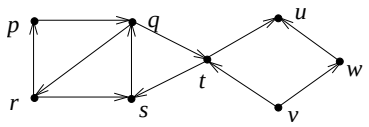
MATH 409, SPRING 2009 HOMEWORK 1 due **Wed, April 8, 11:30 a.m.**

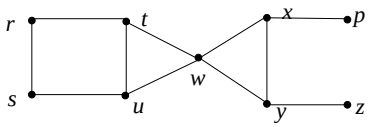
[Homework can be handed in to me or to my mail box in the Math Lounge (opposite the Math main office). Please show your work to receive full credit.]

A. For the digraph shown below:



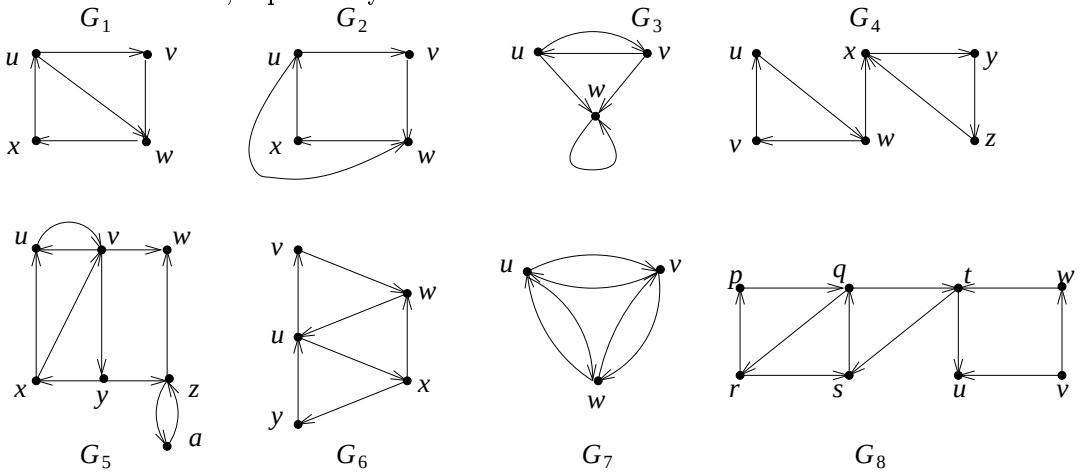
a) Find a path that is not a simple path.
b) Find a closed path having 6 arcs.
c) Find all cycles.
d) Find a longest (i.e., most arcs) simple path.
e) Is the digraph strongly connected? Explain.

B. For the graph shown below:



a) Find a closed path that is not a cycle.
b) Find all cycles.
c) Find a longest (i.e., most arcs) simple path.
d) Find a closed path having 6 arcs.
e) Is the graph connected? If yes, find a path that goes through all vertices.

C. For each digraph shown below, determine if it is strongly connected. If yes, give a closed path that goes through all vertices. If not, explain why not.



D. Exercise 2.2.2 on page 7 of the course notes.

E. Prove that a graph is connected if and only if it has a (undirected) path going through all the vertices.

F. (a) Give an example of a digraph and a path in that digraph which is not a simple path but has no repeated arcs.
(b) Give an example of a graph in which the shortest cycle has 5 arcs and the longest cycle has 8 arcs.
(c) For a digraph (V, A) that is strongly connected and $|V| = n$, what is the least number of arcs? What is the most?
(d) For a graph (V, A) that is connected and $|V| = n$, what is the least number of arcs? What is the most?

G. Let $P: u_1, u_2, ..., u_{p+1}$ be a path in some digraph (V, A). Suppose that $u_i = u_{i+4}$ for some $1 \leq i \leq p-3$ (so $p \geq 4$).
(a) Write down a path from u_1 to u_{p+1} having 4 arcs less than P.
(b) Write down a path having 8 more arcs than P.
(c) Does the digraph *always* have a path with 1 less arc than P? Explain.
(d) Can $|V|$ be less than 4? If yes, give an example. If no, explain why not.

H. Exercise 2.2.3 on page 7 of the course notes.