Math 170E **Homework 5 Solutions**
UCLA, Fall 2026 **due 11/2/2026 at 11:59pm**

Directions. Complete all the problems. Your solutions to the problems should be submitted to Gradescope before the indicated due date above. Please follow rules regarding Gradescope submission as described in the syllabus. If you use any outside source in your solutions, you must cite the source.

Problem 1. A bakery packs fortune cookies in boxes of 300. Each cookie, independently of the others, contains a misprinted fortune with probability 0.01. Let X be the number of cookies with a misprinted fortune in one box.

(a) What is the distribution of X? Explain why the Poisson approximation to the binomial is appropriate here, and state which Poisson random variable Y you would use to approximate X.

Final answer. $X \sim \text{Binom}(300, 0.01)$. The Poisson approximation is appropriate since $n = 300$ is large and $p = 0.01$ is small. Use $Y \sim \text{Poisson}(np) = \text{Poisson}(3)$.

(b) Use the Poisson approximation to estimate:

(i) the probability that a box contains no misprinted fortunes;

(ii) the probability that a box contains at least 2 misprinted fortunes.

Then compute the exact value of the probability in (i) and compare it to your approximation.

Final answer. (i) $P(Y = 0) = e^{-3} \approx 0.0498$.

(ii) $P(Y \geq 2) = 1 - e^{-3} - 3e^{-3} = 1 - 4e^{-3} \approx 0.8009$.

Exact value in (i): $P(X = 0) = (0.99)^{300} \approx 0.0490$, which is close to the approximation $e^{-3} \approx 0.0498$.

(c) The bakery wants to sell a smaller "sampler" box of n cookies, where n is chosen so that the probability of the box containing *at least one* misprinted fortune is at least 0.95. (The misprints are collectibles!) Using the Poisson approximation, find the smallest such n. Is the rule of thumb satisfied for your value of n?

Final answer. We need $1 - e^{-0.01n} \geq 0.95$, i.e. $n \geq 100 \ln 20 \approx 299.57$, so the smallest such n is $n = 300$.

Problem 2. Let X be a continuous random variable with pdf given by

$$f(x) = \begin{cases} c(x^2+1) & \text{if } 0 \le x \le 1 \\ c & \text{if } 2 \le x \le 3 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find c.

Final answer. $c = \dfrac{3}{7}$.

(b) Compute $P(0.5 < X < 2.5)$.

Final answer. $\dfrac{31}{56}$.

(c) Find $\mathbb{E}[X]$.

Final answer. $\mathbb{E}[X] = \dfrac{39}{28}$.

(d) Find $\mathrm{Var}(X)$.

Final answer. $\mathbb{E}[X^2] = \dfrac{103}{35}$, so $\mathrm{Var}(X) = \dfrac{103}{35} - \left(\dfrac{39}{28}\right)^2 = \dfrac{3931}{3920} \approx 1.0028$.

Problem 3. Suppose a continuous random variable X has pdf of the form

$$f(x) = \begin{cases} a & \text{if } x \in [0,1] \\ b(x-1) & \text{if } x \in (1,2] \\ 0 & \text{otherwise.} \end{cases}$$

Suppose also that $\mathbb{E}[X] = 1$. Find a and b.

Final answer. $a = \dfrac{4}{7}$, $b = \dfrac{6}{7}$.

Problem 4. Let X be a continuous random variable which is uniform on $[-1, 1]$.

(a) Compute $P(X^2 > 1/25)$.

Solution. We compute

$$P(X^2 > 1/25) = P(|X| > 1/5) = P(X < -1/5 \text{ or } X > 1/5)$$
$$= \frac{\text{length}[-1, -1/5) + \text{length}(1/5, 1]}{\text{length}[-1, 1]} = \frac{(4/5) + (4/5)}{2} = \frac{4}{5}$$

(b) Compute $P\left(X < -\frac{3}{4} \middle| \, |X| > 1/2\right)$.

Final answer. $\dfrac{1/8}{1/2} = \dfrac{1}{4}$.

Problem 5. Let X be a continuous random variable which is uniform on $[0, 1]$, and let $a < b$ be constants. Find the cdf and pdf of the random variable $Y = (b - a)X + a$. Do you recognize the distribution of Y?

Solution. First, we compute the cdf F_Y. Note that since $X \in [0, 1]$, we have $Y \in [a, b]$. For any $y \in [a, b]$,

$$F_Y(y) = P(Y \leq y) = P((b-a)X+a \leq y) = P((b-a)X \leq y-a) = P(X \leq \frac{y-a}{b-a}).$$

Note that when we divide by $(b-a)$, the inequality stays in the same direction since $b - a > 0$. Now, when $y \in [a, b]$, $(y - a)/(b - a) \in [0, 1]$, so

$$P(X \leq \frac{y-a}{b-a}) = \frac{\text{length}[0, (y-a)/(b-a)]}{\text{length}[0, 1]} = \frac{y-a}{b-a}.$$

Thus, we have

$$F_Y(y) = \begin{cases} 0 & \text{if } y < a \\ \dfrac{y-a}{b-a} & \text{if } a \leq y \leq b \\ 1 & \text{if } y > b. \end{cases}$$

Taking a derivative of F_Y, we get the pdf

$$f_Y(y) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \leq y \leq b \\ 0 & \text{otherwise.} \end{cases}$$

So Y is uniform on $[a, b]$.

Recommended practice. The following exercises from the textbook are not assigned and do not have to be turned in. They are recommended to get more practice.

Section 3.1: 2, 3, 7 (ignore "index of skewness"), 9, 11, 15, 20

Most of the textbook exercises are very good, so you can always get more practice by working on exercises not mentioned in the above list.