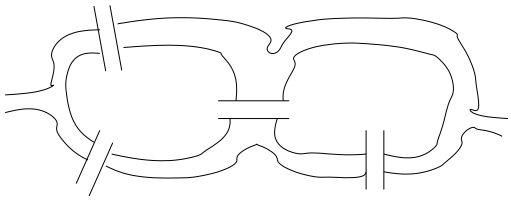
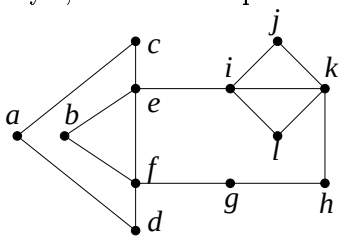
[Homework can be handed in to me or to my mail box in the Math Lounge (opposite the Math main office). Please show your work to receive full credit.]

A. If we have a connected electrical network with m elements (undirected arcs) and n nodes (vertices), what is the minimum number of elements we have to remove to eliminate all cycles in the network? Explain your answer.

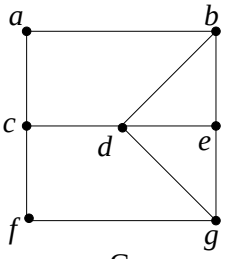
B. Suppose that G is a tree, (u, v) is an arc of G, and H is obtained from G by deleting arc (u, v), but not vertices u and v. Prove that H has exactly two connected components. [Hint: A tree (V, A) has $|A| = |V| - 1$.]

C. (a) For the bridges shown below, build a corresponding graph and determine if your graph has a closed eulerian path or an eulerian path that is not closed. If yes, find one such path.





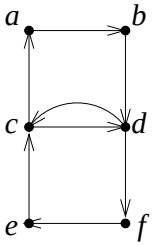
G_1



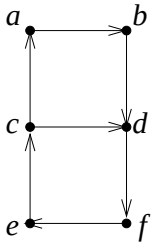
G_2

(b) For each of the two graphs G_1, G_2 shown above, determine if it has a closed eulerian path or an eulerian path that is not closed. If yes, find one such path. If not, explain why not.
(c) Can a tree have a closed eulerian path? If yes, give an example.

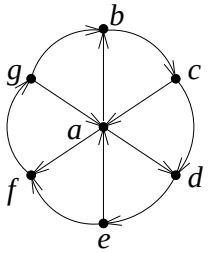
D. (a) For each of the four digraphs G_1, G_2, G_3, G_4 shown below, determine if it has a closed eulerian path or an eulerian path that is not closed. If yes, find one such path. If not, explain why not.



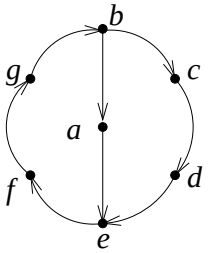
G_1



G_2



G_3



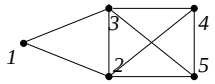
G_4

(b) Does every strongly connected digraph have a closed eulerian path? Explain.

E. Exercise 2.6.1 on page 21 of course notes.

F. Exercise 2.6.2 on page 22 of course notes.

G. Consider the graph shown:



(a) Apply the DFS (Depth First Search) version of TREE to find a spanning tree for this graph, starting at vertex 1. Write down the vertex set W and arc set B at each iteration.
(b) Apply the BFS (Breadth First Search) version of TREE to find a spanning tree for this graph, starting at vertex 1. Write down the vertex set W and arc set B at each iteration.