Math 170E **Homework 8**
UCLA, Fall 2026 **due 11/23/2026 at 11:59pm**

Directions. Complete all the problems. Your solutions to the problems should be submitted to Gradescope before the indicated due date above. Please follow rules regarding Gradescope submission as described in the syllabus. If you use any outside source in your solutions, you must cite the source.

Problem 1. Let X and Y be continuous random variables with joint pdf

$$f(x,y) = \begin{cases} 3x & \text{if } 0 \le y \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Compute the correlation coefficient of X and Y.

Problem 2. Let X and Y be discrete random variables with joint pmf

$$f(x,y) = \frac{1}{4} \text{ for } (x,y) \in \{(0,0),(1,1),(1,-1),(2,0)\}.$$

(a) Are X and Y independent?

(b) Calculate $\mathrm{Cov}(X,Y)$.

Problem 3. Let $X_1, X_2 \ldots, X_n$ be independent continuous random variables, each of which is distributed uniformly on $[0,1]$. Define random variables

$$Y = \text{the number of } i \text{ such that } X_i \ge 1/2,$$
$$Z = \text{the number of } i \text{ such that } X_i \ge 3/4.$$

(a) Compute $\mathrm{Cov}(Y,Z)$.

Hint. Write both Y and Z as sums of indicator random variables, then use the bilinearity of covariance.

(b) Are Y and Z independent? Justify your answer.

Problem 4. Fix $n \in \{1, 2, \dots\}$ and $p \in (0, 1)$. A row of seats in a theater has $n + 2$ seats (2 aisle seats and n non-aisle seats). A seat will be occupied by a person with probability p and will be left vacant with probability $1 - p$, independent of the other seats. A non-aisle seat is considered to be occupied by a comfortable person if (1) the seat is occupied and (2) both seats next to that seat are vacant. Let X be the number of non-aisle seats occupied by a comfortable person. The aim of the following exercises is to compute the variance of X.

Define indicator random variables $X_1, \dots, X_n$ by

$$X_i = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ non-aisle seat is occupied by a comfortable person,} \\ 0 & \text{otherwise.} \end{cases}$$

Note that $X = X_1 + \cdots + X_n$.

(a) For $1 \le i \le n$, compute $P(X_i = 1)$ and $\mathrm{Var}(X_i)$.

(b) For $1 \le i \le n - 1$, compute $\mathrm{Cov}(X_i, X_{i+1})$.

(c) For $1 \le i \le n - 2$, compute $\mathrm{Cov}(X_i, X_{i+2})$.

(d) For $1 \le i < j \le n$ with $|i - j| > 2$, compute $\mathrm{Cov}(X_i, X_j)$.

(e) Compute $\mathrm{Var}(X)$.

Hint. First, notice that

$$\mathrm{Var}(X) = \mathrm{Var}(X_1 + \cdots + X_n) = \mathrm{Cov}(X_1 + \cdots + X_n, X_1 + \cdots + X_n).$$

Using bilinearity of the covariance, you can write this as a sum of terms of the form $\mathrm{Cov}(X_i, X_j)$. Your answers to the previous parts will tell you the values of these terms.

Recommended practice. The following exercises from the textbook are not assigned and do not have to be turned in. They are recommended to get more practice.

Section 4.2: 3(a), 6

Most of the textbook exercises are very good, so you can always get more practice by working on exercises not mentioned in the above list.