 2nd Seminar, Moscow, October 1981), Inst. Nucl. Res. USSR Acad. Sci., Moscow (1982), p. 39.
8A. Vilenkin, Phys. Lett. B 117, 25 (1982).
9A. Vilenkin, Phys. Rev. D 27, 2848 (1983).
10A. A. Starobinskii, Phys. Lett. B 91, 99 (1980).
11A. H. Guth, Phys. Rev. D 23, 347 (1981).
12A. D. Linde, Phys. Lett. B 108, 389 (1982).
13B. S. DeWitt, Phys. Rev. 160, 1113 (1967).
14A. A. Starobinskii, Pis'ma Zh. Eksp. Teor. Fiz. [JETP Lett.] (1984, in press).
15A. A. Starobinskii, Pis'ma Zh. Eksp. Teor. Fiz. 37, 55 (1983) [JETP Lett. 37, 66 (1983)].
16A. A. Starobinskii, in: Classical and Quantum Gravitation Theory [in Russian] (abstracts, 4th All-Union Gravitation Conf., Minsk, July 1976), Inst. Phys. Beloruss. Acad. Sci., Minsk (1976), p. 110.
17S. G. Mamaev and N. N. Trunov, Teor. i Mat. Fiz. 38, 345 (1979) [Theor. and Math. Phys. 38–41, 228 (1979)].
18S. G. Mamaev and N. N. Trunov, Izv. Vyssh. Uchebn. Zaved. Fizika (1979), No. 9, 51 [Sov. Phys. J. 22, 966 (1980)].
19R. Banach and J. S. Dowker, J. Phys. A 12, 2545 (1979).
20B. S. DeWitt, C. F. Hart, and C. J. Isham, Physica A 96, 197 (1979).
21L. H. Ford, Phys. Rev. D 21, 933 (1980).


# Intracluster gravitational separation of deuterium and helium in rich galaxy clusters


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**Diffusion of elements in the intergalactic gas may have significantly enhanced the deuterium, helium, and lithium abundance in the core regions of rich clusters of galaxies.**


## 1. INTRODUCTION

X-ray observations have revealed[1] that rich clusters of galaxies contain hot ($T_e \approx 10^7$–$10^8$°K), rarefied ($N_e \approx 10^{-3}$–$10^{-2}$ cm$^{-3}$) intergalactic gas. The x-ray lines of hydrogen- and heliumlike iron ions have been detected.[2,3]

Gravitational separation of elements may occur in the gravitational field of a cluster. Fabian and Pringle[4] and Rephaeli[5] have discussed this sedimentation mechanism with reference to iron. In this letter we consider the dynamics of the diffusion process. We show that the sedimentation mechanism will operate most efficiently (on a time scale of order $10^{16}$ sec) for the light elements — deuterium, helium, lithium; for elements with $Z \geqslant 16$, changes in distribution will be 10–20 times slower. Gravitational settling could enrich the central, dominant galaxy of a cluster with light elements, especially helium. Moreover, helium enhancement might affect the brightness profile of the intracluster x rays and could weaken the cosmic background radiation observed toward clusters.

One other very interesting possibility is that galaxies may have become enriched with light elements during the era when the protoclusters were developing.

## 2. INTRACLUSTER GRAVITATIONAL POTENTIAL

In the central part [$r \leqslant (5$–$6)a$, where $a$ is the radius of the core region] of a rich cluster, the galaxy distribu-