2026-02-02

Applications of Analog and Mixed-Signal Circuits in Electronic Systems

- Power and Energy: DC–DC converters, LDOs, battery management ICs.
- Signal Conditioning: Amplifiers, filters, analog front-ends for sensors.
- Data Conversion: ADCs, DACs for bridging analog digital domains.
- Timing and Clocking: PLLs/DLLs, crystal oscillators, clock distribution.
- High-Speed I/O: SerDes transceivers, equalizers, PHY interfaces.
- RF and Wireless: Mixers, RF front-end modules, modulators/demodulators.
- Sensing Interfaces: MEMS sensors, photodiodes, health monitoring front-ends.
- Automotive & Industrial: ADAS sensor fusion, process control, motor drives.
- Consumer and IoT: Audio codecs, touch interfaces, low-power radios.
- Healthcare Electronics: Patient monitors, diagnostic instrumentation.
- Edge and Embedded Computing: Mixed-signal accelerators, near-sensor processing.

In the future, with the growth of artificial intelligence and high-performance computing, there will be increasing demand for analog and mixed-signal circuits in high-speed data conversion, energy-efficient edge AI processors, sensor-rich autonomous systems, and ultra-low-power IoT devices, as these technologies require tight integration of analog physics with digital processing to maximize performance and efficiency.:contentReference[oaicite:7]index=7

Differnt Opteration Regions of MOSFET

(a) Linear (Ohmic) Region ($V_T < V_{GS}$):
Inversion channel formed. For small V_{DS}, I_D increases linearly with V_{DS}.

$$\begin{aligned} I_D &= Q_{\text{inv}} \cdot \mu_n E \cdot W \approx C_{\text{ox}}(V_{GS} - V_T) \cdot \mu_n \cdot \frac{V_{DS}}{L} \cdot W \\ &= \mu_n C_{\text{ox}} \left(\frac{W}{L}\right) (V_{GS} - V_T) V_{DS} \end{aligned} \tag{1}$$

(b) Nonlinear Region ($V_T < V_{GS}$, V_{DS} less than but close to $V_{GS} - V_T$):
Channel is formed, I_D no-longer rises linearly as V_{DS} increases. Channel begins to taper; device is transitioning toward saturation. Channel charge varies along the length:

$$Q_{\text{inv}}(y) = -C_{\text{ox}} \left(V_{GS} - V_{CS}(y) - V_T\right) \tag{2}$$

Channel charge density varies along the channel.

$$\begin{aligned} \int_0^L I dy &= \int_0^L C_{\text{ox}} \left(V_{GS} - V_{cS}(y) - V_T\right) \cdot \mu_n \frac{dV_{cs}(y)}{dy} \cdot dy \cdot W \\ \rightarrow I &= \mu_n C_{\text{ox}} \left(\frac{W}{L}\right) \left[(V_{GS} - V_T)V_{DS} - \frac{1}{2} V_{DS}^2\right] \end{aligned} \tag{3}$$

(c) Saturation Region ($V_T < V_{GS}$, $V_{GS} - V_T < V_{DS}$):
Channel pinched off near drain, I_D saturates and becomes independent of V_{DS} and is given by:

$$I_D = \frac{1}{2} \mu_n C_{\text{ox}} \left(\frac{W}{L}\right) (V_{GS} - V_T)^2 \tag{4}$$

(d) Subthreshold Region ($V_{GS} < V_T$):
A weak inversion layer begins to form under the gate, enabling a small diffusion current. There is no strong inversion, and the current flows primarily due to carrier diffusion rather than drift. This exponential behavior is characteristic here, where η is the subthreshold slope factor and D_N is the electron diffusion coefficient.

$$I_D \approx \frac{W}{L} Q_{N_0} D_N \exp\left(\frac{V_{GS} - V_T}{\eta \frac{kT}{q}}\right) \tag{5}$$

(e) Threshold Point ($V_{GS} \approx V_T$):
Weak inversion just formed under the gate, I_D starts to rise. Since $V_{GS} - V_T$ is very small, $V_{DS} > V_{GS} - V_T$ so the device is actually in the saturation region

$$I_D = \frac{1}{2} \mu C_{\text{ox}} \left(\frac{\text{W}}{L}\right) (V_{GS} - V_T)^2 \tag{6}$$

(f) Triode Region ($V_T < V_{GS}$, $0 < V_{DS} \leq V_{GS} - V_T$):
Well-formed inversion channel, I_D increases quadratically or exponentially depending on V_{DS}.

$$-I = \mu_n C_{\text{ox}} \left(\frac{W}{L}\right) \left((V_{GS} - V_T) - \frac{\text{V}_{\text{DS}}}{2}\right) V_{DS} \tag{7}$$

(Note: The term triode region ($0 < V_{DS} \leq V_{GS} - V_T$) refers to both the linear (ohmic) and nonlinear subregion combined.)

MOSFET Operating Regions from Channel Formation and Pinch-Off Conditions

Inversion exists if $V_T < V_{GS}$; Pinch-off occurs if $V_{GS} - V_T < V_{DS}$.

Gate Overdrive : V_{GS}, compare to V_T

Drain Perturbation: V_{DS}, compare to $V_{GS} - V_T$

	$V_{GS} < V_T$	$V_{GS} = V_T$	$V_T < V_{GS}$
$V_{GS} - V_T < V_{DS}$	cutoff	Threshold / Boundary	Saturation
$0 \ll V_{DS} \lesssim V_{GS} - V_T$	cutoff	Threshold / Boundary	Triode (Nonlinear)
$0 < V_{DS} \ll V_{GS} - V_T$	cutoff	Threshold / Boundary	Triode (Linear)
$V_{DS} = 0$	cutoff	Threshold / Boundary	Triode ($V_{DS} = 0$)

pinch-off occurs

Column Explanation (V_{GS}, compare to V_T)

- Column I ($V_{GS} < V_T$): No inversion channel exists. Drain voltage cannot create a channel. Therefore, always cutoff, regardless of V_{DS}.
- Column II ($V_{GS} = V_T$): This column is a boundary / threshold condition for discussion, not triode or saturation.
- Column III ($V_T < V_{GS}$): Region classification is about channel topology, not current magnitude.

Conclusion:

- If $V_T < V_{GS}$, a strong inversion layer forms at the surface: the depletion region is established first, and additional gate-induced electrons accumulate at the oxide–semiconductor interface to form a conductive channel.
- When $V_{GS} - V_T < V_{DS}$, the inversion charge density at the drain end goes to zero, and the channel pinches off at that point.
- "Linear" and "nonlinear" are subcases within the triode region and do not represent distinct physical operating regions.

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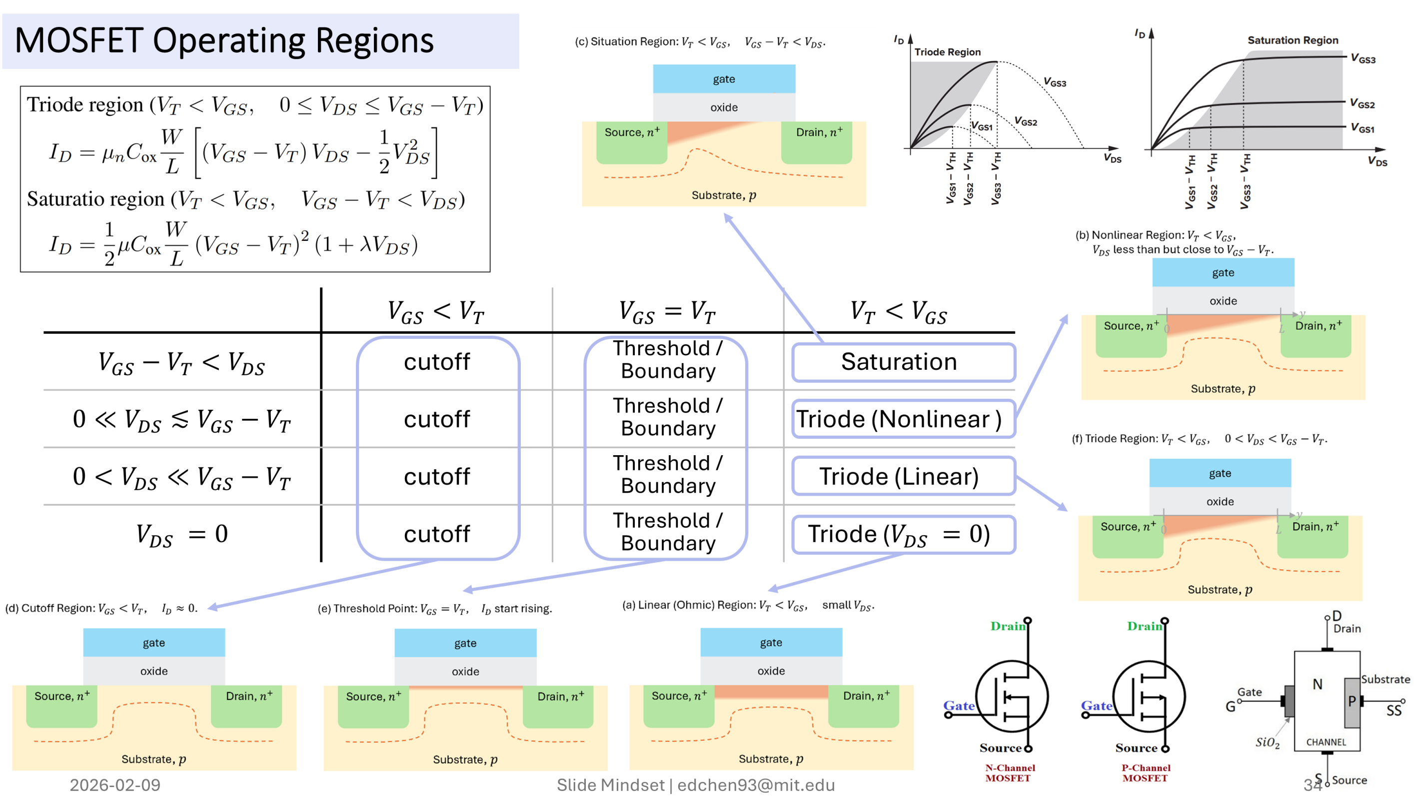

Figure 1: MOSFET Operating Regions.

Current Calculation in a MOSFET

We consider a long-channel nMOS transistor operating in strong inversion ($V_{GS} > V_T$) and the triode region.

Device geometry and coordinates

The MOSFET consists of a gate (G), source (S), and drain (D). We define the coordinate system as follows:

- y-axis: along the channel from source to drain, with channel length L
- z-axis: along the device width, with width W
- x-axis: perpendicular to the surface, pointing downward into the substrate

Carrier transport and current density

Electrons in the inversion channel move due to the lateral electric field $\varepsilon_y(y)$, with drift velocity v_{drift}, where μ_n is the electron mobility.

$$\boxed{v_{\text{drift}} = \mu_n \varepsilon_y,} \tag{8}$$

The local drain current density is $J(x, y)$, where q is the elementary charge and $n(x, y)$ is the electron concentration in the channel.

$$\boxed{\begin{aligned} J(x, y) &= q\, n(x, y)\, v_{\text{drift}} \\ &= q\, n(x, y)\, \mu_n \varepsilon_y, \end{aligned}} \tag{9}$$

Drain current

The drain current I_D is obtained by integrating the current density over the channel cross-section:

$$\begin{aligned} I_D &= \iint J(x, y)\, dx\, dz \\ &= W \int J(x, y)\, dx \\ &= W \mu_n \varepsilon_y \int q\, n(x, y)\, dx. \end{aligned} \tag{10}$$

The integral of charge density along the x-direction defines the mobile inversion charge per unit area,

$$Q_m(y) \equiv \int q\, n(x, y)\, dx.$$

Electrostatics of the inversion channel

Under the gradual channel approximation (GCA), the inversion channel is assumed to vary slowly along the transport direction, allowing the electrostatics in the vertical (x) direction to be treated independently at each position y. The mobile inversion charge density per unit area is therefore approximated as

$$Q_m(y) \approx C_{\text{ox}}\big(V_{GS} - V(y) - V_T\big),$$

where

- $C_{\text{ox}} = \dfrac{\varepsilon_{\text{ox}}}{t_{\text{ox}}}$ is the gate oxide capacitance per unit area,
- $V(y)$ is the local channel potential referenced to the source,
- V_T is the threshold voltage.

The lateral electric field driving carrier transport is related to the channel potential by

$$\varepsilon_y = \frac{dV(y)}{dy}.$$

Combining charge control through $Q_m(y)$ with drift–diffusion transport and integrating along the channel leads to the classical long-channel MOSFET current expressions. In saturation, this results in the Shichman–Hodges model, which describes the drain current as follow, where λ is the channel-length modulation parameter accounting for the finite output resistance in saturation.

$$\boxed{I_D = \frac{1}{2}\, \mu_n\, C_{\text{ox}}\, \frac{W}{L}\, (V_{GS} - V_T)^2\, (1 + \lambda V_{DS}),} \tag{11}$$

This model provides a compact, first-order link between MOSFET electrostatics, carrier transport, and the observed square-law I–V behavior of long-channel devices.

Conclusion

This analysis shows that MOSFET drain current arises from gate-controlled inversion charge transported by a lateral electric field. Within the gradual channel approximation, electrostatics and drift transport together provide a direct physical basis for the Shichman–Hodges model and its square-law I–V behavior.

Drain Current at Triode Region

We now derive the drain current of a long-channel nMOS transistor operating in the triode (linear) region. This section is self-contained and uses the same electrostatic and drift-transport framework as the saturation derivation, but with triode-region boundary conditions.

Assumptions

- Long-channel device operating in strong inversion.
- Pure drift transport with constant electron mobility μ_n.
- Gradual channel approximation (GCA).
- Negligible diffusion current, velocity saturation, mobility degradation, and DIBL.
- Triode-region condition defined by $0 \leq V_{DS} < V_{GS} - V_T$.

Channel charge and current under GCA

Let y denote the coordinate along the channel from source ($y = 0$) to drain ($y = L$), and let $V(y)$ be the local channel potential referenced to the source, so that

$$V(0) = 0, \qquad V(L) = V_{DS}.$$

Under GCA in strong inversion, the mobile inversion charge per unit area is

$$Q_m(y) = C_{\text{ox}}\big(V_{GS} - V(y) - V_T\big). \tag{12}$$

The local lateral electric field along the channel is

$$\varepsilon_y(y) = \frac{dV(y)}{dy}, \tag{13}$$

and the electron drift velocity is $v_{\text{drift}} = \mu_n \varepsilon_y$. The drain current (constant along y at steady state) is the width-scaled sheet-charge flux:

$$\begin{aligned} I_D &= W\, Q_m(y)\, v_{\text{drift}}(y) \\ &= W\, \mu_n\, Q_m(y)\, \varepsilon_y(y) \\ &= W\, \mu_n\, C_{\text{ox}}\big(V_{GS} - V(y) - V_T\big)\frac{dV(y)}{dy}. \end{aligned} \tag{14}$$

Integration along the channel (triode boundary condition)

Rearrange and integrate from source to drain. Since $V(y)$ increases monotonically from 0 to V_{DS} in the triode region,

$$I_D \int_0^L dy = W \mu_n C_{\text{ox}} \int_0^{V_{DS}} \big(V_{GS} - V - V_T\big)\, dV. \tag{15}$$

Evaluating the integral gives the following, and we can solve I_D to find the classical long-channel triode-region current.

$$I_D L = W \mu_n C_{\text{ox}} \left[(V_{GS} - V_T)\, V_{DS} - \frac{1}{2} V_{DS}^2\right]. \tag{16}$$

Final triode-region current expression

With $0 \leq V_{DS} < V_{GS} - V_T$, the drain current I_D in triode region is:

$$\boxed{I_D = \mu_n C_{\text{ox}} \frac{W}{L}\left[(V_{GS} - V_T)\, V_{DS} - \frac{1}{2} V_{DS}^2\right]} \tag{17}$$

As illustrated in Figure 2, the triode-region I_D–V_{DS} characteristics rise approximately linearly at small V_{DS}, where the device behaves as a voltage-controlled resistor with $I_D \approx \mu_n C_{\text{ox}} \frac{W}{L}(V_{GS} - V_T)V_{DS}$, corresponding to the near-linear slopes at the origin. As V_{DS} increases, the $-\frac{1}{2}V_{DS}^2$ term introduces the observed downward curvature, reflecting the gradual reduction of inversion charge toward the drain. The current reaches its maximum at $V_{DS} = V_{GS} - V_T$, marking the transition boundary to saturation for each V_{GS} curve shown.

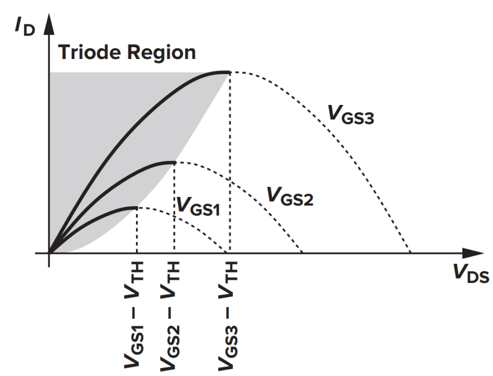

Figure 2: Drain current versus drain-source voltage in the triode region.[1]

Drain Current at Saturation Region

We now derive the drain current of a long-channel nMOS transistor operating in the saturation region. This section is self-contained and follows the same electrostatic and transport framework established previously.

Assumptions

- Long-channel device operating in strong inversion.
- Pure drift transport with constant electron mobility μ_n.
- Gradual channel approximation (GCA).
- Negligible diffusion current, velocity saturation, mobility degradation, and DIBL.
- Saturation condition defined by $V_{DS} \geq V_{GS} - V_T$.

Channel charge and current under GCA

At a position y along the channel, the mobile inversion charge per unit area $Q_m(y)$ is

$$Q_m(y) = C_{\text{ox}}\big(V_{GS} - V(y) - V_T\big). \tag{18}$$

The local lateral electric field $\varepsilon_y(y)$ along the channel from source to drain is

$$\varepsilon_y(y) = \frac{dV(y)}{dy}, \tag{19}$$

and the drift velocity of electrons in the channel v_{drift} is

$$v_{\text{drift}} = \mu_n \varepsilon_y. \tag{20}$$

The drain current I_D can be expressed as the product of the carrier drift velocity, the mobile inversion charge per unit area, and the effective cross-sectional area available for current flow. Specifically, the channel width W converts the sheet charge density into total charge per unit length, while the lateral electric field $\varepsilon_y(y) = dV(y)/dy$ drives carrier motion along the channel.

$$\begin{aligned} I_D &= W\mu_n Q_m(y)\varepsilon_y(y) \\ &= W\mu_n C_{\text{ox}}\big(V_{GS} - V(y) - V_T\big)\frac{dV(y)}{dy}. \end{aligned} \tag{21}$$

Integration along the channel

Rearranging and integrating from source to drain gives

$$I_D \int_0^L dy = W\mu_n C_{\text{ox}} \int_0^{V_{DS}} \big(V_{GS} - V - V_T\big)\, dV. \tag{22}$$

In saturation, the channel pinches off at the drain when

$$V_{DS} = V_{GS} - V_T,$$

so the upper limit of the voltage integral is $V_{GS} - V_T$. Performing the integration yields

$$\begin{aligned} I_D L &= W\mu_n C_{\text{ox}} \int_0^{V_{GS}-V_T} \big(V_{GS} - V - V_T\big)\, dV \\ &= W\mu_n C_{\text{ox}} \left[\frac{1}{2}\left(V_{GS} - V_T\right)^2\right]. \end{aligned} \tag{23}$$

Solving for the drain current gives the ideal long-channel saturation current:

$$I_{D,\text{sat}} = \frac{1}{2}\,\mu_n\, C_{\text{ox}}\, \frac{W}{L}\left(V_{GS} - V_T\right)^2. \tag{24}$$

Empirical channel-length modulation

In practice, the drain current in saturation exhibits a weak dependence on V_{DS} due to channel-length modulation (CLM), where the effective channel length shortens as the drain depletion region extends toward the source. Rather than deriving this effect from detailed electrostatics, it is commonly modeled empirically by introducing a first-order correction factor:

$$I_D = I_{D,\text{sat}}\left(1 + \lambda V_{DS}\right), \tag{25}$$

where λ is the channel-length modulation parameter (typically ranging from $0.01 \sim\ 0.1\ V^{-1}$), much smaller than unity for long-channel devices.

Final saturation current expression

Combining the ideal saturation current with the empirical CLM correction yields the classical Shichman–Hodges saturation-region expression:

$$\boxed{I_D = \frac{1}{2}\,\mu_n\, C_{\text{ox}}\, \frac{W}{L}\left(V_{GS} - V_T\right)^2\left(1 + \lambda V_{DS}\right).} \tag{26}$$

This equation provides a compact first-order description of MOSFET operation in saturation, linking gate-controlled inversion charge, drift transport, and the observed finite output resistance of real devices.

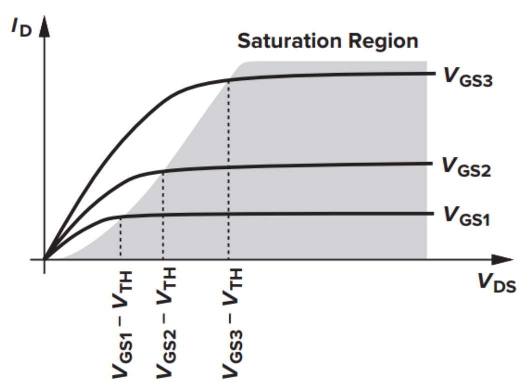

Figure 3: Saturation of drain current.[1]

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- **Linear (Ohmic) Region:** $V_T < V_{GS}$, small V_{DS}.
- **Nonlinear Region:** $V_T < V_{GS}$, V_{DS} less than but close to $V_{GS} - V_T$.
- **Saturation Region:** $V_T < V_{GS}$, $V_{GS} - V_T < V_{DS}$.
- **Cutoff Region:** $V_{GS} < V_T$, $I_D \approx 0$.
- **Threshold Point:** $V_{GS} = V_T$, I_D starts rising.
- **Triode Region:** $V_T < V_{GS}$, $0 < V_{DS} < V_{GS} - V_T$.

Important regios in MOSFET

- **Cutoff (Subthreshold)**: The gate voltage is below threshold ($V_{GS} < V_{TH}$), so no strong inversion channel is formed. Drain current is ideally zero in circuit models, but physically consists of exponentially small diffusion current.
- **Strong Inversion**: The gate voltage exceeds threshold ($V_{TH} < V_{GS}$), creating a continuous inversion layer at the surface. This is a gate-controlled condition and serves as an umbrella regime under which triode and saturation are defined.
  - **Linear Region (Triode Region)**: The transistor is in strong inversion and the drain voltage is small ($V_{DS} < V_{GS} - V_{TH}$). An inversion channel exists along the entire length, and the device behaves as a voltage-controlled resistor.
  - **Saturation Region (Active Region)**: The transistor is in strong inversion and the drain voltage is large enough to pinch off the channel near the drain ($V_{GS} - V_{TH} \le V_{DS}$). The drain current is primarily controlled by V_{GS} and is ideally independent of V_{DS}, enabling voltage gain.

I–V Characteristic of MOSFET in the Triode Region

We again begin from the local drain current density $J(x,y)$ in the inversion channel. Electron transport is assumed to be dominated by drift under the lateral electric field $\varepsilon_y(y)$, with electron mobility μ_n.

The local current density $J(x,y)$ is as follow, where $n(x,y)$ is the electron concentration in the inversion layer, and v_{drift} is the carrier drift velocity.

$$J(x,y) = q\,n(x,y)\,v_{\text{drift}} = q\,n(x,y)\,\mu_n \varepsilon_y(y), \tag{27}$$

The total drain current I_D is obtained by integrating the current density over the channel cross section, where W is the device width. Substituting the expression for $J(x,y)$ gives

$$\begin{aligned} I_D &= \iint J(x,y)\,dx\,dz = W \int J(x,y)\,dx \\ &= W\,\mu_n\,\varepsilon_y(y) \int q\,n(x,y)\,dx. \end{aligned} \tag{28}$$

The integral of the charge density along the vertical (x) direction defines the mobile inversion charge per unit area $Q_m(y)$ is:

$$Q_m(y) \equiv \int q\,n(x,y)\,dx. \tag{29}$$

Under the gradual channel approximation and strong inversion, it can be rewrite as follow, where C_{ox} is the gate oxide capacitance per unit area, V_{GS} is the gate–source voltage, V_T is the threshold voltage, and $V(y)$ is the local channel potential measured from the source.

$$Q_m(y) = C_{\text{ox}}\big(V_{GS} - V_T - V(y)\big), \tag{30}$$

Using the relation between lateral electric field and channel potential, $\varepsilon_y(y) = \frac{dV(y)}{dy}$, the drain current in 28 can be written as:

$$\rightarrow I_D = W\,\mu_n\,C_{\text{ox}}\big(V_{GS} - V_T - V(y)\big)\frac{dV(y)}{dy}. \tag{31}$$

In the triode (linear) region, the channel remains inverted along its entire length, so charge transport occurs through a continuous inversion layer from source to drain. The drain current is therefore obtained by integrating the local current contribution along the channel from $y = 0$ (source, $V = 0$) to $y = L$ (drain, $V = V_{DS}$), where L is the channel length and V_{DS} is the drain-source voltage. Once V_{GS} exceeds the threshold voltage V_T, the resulting $I_D - V_{DS}$ relationship can be expressed in closed form, yielding the following current expressions for the triode and saturation regions:

$$\boxed{\begin{aligned} &\text{Triode region } (V_T < V_{GS}, \quad 0 \le V_{DS} \le V_{GS} - V_T) \\ &\quad I_D = \mu_n C_{\text{ox}} \frac{W}{L}\left[(V_{GS} - V_T)\,V_{DS} - \frac{1}{2}V_{DS}^2\right] \\ &\text{Saturatio region } (V_T < V_{GS}, \quad V_{GS} - V_T < V_{DS}) \\ &\quad I_D = \frac{1}{2}\mu C_{\text{ox}} \frac{W}{L}\,(V_{GS} - V_T)^2\,(1 + \lambda V_{DS}) \end{aligned}} \tag{32}$$

In the triode region, the MOSFET drain current results from gate-controlled inversion charge transported by drift under a lateral electric field. The resulting I–V relationship is linear in V_{DS} for small drain bias, with a quadratic correction that reflects the gradual voltage drop along the channel.

Strong Inversion MOSFET

What are the difference between Strong Inversion, Linear region, Saturation region, and Triode Regio. They are probabiliy umbrella terms under each other. Try to follow the definition in "Design of Analog CMOS Integrated Circuits", Book by Behzad Razavi.

Relation between V_{GS} and I_D in strong inversion Linear region (small V_{DS})

$$I_D = \mu C_{\text{ox}} \frac{W}{L} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

Saturation region

$$I_D = \frac{1}{2} \mu C_{\text{ox}} \frac{W}{L} (V_{GS} - V_{TH})^2$$

Strong inversion MOSFET

In the strong inversion regime, the surface potential at the semiconductor–oxide interface is sufficiently large that a well-defined inversion layer is formed, and carrier transport along the channel is dominated by drift. For $V_{GS} > V_T$, the inversion charge density increases approximately linearly with the gate overdrive $(V_{GS} - V_T)$, leading to the familiar square-law dependence of drain current on gate voltage in long-channel devices. In saturation, the pinch-off condition occurs near the drain when $V_{DS} = V_{GS} - V_T$, and channel length modulation introduces a weak dependence of I_D on V_{DS} through the factor $(1 + \lambda V_{DS})$.

Physically, this regime can be interpreted using a voltage-controlled resistor or current-source picture, where the gate voltage controls the amount of mobile charge in the channel and the lateral electric field sets the drift velocity. This strong-inversion description forms the basis of the classical Shichman–Hodges model and is most accurate for long-channel MOSFETs under moderate electric fields.

Subthreshold Region

The subthreshold (or weak inversion) region corresponds to gate voltages slightly below threshold, typically within a few kT/q of V_T, where no strong inversion layer is formed but a finite drain current still flows. In this regime, carrier transport is dominated by diffusion rather than drift, and the drain current exhibits an exponential dependence on gate voltage,

$$I_D = I_0 e^{\phi_s / V_T} = I_0 e^{V_{GS}/(nV_T)} (1 + \lambda V_{DS}),$$

where ϕ_s is the surface potential, $V_T = kT/q$ is the thermal voltage, and $n = 1 + C_B/C_{\text{ox}}$ is the subthreshold slope factor accounting for body coupling.

On a semilogarithmic I_D–V_{GS} plot, this exponential behavior appears as a straight line, with an ideal slope limit of 60 mV/dec at room temperature and larger values (e.g. $\sim 100 \text{ mV/dec}$) in practical devices. In this sense, a MOSFET operating in subthreshold behaves mathematically and physically similar to a bipolar transistor, with gate voltage playing a role analogous to the base–emitter voltage.

Subthreshold Region (Weak Inversion)

The subthreshold (or weak inversion) region corresponds to gate voltages slightly below threshold, typically within a few thermal voltages of V_T, where no strong inversion layer exists but a finite drain current still flows:

$$V_{GS} V_T, \quad I_D \neq 0 \tag{33}$$

In this regime, carrier transport is dominated by <u>diffusion</u> rather than drift. The surface electron concentration varies exponentially with surface potential, leading to an exponential dependence of drain current on gate voltage. The drain current can be expressed as

$$\boxed{I_D = I_0 \exp\left(\frac{\phi_s}{V_T}\right) = I_0 \exp\left(\frac{V_{GS}}{nV_T}\right) \left(1 - e^{-V_{DS}/V_T}\right),} \tag{34}$$

where

- $V_T = kT/q$ is the thermal voltage,
- $n = 1 + \dfrac{C_B}{C_{\text{ox}}}$ is the subthreshold slope factor,
- C_B is the depletion (body) capacitance per unit area.

For $V_{DS} 3V_T$, the current saturates with respect to V_{DS} and the expression simplifies to

$$\boxed{I_D \approx I_0 \exp\left(\frac{V_{GS}}{nV_T}\right).} \tag{35}$$

On a semilogarithmic I_D–V_{GS} plot, this appears as a straight line. The ideal subthreshold swing is

$$\boxed{S \equiv \frac{dV_{GS}}{d(\log_{10} I_D)} = n\, V_T \ln 10 \geq 60 \text{ mV/dec} \quad (300 \text{ K}),} \tag{36}$$

with equality only in the ideal case $n = 1$.

Cutoff vs. Subthreshold. In circuit-level abstraction, <u>cutoff</u> assumes $I_D = 0$ for $V_{GS} < V_T$. Physically, this corresponds to ignoring the subthreshold diffusion current. Thus, cutoff is a modeling idealization, whereas subthreshold is a real operating regime.

Strong Inversion as an Umbrella Regime

Strong inversion is defined solely by channel formation:

$$\boxed{V_{GS} > V_T \quad \Rightarrow \quad \text{inversion layer exists.}}$$

Once inversion exists, region classification is determined by the drain perturbation relative to the gate overdrive:

$$V_{DS} \text{ compared to } V_{GS} - V_T.$$

- **Triode region** ($0 \le V_{DS} < V_{GS} - V_T$): The channel exists along the entire length; current is controlled by both V_{GS} and V_{DS}.
- **Saturation region** ($V_{DS} \ge V_{GS} - V_T$): The inversion charge at the drain end goes to zero; the channel pinches off and current becomes primarily controlled by V_{GS}.

Thus, triode and saturation are subcases of strong inversion, not independent regimes.

Linear vs. Nonlinear Triode Operation

Within the triode region, it is useful (but not fundamental) to distinguish:

- **Linear triode**: $V_{DS} \ll V_{GS} - V_T$, where

  $$I_D \approx \mu_n C_{\text{ox}} \frac{W}{L} (V_{GS} - V_T) V_{DS},$$

  and the MOSFET behaves as a voltage-controlled resistor.
- **Nonlinear triode**: V_{DS} approaches $V_{GS} - V_T$, where channel charge varies significantly along y and the quadratic term becomes important.

"Linear" and "nonlinear" do not define distinct physical regions; they are operating limits within triode operation.

Moderate Inversion (Transition Region)

Moderate inversion bridges weak and strong inversion:

$$\boxed{V_{GS} \approx V_T \pm \text{a few } V_T.}$$

In this region, neither pure diffusion nor pure drift dominates, and no single closed-form expression accurately describes I_D. Nevertheless, this regime is extremely important for analog design due to its favorable transconductance efficiency.

Transconductance Efficiency Across Regions

A unifying design metric is the transconductance efficiency $\frac{g_m}{I_D}$.

$$\boxed{\frac{g_m}{I_D} = \begin{cases} \dfrac{1}{nV_T}, & \text{subthreshold (weak inversion)} \\ \dfrac{2}{V_{GS} - V_T}, & \text{strong inversion (saturation)} \end{cases}} \tag{37}$$

This explains why weak and moderate inversion are preferred for low-power analog design, while strong inversion is favored for speed and large signal swing.

Velocity Saturation and Breakdown of the Square-Law

In short-channel devices, the carrier drift velocity saturates at high lateral electric fields:

$$v_{\text{drift}} \to v_{\text{sat}},$$

causing the drain current to deviate from the square-law behavior. In this case, the saturation current scales approximately linearly with gate overdrive:

$$\boxed{I_D \propto W\, C_{\text{ox}} (V_{GS} - V_T)\, v_{\text{sat}}.} \tag{38}$$

As a result, classical long-channel models overestimate both I_D and g_m in deep-submicron technologies.

Summary of Region Classification (Channel Perspective)

$$\boxed{\begin{aligned} & V_{GS} < V_T \\ & \qquad \to \text{No inversion (cutoff / subthreshold)} \\ & V_T < V_{GS},\ V_{DS} < V_{GS} - V_T \\ & \qquad \to \text{Strong inversion, triode} \\ & V_T < V_{GS},\ V_{GS} - V_T \le V_{DS} \\ & \qquad \to \text{Strong inversion, saturation (pinch-off)} \end{aligned}} \tag{39}$$

References

1. Razavi, B. Design of Analog CMOS Integrated Circuits 2nd ed. Accessed: 2026-02-06. ISBN: 978-0072524932. https://www.academia.edu/76121731/Design_of_Analog_CMOS_Integrated_Circuits_Behzad_Razavi_z_lib_org_ (McGraw-Hill Education, New York, NY, 2017).