Topics in Reinforcement Learning: Rollout and Approximate Policy Iteration

ASU, CSE 691, Spring 2021

Links to Class Notes, Videolectures, and Slides at
http://web.mit.edu/dimitrib/www/RLbook.html

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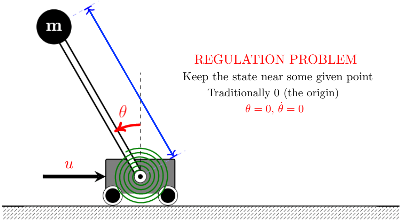
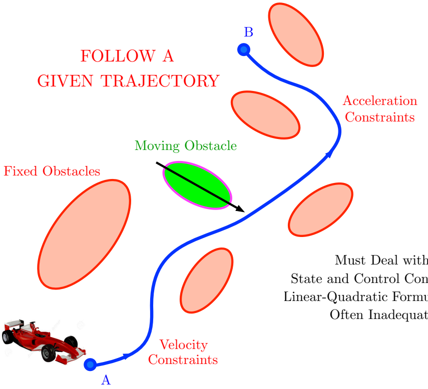
Lecture 6
Model Predictive Control, Multiagent Rollout

Outline

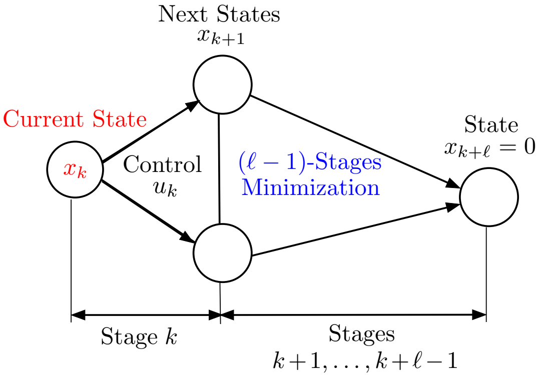
Classical Control Problems - Infinite Control Spaces





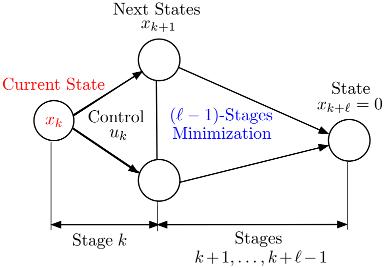
Must Deal with
State and Control Constraints
Linear-Quadratic Formulation is
Often Inadequate

Model Predictive Control (MPC) for Regulation to the Origin Problems



- System: $x_{k+1} = f(x_k, u_k)$; 0 is an absorbing state, $f(0, u) \equiv 0$.
- Cost per stage: $g(x_k, u_k) > 0$, except that 0 is cost-free, $g(0, u) \equiv 0$.
- Control constraints: $u_k \in U(x_k)$ for all k. Perfect state information.
- MPC: At x_k solve an ℓ-step lookahead version of the problem, requiring $x_{k+\ell} = 0$ (ℓ: fixed and sufficiently large to allow the transfer to 0).
- If $\{\tilde{u}_k, \dots, \tilde{u}_{k+\ell-1}\}$ is the control sequence so obtained, apply $\tilde{u}_k$, discard $\tilde{u}_{k+1}, \dots$

MPC Relation to Rollout



- MPC is rollout w/ base heuristic the $(\ell-1)$-step min to 0 (and stay at 0).
- Let $H(x)$ denote the optimal cost of the $(\ell-1)$-step min, starting from x.
- This heuristic is sequentially improving (not sequentially consistent), i.e.,

$$\underbrace{\min_{u\in U(x)}\big[g(x,u)+H\big(f(x,u)\big)\big]}_{\substack{\text{opt cost from } x \text{ to 0 in } \ell \text{ steps}\\ \text{then stay at 0 for additional steps}}} \leq \underbrace{H(x)}_{\substack{\text{opt cost from } x \text{ to 0 in } (\ell-1) \text{ steps}\\ \text{then stay at 0 for additional steps}}}$$

  because (opt. cost to reach 0 in ℓ steps) $\leq$ (opt. cost to reach 0 in $\ell-1$ steps)

- Sequential improvement $\rightarrow$ "stability": For all N, $\sum_{k=0}^{N} g(\tilde{x}_k,\tilde{u}_k)\leq H(x_0)<\infty$, where $\{x_0,\tilde{u}_0,\tilde{x}_1,\tilde{u}_1,\ldots\}$ is the state and control sequence generated by MPC from x_0.

A Major Variant: MPC with Terminal Cost (Applies also for $\ell = 1$)

- At state x_0, instead of requiring that $x_\ell = 0$, we solve

$$\min_{u_i,\, i=0,\ldots,\ell-1} \left[G(x_\ell) + \sum_{i=0}^{\ell-1} g(x_i, u_i) \right],$$

subject to $u_i \in U(x_i)$ and $x_{i+1} = f(x_i, u_i)$. We assume that for all x,

$$\min_{u \in U(x)} \Big[g(x, u) + G\big(f(x, u)\big) \Big] \leq G(x) \qquad \text{(a “Lyapunov condition")}$$

- This heuristic is sequentially improving, implying stability. Proof: Given x_0, for some control and state sequences $(\bar{u}_0, \ldots, \bar{u}_{\ell-1})$ and $(\bar{x}_1, \ldots, \bar{x}_\ell)$,

$$H(x_0) = G(\bar{x}_{\ell-1}) + g(x_0, \bar{u}_0) + \sum_{i=1}^{\ell-2} g(\bar{x}_i, \bar{u}_i) \quad \text{(Use the definition of } H\text{)}$$

$$\geq G(\bar{x}_\ell) + g(x_0, \bar{u}_0) + \sum_{i=1}^{\ell-1} g(\bar{x}_i, \bar{u}_i) \quad \text{(Apply the Lyapunov cond. for } x = \bar{x}_{\ell-1}\text{)}$$

$$\geq \min_{u_i,\, i=0,\ldots,\ell-1} \left[G(x_\ell) + g(x_0, u_0) + \sum_{i=1}^{\ell-1} g(x_i, u_i) \right] \quad \text{(Opt. cost of } \ell\text{-step probl.)}$$

$$= \min_{u \in U(x_0)} \Big[g(x_0, u_0) + H\big(f(x_0, u_0)\big) \Big] \quad \text{(Use the definition of } H\text{)}$$

Other Variants of MPC

MPC with state/safety/tube constraints: $x_k \in X$ for all k

- Special difficulties arise because the tube constraint may be impossible to satisfy for some states $x_0 \in X$
- This leads to the methodology of reachability of target tubes, i.e., constructing an inner tube from within which the state constraints can be met (see video lecture 5 of 2019 course offering)

Simplified MPC

- Since MPC can be viewed in the context of rollout, the methodology of simplified rollout can be used
- Assuming a heuristic cost $H(x)$ that satisfies the sequential improvement condition

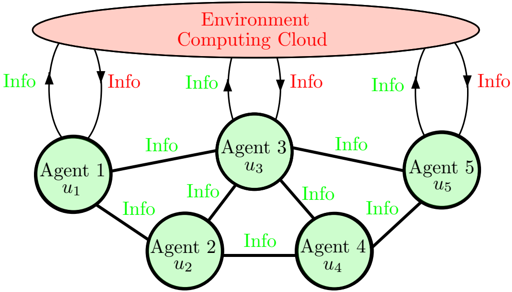
$$\min_{u \in U(x)} \big[g(x,u) + H\big(f(x,u)\big)\big] \leq H(x), \qquad \text{for all } x,$$

use at state x any control $\tilde{\mu}(x)$ such that

$$g\big(x, \tilde{\mu}(x)\big) + H\Big(f\big(x, \tilde{\mu}(x)\big)\Big) \leq H(x)$$

Other variants: Time-varying system, stochastic problem, multiagent, etc

Multiagent Problems - A Very Old (1960s) and Well-Researched Field

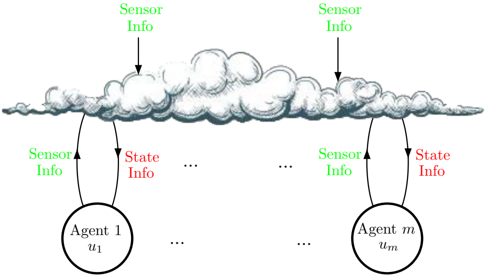


- Multiple agents collecting and sharing information selectively with each other and with an environment/computing cloud
- Agent i applies decision u_i sequentially in discrete time based on info received

The major mathematical distinction between structures

- The classical information pattern: Agents are fully cooperative, fully sharing and never forgetting information. Can be treated by Dynamic Programming (DP)
- The nonclassical information pattern: Agents are partially sharing information, and may be antagonistic. HARD because it cannot be treated by DP

Our Starting Point: A Classical Information Pattern ... but we will Generalize



The agents have exact state info, and choose their controls as functions of the state

Model: Stochastic DP (finite or infinite horizon) with state x and control u

- Decision/control has m components $u = (u_1, \ldots, u_m)$ corresponding to m "agents"
- "Agents" is just a metaphor - the important math structure is $u = (u_1, \ldots, u_m)$
- We apply approximate DP/rollout ideas, aiming at faster computation in order to:
  - Deal with the exponential size of the search/control space
  - Be able to compute the agent controls in parallel (in the process we will deal in part with nonclassical info pattern issues)

To simplify notation, consider infinite horizon setting. The standard rollout operation is

$$\big(\tilde\mu_1(x), \ldots, \tilde\mu_m(x)\big) \in \arg \min_{(u_1,\ldots,u_m)} E_w\Big\{g(x, u_1, \ldots, u_m, w) + \alpha J_\mu\big(f(x, u_1, \ldots, u_m, w)\big)\Big\};$$

the search space is exponential in m (μ is the base policy, seq. consistency holds)

Multiagent rollout (a form of simplified rollout; implies cost improvement)

Perform a sequence of m successive minimizations, one-agent-at-a-time

$$\tilde\mu_1(x) \in \arg\min_{u_1} E_w\Big\{g(x, u_1, \mu_2(x), \ldots, \mu_m(x), w) + \alpha J_\mu\big(f(x, u_1, \mu_2(x), \ldots, \mu_m(x), w)\big)\Big\}$$

$$\tilde\mu_2(x) \in \arg\min_{u_2} E_w\Big\{g(x, \tilde\mu_1(x), u_2, \mu_3(x) \ldots, \mu_m(x), w) + \alpha J_\mu\big(f(x, \tilde\mu_1(x), u_2, \mu_3(x), \ldots, \mu_m(x), w)\big)$$

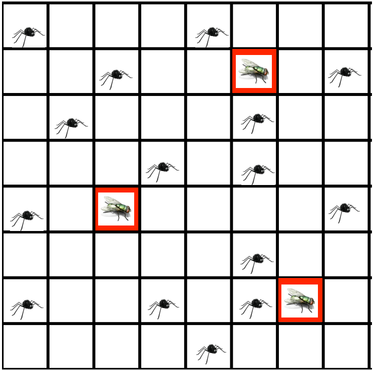
$$\cdots \qquad \cdots \qquad \cdots \qquad \cdots$$

$$\tilde\mu_m(x) \in \arg\min_{u_m} E_w\Big\{g(x, \tilde\mu_1(x), \tilde\mu_2(x), \ldots, \tilde\mu_{m-1}(x), u_m, w) + \alpha J_\mu\big(f(x, \tilde\mu_1(x), \tilde\mu_2(x), \ldots, \tilde\mu_{m-1}(x), u$$

- Has a search space with size that is linear in m; ENORMOUS SPEEDUP!

Survey reference: Bertsekas, D., "Multiagent Reinforcement Learning: Rollout and Policy Iteration," IEEE/CAA J. of Aut. Sinica, 2021 (and earlier papers quoted there).

Spiders-and-Flies Example
(e.g., Delivery, Maintenance, Search-and-Rescue, Firefighting)



15 spiders move in 4 directions with perfect vision

3 blind flies move randomly

- Objective is to catch the flies in minimum time
- At each time we must select one out of $\approx 5^{15}$ joint move choices
- Multiagent rollout reduces this to $5 \cdot 15 = 75$ choices (while maintaining cost improvement); applies a sequence of one-spider-at-a-time moves
- Later, we will introduce “precomputed signaling/coordination" between the spiders, so the 15 spiders will choose moves in parallel (extra speedup factor of up to 15)

Four Spiders and Two Flies: Illustration of Various Forms of Rollout

Video: Base Policy

Video: Standard Rollout

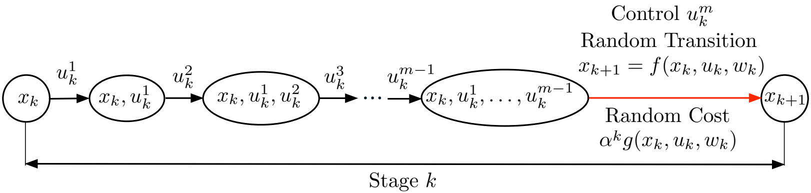
Video: Mutiagent Rollout

Base policy: Move along the shortest path to the closest surviving fly (in the Manhattan distance metric). No coordination.

Time to catch the flies

- Base policy (each spider follows the shortest path): Capture time = 85
- Standard rollout (all spiders move at once, $5^4 = 625$ move choices): Capture time = 34
- Agent-by-agent rollout (spiders move one at a time, $4 \cdot 5 = 20$ move choices): Capture time = 34

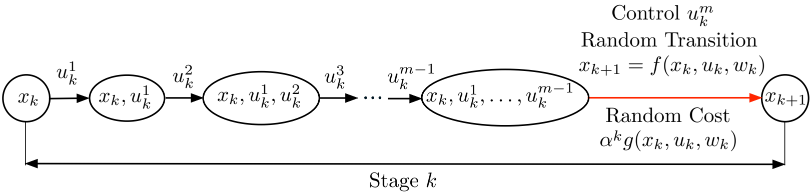
Let's Take a Working Break



Think about an equivalent problem reformulation for multiagent rollout

- "Unfold" the control action
- Consider standard (not multiagent) rollout for the reformulated problem
- What about cost improvement?

Justification of Cost Improvement through Reformulation: Trading off Control and State Complexity (NDP Book, 1996)



An equivalent reformulation - “Unfolding" the control action

- The control space is simplified at the expense of $m - 1$ additional layers of states, and corresponding $m - 1$ cost functions

$$J^1(x, u_1), J^2(x, u_1, u_2), \ldots, J^{m-1}(x, u_1, \ldots, u_{m-1})$$

- Multiagent rollout is just standard rollout for the reformulated problem
- The increase in size of the state space does not adversely affect rollout (only one state per stage is looked at during on-line play)
- Complexity reduction: The one-step lookahead branching factor is reduced from n^m to $n \cdot m$, where n is the number of possible choices for each component u_i

Multiagent MPC (A Form of Simplified MPC)

Consider MPC where u_k consists of both discrete and continuous components

$$u_k = (y_k^1, \ldots, y_k^m, v_k),$$

where $y_k^1, \ldots, y_k^m$ are discrete, and v_k is continuous.

- For example $y_k^1, ..., y_k^m$ may be system configuration variables, and v_k may be a multidimensional vector with real components (e.g., as in linear quadratic control).
- The base policy may consist of a "nominal configuration" $\bar{y}_k^1, ..., \bar{y}_k^m$ (that depends on the state x_k), and a continuous control policy that drives the state to 0 in $(\ell - 1)$ steps with minimum cost.
- In a component-by-component version of MPC, at state x_k:
  - $y_k^1, \ldots, y_k^m$ are first chosen one-at-a-time, and with all future components fixed at the values determined by the nominal configuration/base policy.
  - Then the continuous component v_k is chosen to drive the state to 0 in ℓ steps at minimum cost with the discrete components fixed.
- This simplifies lookahead minimization by:
  - Separating the "difficult" minimization over $y_k^1, \ldots, y_k^m$ from the continuous minimization over v_k
  - Optimizing over $y_k^1, \ldots, y_k^m$ one-at-a-time (simpler integer programming problem).
- Maintains the cost improvement/stability property of MPC.

Parallelization of Agent Actions in Multiagent Rollout: Allowing for Agent Autonomy

Multiagent rollout/policy improvement is an inherently serial computation. How can we parallelize it, to get extra speedup, and also deal with agent autonomy?

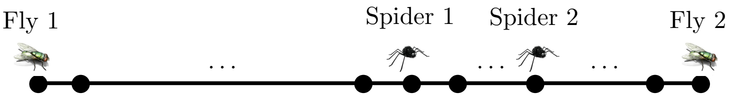
Precomputed signaling

- Obstacle to parallelization: To compute the agent ℓ rollout control we need the rollout controls of the preceding agents $i < \ell$
- Signaling remedy: Use precomputed substitute "guesses" $\widehat{\mu}_i(x)$ in place of the preceding rollout controls $\tilde{\mu}_i(x)$

Signaling possibilities

- Use the base policy controls for signaling $\widehat{\mu}_i(x) = \mu_i(x)$, $i = 1, \ldots, \ell - 1$ (this may work poorly)
- Use a neural net representation of the rollout policy controls for signaling $\widehat{\mu}_i(x) \approx \tilde{\mu}_i(x)$, $i = 1, \ldots, \ell - 1$ (this requires training/off-line computation)
- Other, problem-specific possibilities

The Pitfall of Using the Base Policy for Signaling



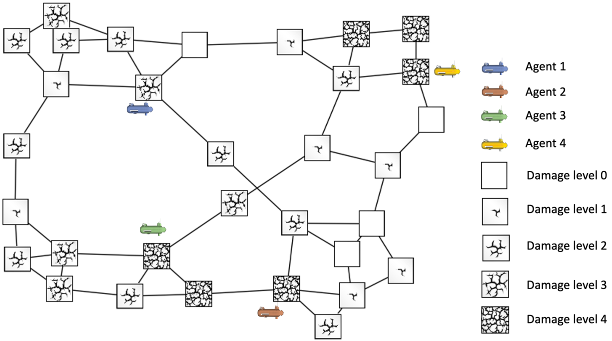
Two spiders trying to catch two stationary flies in minimum time

- The spiders have perfect vision/perfect information. The flies do not move.
- Base policy for each fly: Move one step towards the closest surviving fly

Performance of various algorithms

- Optimal policy: Split the spiders towards their closest flies
- Standard rollout is optimal for all initial states (it can be verified)
- Agent-by-agent rollout is also optimal for all initial states (it can be verified)
- Agent-by-agent rollout with base policy signaling is optimal for “most" initial states, with A SIGNIFICANT EXCEPTION
- When the spiders start at the same location, the spiders oscillate and never catch the flies

Multirobot Repair of a Network of Damaged Sites (2020 Paper by Bhatacharya, Kailas, Badyal, Gil, DPB, from my Website)



- Damage level of each site is unknown, except when inspected. It deteriorates according to a known Markov chain unless the site is repaired (this is a POMDP)
- Control choice of each robot: Inspect and repair (which takes one unit time), or inspect and move to a neighboring site
- State of the system: The set of robot locations, plus the belief state of the site damages
- Stage cost at each unrepaired site: Depends on the level of its damage

Videos: Multirobot Repair in a Network of Damaged Sites (Agents Start from the Same Location)

Video: Base Policy (Shortest Path/No Coordination)

Video: Multiagent Rollout

Video: Multiagent with Base Policy Signaling

Video: Multiagent with Policy Network Signaling

Cost comparisons

- Base policy cost: 5294 (30 steps)
- Multiagent rollout : 1124 (9 steps)
- Multiagent Rollout with base policy signaling: 31109 (Never stops)
- Multiagent Rollout with neural network policy signaling: 2763 (15 steps)

We will return to this problem in the future (in the context of infinite horizon policy iteration)

About the Next Lecture

We will cover:

- Rollout algorithms for constrained deterministic problems
- Applications in combinatorial and discrete optimization

Next Homework

- Exercise 2.3 of latest version of class notes
- Due Sunday, Feb. 28

Optional/Recommended Homework

- Verify the statements on the “Pitfall" slide for the two-spider-two-flies example

About questions on your project

- Send me email (dbertsek@asu.edu)
- Make appointment to talk by zoom (there are no fixed office hours in this course)