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Electrokinetics and Electrohydrodynamics of Microsystems

Induced-Charge Electrokinetic Phenomena

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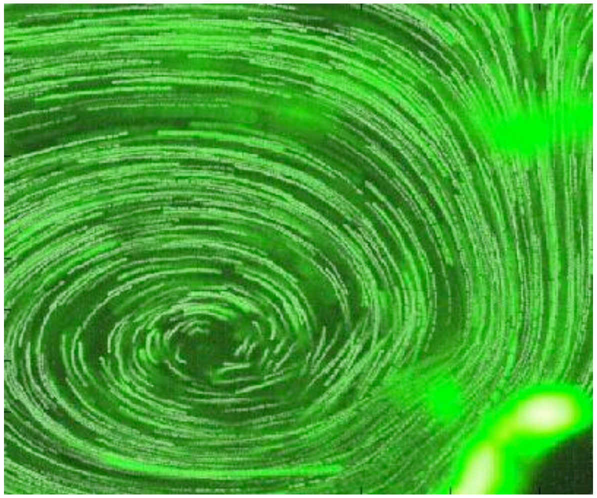
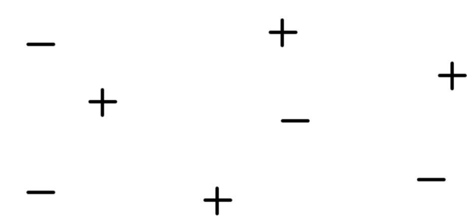
Lectures

1. Introduction
2. Low-voltage theory
3. Particle motion
4. Fluid motion
5. Large-voltage theory



Standard Model

$$\begin{aligned} c_i &= \text{ion concentrations} \\ \phi &= \text{electrostatic potential} \\ \vec{u} &= \text{fluid velocity} \end{aligned}$$



Dilute solution / mean field

Poisson-Nernst-Planck equations

$$\begin{aligned} \frac{\partial c_i}{\partial t} + \text{Pe}\, \vec{u} \cdot \vec{\nabla} c_i &= D_i \vec{\nabla} \cdot (\vec{\nabla} c_i + z_i c_i \vec{\nabla} \phi) \\ -\epsilon^2 \nabla^2 \phi &= \sum z_i c_i \end{aligned}$$

$$\epsilon = \lambda / L \ll 1$$

Singular perturbation

λ = screening length = 1-100nm

Navier-Stokes equations with electrostatic stress

$$\begin{aligned} \text{Sc}^{-1} \frac{\partial \vec{u}}{\partial t} + \text{Re}\, \vec{u} \cdot \vec{\nabla} \vec{u} &= -\vec{\nabla} p + \vec{\nabla}^2 \vec{u} + \text{Ra}_e\, \vec{\nabla} |\vec{\nabla} \phi|^2 \\ \vec{\nabla} \cdot \vec{u} &= 0 \end{aligned}$$

Regime of interest: $\text{Re} \ll 1, \text{Sc} \gg 1, \text{Ra}_e \approx 1$

Charge relaxation and ICEO flow

Electric field

ICEO velocity

Movies: numerical solution of the Poisson-Nernst-Planck/Navier-Stokes equations by Y. Ben, 2005 ($\lambda/a=0.005$)

We will now show how the same problem can be solved analytically in the (typical) limit of thin double layers ($\varepsilon \rightarrow 0$). Calculations will be done at the board following Squires & Bazant, J Fluid Mech 2004, 2006.

Thin-double-layer, low-voltage theory

Electrostatic Problem

Ohm's law, neutral bulk

$$\nabla^2 \phi = 0$$

"RC circuit" boundary condition

$$\frac{dq}{dt} = C\frac{d(\phi - \phi_0)}{dt} = \hat{n} \cdot \sigma \nabla \phi$$

Diffuse-layer and Stern-layer capacitances in series

$$C = \frac{C_d}{1+\delta}, \quad \delta = \frac{C_d}{C_s}, \quad C_d = \frac{\varepsilon}{\lambda}$$

Fix surface potential, ϕ_0
or total charge $Q = \oint q\, da$

Fluid Problem

Stokes flow

$$\nabla p = \eta \nabla^2 \vec{u}, \quad \nabla \cdot \vec{u} = 0$$

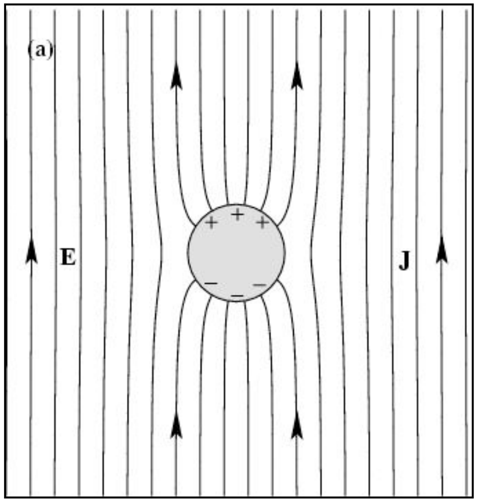
"Induced-charge" electro-osmotic slip boundary condition

$$\vec{u}_s = \vec{u} - \vec{u}_0 = -\frac{\varepsilon \zeta}{\eta} \vec{E}_{\parallel}$$

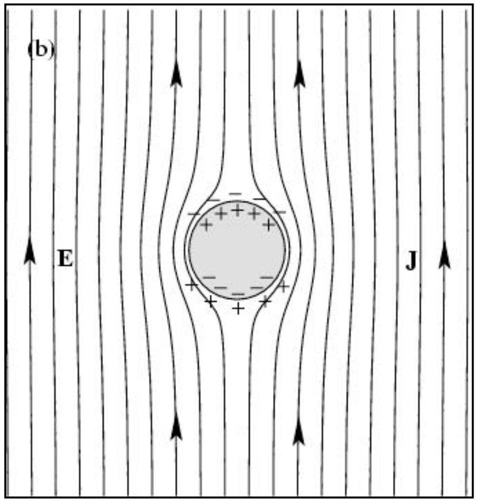
$$\zeta = \frac{\phi_0 - \phi}{1+\delta}$$

For a freely suspended particle, determine velocity & rotation by force=torque=0.

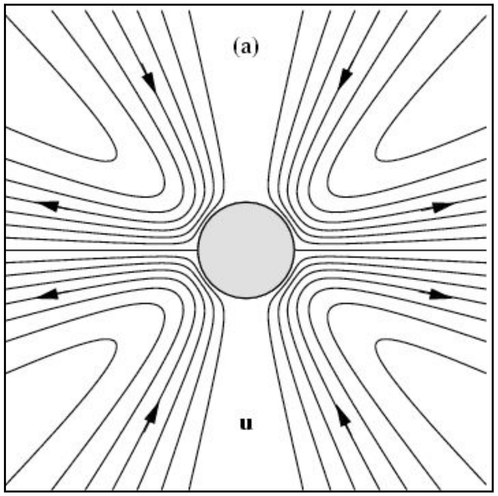
Example 1: ICEO flow around an uncharged metal sphere in an AC field



$E(\omega >> \omega_c)$



$E(\omega << \omega_c)$



$\langle u \rangle(\omega << \omega_c)$

Murtsovkin, Colloid Journal (1996); Squires & Bazant, J Fluid Mech, (2004).

Dimensionless Equations

$$\nabla^2 \phi = 0$$

$$\nabla p = \nabla^2 \vec{u}, \quad \nabla \cdot \vec{u} = 0$$

$$\frac{d(\phi - \phi_0)}{dt} = \hat{n} \cdot \nabla \phi$$

$$\vec{u}_s = \frac{\phi - \phi_0}{1 + \delta} \vec{E}$$

"RC" time scale

$$\frac{\tau_c}{(1+\delta)} = (1+\delta)\omega_c^{-1} = \frac{Ca}{\sigma} = \frac{\varepsilon}{\sigma}\frac{a}{\lambda} = \frac{\lambda a}{D}$$

ICEO velocity scale

$$U_0 = \frac{\varepsilon L E_0^{\,2}}{\eta(1+\delta)}$$

Linear response to AC field

$$\phi = \mathrm{Re}[e^{i\omega t}(\Phi - \phi_0)], \quad i\omega\Phi = \hat{n} \cdot \nabla\Phi, \quad \langle u_s \rangle = -\frac{1}{4}\nabla_{\parallel} |\Phi|^2$$

Solution

$$E(r \rightarrow \infty) = \cos\omega t = \mathrm{Re}[e^{i\omega t}]$$

$$\Phi(r,\theta,\omega) = -r\cos\theta\left(1 + G(\omega)\left(\frac{a}{r}\right)^3\right)$$

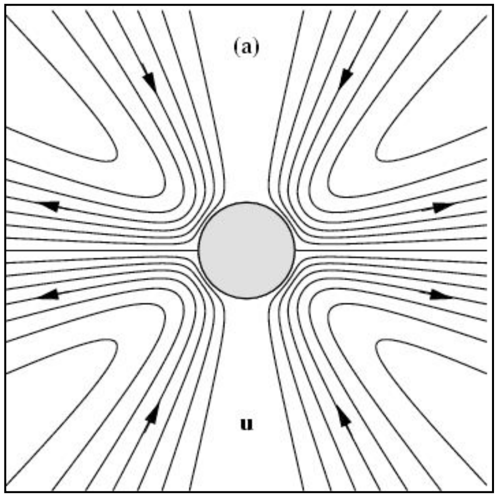
G=induced dipole moment
=1/2 insulator, =-1 conductor

$$i\omega\Phi(a,\theta) = E_r(a,\theta)$$

$$\Rightarrow \quad G = \frac{1-i\omega}{2+i\omega}$$

$$u_\theta(a,\theta) = \mathrm{Re}[\Phi(a,\theta)]\cdot\mathrm{Re}[E_\theta(a,\theta)]$$

$$= \frac{9}{2}\sin 2\theta\,\mathrm{Re}\left[\frac{e^{i\omega t}}{2+i\omega}\right]^2$$



Quadrupolar flow decays above RC frequency

$$\langle u_\theta(a,\theta)\rangle = \frac{9}{16}\frac{\sin 2\theta}{[1+(\omega/2)^2]} = U_s\sin 2\theta$$

$$u_\theta(r,\theta) = U_s\left(\frac{a}{r}\right)^4\sin 2\theta$$

$$u_r(r,\theta) = \frac{U_s}{2}(1+3\cos 2\theta)\left[\left(\frac{a}{r}\right)^4 - \left(\frac{a}{r}\right)^2\right]$$

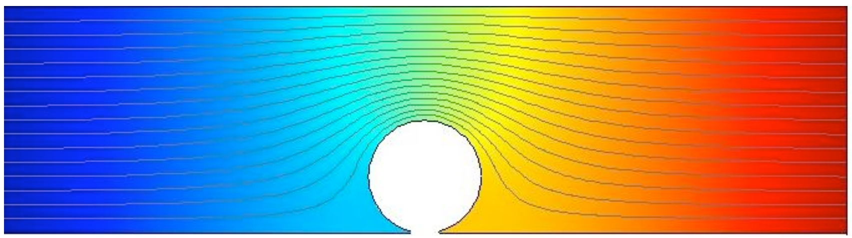
Example 2: Our first experiment

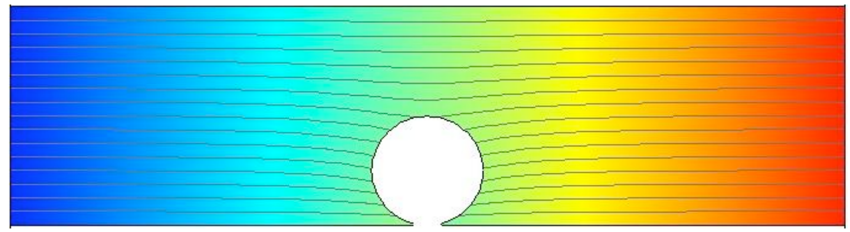
ICEO around a 100 micron platinum cylinder in 0.1 mM KCl

Levitan, ... Y. Ben,… Colloids and Surfaces (2005).

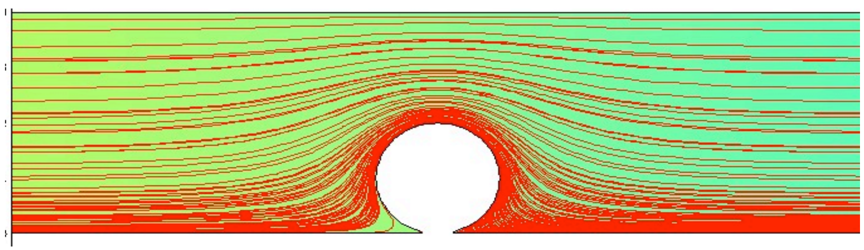
Low frequency DC limit *At the “RC” frequency*

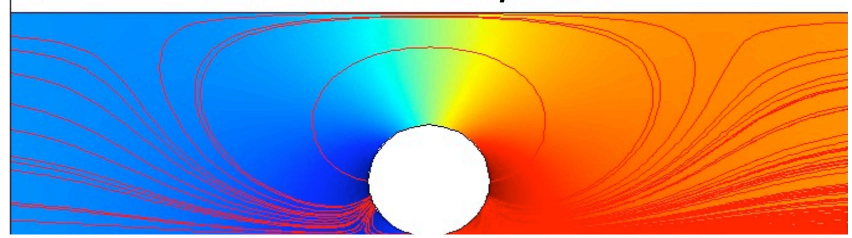
In-phase E field (insulator) $-\mathrm{Re}(\nabla\Phi)$ *Normal current*



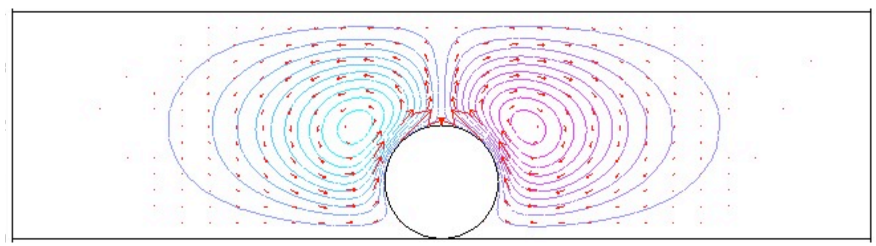


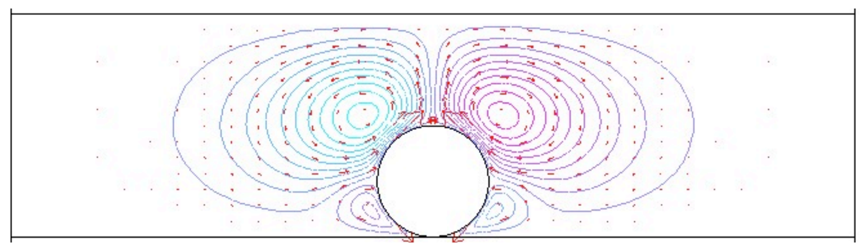
Out-of-phase E (negligible) $-\mathrm{Im}(\nabla\Phi)$ *Induced dipole*





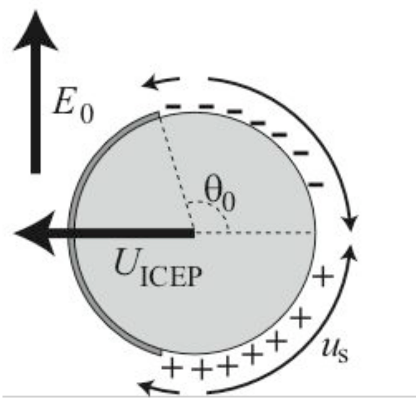
Time-averaged velocity



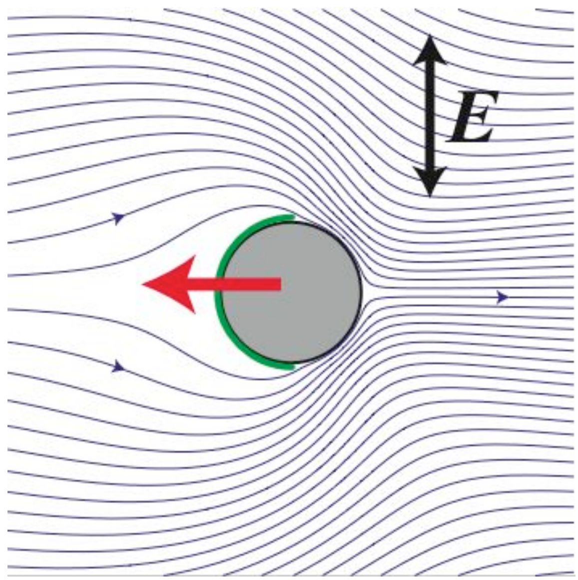


Example 3:
Induced-charge electrophoresis of a metal/insulator Janus sphere transverse to a uniform AC field

Theory: Squires and Bazant, J Fluid Mech, (2006); Experiment: Gangwal et al, Phys Rev Lett (2008).



stable orientation



$$\langle U_{ICEP} \rangle = \frac{9}{128} \frac{\varepsilon a E^2}{\eta(1+\delta)}$$

= surface average slip
(Stone & Samuels 1996)

- Metal side acts like a "jet engine".
-Stable motion transverse to a uniform DC or AC field

Conclusion
Lecture 2: Low-voltage Theory

The Standard Model is convenient to qualitatively predict many nonlinear electrokinetic phenomena in colloids (lecture 3) and microfluidics (lecture 4), but is still incomplete and may require modifications (lecture 5).