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Here are more type A and B problems.

A7. (Primal-dual interior-point method for SDP). Show that the Monteiro-Zhang Newton equation for SDP:

$$\mathrm{Sym}[PUYP^{-1}] + \mathrm{Sym}[PXVP^{-1}] = \epsilon I - \mathrm{Sym}[PXYP^{-1}]$$

where $X, Y \succ 0$ and $P \in \mathbb{R}^{n\times n}$ is nonsingular, reduces to the Nesterov-Todd Newton equation

$$(X \circ Y)^{1/2}(X^{-1} \circ U)(X \circ Y)^{1/2} + X \circ V = \epsilon I - X \circ Y$$

when $P = W^{1/2}$ and $W = X^{-1/2}(X \circ Y)^{1/2}X^{-1/2}$. Here $X \circ Y = X^{1/2}YX^{1/2}$. (Hint: Multiply the MZ equation left and right by P^T and P^{-T}. Also, show that W satisfies $WXW = Y$. In fact, W is the unique solution of this equation and is called the metric geometric mean of X^{-1} and Y.)

B4. (AHO direction for SDP). Assume $\mathrm{Null}(\mathcal{A}^*) = \{0\}$. Show that the AHO Newton equation

$$\mathcal{A}U = 0, \quad V = \mathcal{A}^* w, \quad L_X U + L_Y V = R$$

has a unique solution U, V for any $X, Y \succ 0$ with $\mathrm{Sym}[XY] \succeq 0$ and any $R \in \mathbb{S}^n$, where $L_X U = \mathrm{Sym}[UY]$ and $L_Y V = \mathrm{Sym}[XV]$. (Hint: Show that the linear mapping $L_X^{-1} L_Y$ is positive definite (and hence invertible), i.e., $\langle V, L_X^{-1} L_Y V\rangle > 0$ whenever $V \neq 0$.)

B5. (NT and HRVW/M/KSH directions for SDP). This is the same question as B4, except that $L_X U = \mathrm{Sym}[(X \circ Y)^\tau (X^{-1} \circ U)(X \circ Y)^{1-\tau}]$ $(0 \leq \tau \leq \frac{1}{2})$ and $L_Y V = X \circ V$ instead. Here we do not assume $\mathrm{Sym}[XY] \succeq 0$.