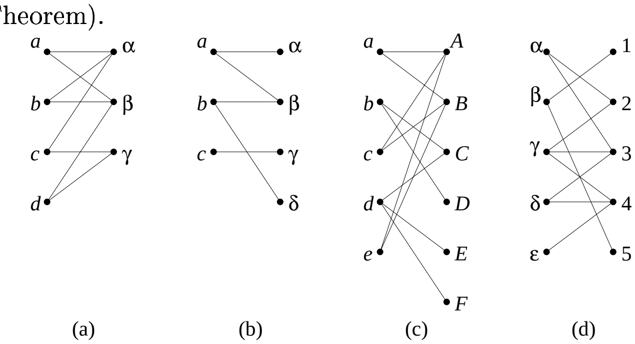
[Homework can be handed in to me or to my mail box in the Math Lounge (opposite the Math main office). Please show your work to receive full credit.]

A. For each bipartite graph $(X \cup Y, A)$ of shown below (X is the subset of vertices on the left), determine if it has an X-saturating matching (i.e., a matching of cardinality $|X|$). If yes, find such a matching. If no, explain why (using Hall's Theorem).



B. [Applied Combinatorics, Sec. 12.2 # 9] There are 6 committees of a state legislature, shown in the columns below. There are 10 legislators who need to be assigned to chair the committees. The following matrix has its (i, j)th entry equal to 1 iff the ith legislator can chair the jth committee.

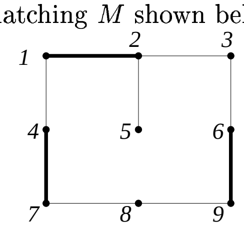
	Finance	*Environment*	*Health*	*Transportation*	*Education*	*Housing*
Allen	1	1	1	0	0	0
Barnes	1	1	0	1	1	0
Cash	1	1	1	0	0	0
Dunn	1	0	0	1	1	1
Ecker	0	1	1	0	0	0
Frank	1	1	0	0	0	0
Graham	1	1	1	0	0	0
Hall	1	0	0	0	0	0
Inman	1	1	1	0	0	0
Johnson	1	1	0	0	0	0

Suppose we wish to choose exactly one legislator to chair each committee, and no legislator can chair more than one committee. Is this possible? Explain your answer.

C. To improve computer security, a company has put in 20 special passwords. Each password is known by exactly two people in the company. (Which two people know the same password is *given*.) We wish to find the smallest set of people who together know all the passwords. Describe how this problem can be formulated as a minimum vertex covering problem. The graph in your answer need not be bipartite, however.

D. Consider the bipartite graph G in (c) of Problem A. (It has 11 vertices.)
(a) Find the associated capacitated digraph G'.
(b) Find an integer s-t flow in G' of maximum value (say, using the MAXFLOW algorithm) and its corresponding matching in G.
(c) Find an s-t cut in G' of minimum capacity and its corresponding covering in G.

E. Consider the bipartite graph G and the matching M shown below.



(a) Find the associated capacitated digraph G' and the integer s-t flow corresponding to M.

(b) Is M of maximum cardinality? If not, use flow augmentation (starting at the flow found in (a)) to find an integer s-t flow in G' of maximum value and the corresponding matching in G.

(c) Find an s-t cut in G' of minimum capacity and the corresponding covering in G.

F. Consider a bipartite graph $G = (V, A)$ with $V = X \cup Y$. Suppose every vertex has degree k (k is a positive integer). Prove, using Hall's theorem, that there exists a matching M that is X-saturating, i.e., $|M| = |X|$. (Hint: For any $U \subseteq X$, the arcs joined to U form a subset of the arcs joined to $N_A(U)$.)