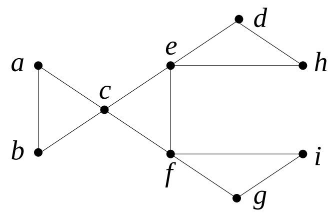
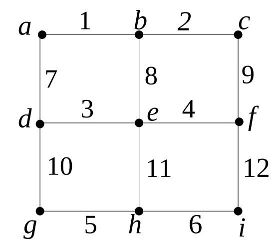
[Homework can be handed in to me or to my mail box in the Math Lounge (opposite the Math main office). Please show your work to receive full credit.]

A. Apply EPATH to find a closed eulerian path for the graph shown below on the left. Start at vertex a and break ties by choosing the lowest vertex (in alphabetical order). Indicate the vertex r and the path P at each iteration.





B. Consider the weighted graph shown above on the right.

(a) Apply MST-Prim (Prim's algorithm) to find a MST. Indicate the order in which arcs are added.

(b) Apply MST-Kruskal (Kruskal's algorithm) to find a MST. Indicate the order in which arcs are added.

(c) Apply the arc-swapping algorithm instead, starting at the spanning tree with arcs $(a, b), (b, c), (a, d)$, $(d, g), (d, e), (e, h), (c, f), (f, i)$. Indicate the order in which arcs are swapped.

(d) What is the range of ω_{ab} over which the current MST remains a MST? What is the range of ω_{eh} over which the current MST remains a MST?

C. Exercise 3.1.1 on page 25 of course notes.

Can the following three problems be transformed into a MST problem? Explain your answer.

(a) $G = (V, A)$ is a connected graph with arc weights $\{\omega_{uv}\}_{(u,v)\in A}$. Find a spanning tree $T = (V, B)$ of G that minimizes the Euclidean weight $\left[\sum_{(u,v)\in B} (\omega_{uv})^2\right]^{1/2}$.

(b) $G = (V, A)$ is a connected graph with each arc (u, v) having a probability $p_{uv} > 0$ of not failing. Assuming the arcs fail independently, the probability that no arc in a spanning tree $T = (V, B)$ fails is $\prod_{(u,v)\in B} p_{uv}$.
Find a spanning tree of G that maximizes the probability of no arc failing. [Hint: Use log.]

(c) $G = (V, A)$ is a connected graph with every arc colored either red or blue. Find a spanning tree with the maximum number of red arcs.

D. Exercise 3.3.1 on page 31 of course notes. (Note that a spanning tree has at least 2 vertices of degree 1; see page 17 of course notes.)

E. Exercise 3.3.2 on page 31 of course notes.

F. Prove that a MST T is unique if and only if any arc (u, v) not in T has larger weight than any arc on the cycle created by adding (u, v) to T. [Hint for the "if" direction: Argue by contradiction. If there is another MST T', show that we can replace some arc in T' with some arc in T to get another spanning tree of lower weight.]

Bonus: Exercise 3.3.3 on page 31 of course notes.