March 8, 2009

Here are more type A and B problems.

A10. Verify that the dual of

$$\min_{u\in\mathbb{R}^{N^2}} \ \frac{1}{2}\|u\|_2^2 + \tau' \sum_{1\le i,j\le N} \|A^{ij}u\|_2.$$

is

$$\begin{array}{ll} \max_{x\in\mathbb{R}^{2N^2}} & -\frac{1}{2}\left\| \sum_{1\le i,j\le N} (A^{ij})^* x^{ij} \right\|_2^2 - \left\langle b, \sum_{1\le i,j\le N} (A^{ij})^* x^{ij} \right\rangle \\ \text{s.t.} & \|x^{ij}\|_2 \le \tau' \quad \forall 1\le i,j\le N. \end{array}$$

A11. (SDP representation of nucleare norm). For any $X\in\mathbb{S}^n$, consider the SDP:

$$\begin{array}{ll} \min_{W,Z} & \frac{1}{2}(\mathrm{tr}[W] + \mathrm{tr}[Z]) \\ \text{s.t.} & \begin{bmatrix} W & X \\ X & Z \end{bmatrix} \succeq 0. \end{array}$$

(a) Show that $W = Z = (X^2)^{1/2}$ is feasible for this SDP.

(b) Find the dual problem.

(c) Show that $\mathrm{Sign}(X)$ is feasible for the dual problem with the same objective function value as in (a), where $\mathrm{Sign}(X)$ is obtained from X by replacing the eigenvalues in its eigen-decomposition by their signs.

B7. (logistic regression). Consider the logistic regression function $f(x) = \ell(Ax)$, where $\ell(u) = \sum_{i=1}^m \log(1+e^{u_i}) - b_i u_i$.

(a) Show that $\nabla\ell$ is Lipschitz continuous with constant $\frac{1}{4}$.

(b) Show that ∇f is Lipschitz continuous with constant $\frac{\lambda_{\max}(A^TA)}{4}$.

B8. Show that when the proximal gradient method is applied to solve $\min_{x\le 0} e^x$, starting at any $x_0\le 0$, the number of iterations k to achieve $e^{x_k}\le\epsilon$ is at least in the order of $1/\epsilon$ for any $\epsilon>0$ sufficiently small.