Quantum One-Time Programs, Revisited

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One-Time Programs (OTP)

[Goldwasser-Kalai-Rothblum'08]

I grant you one evaluation!

Image generated by ChatGPT

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Program

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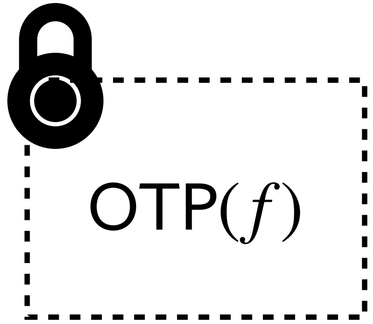

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- Classically impossible 😢 (without hardware assumptions)

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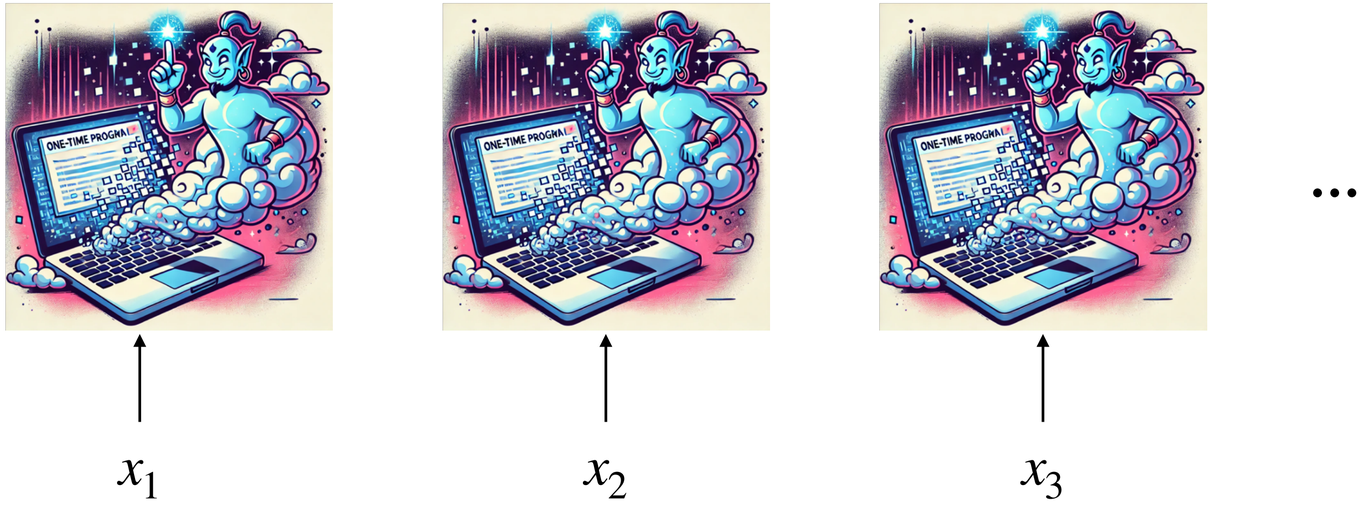
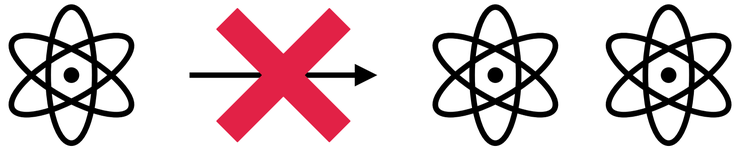
  - Can copy program many times, evaluate each copy



Quantum One-Time Programs?

Quantum No-cloning Theorem: 

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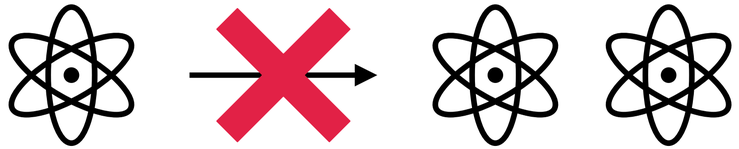
Quantum No-cloning Theorem: 

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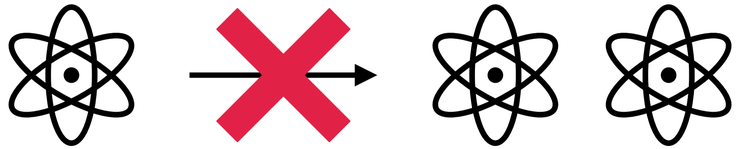
Quantum No-cloning Theorem: 

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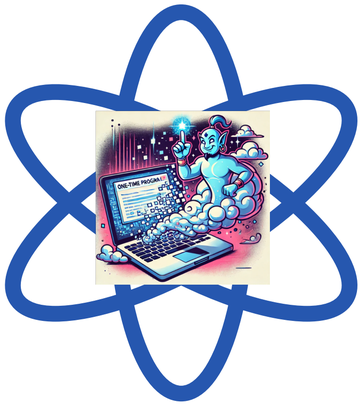
Classical program

$f : X \to Y$

$|\mathsf{OTP}_f\rangle$



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No 😢

[Broadbent-Gutoski-Stebila '13]

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$|x\rangle_{\text{in}} |\text{OTP}_f\rangle$

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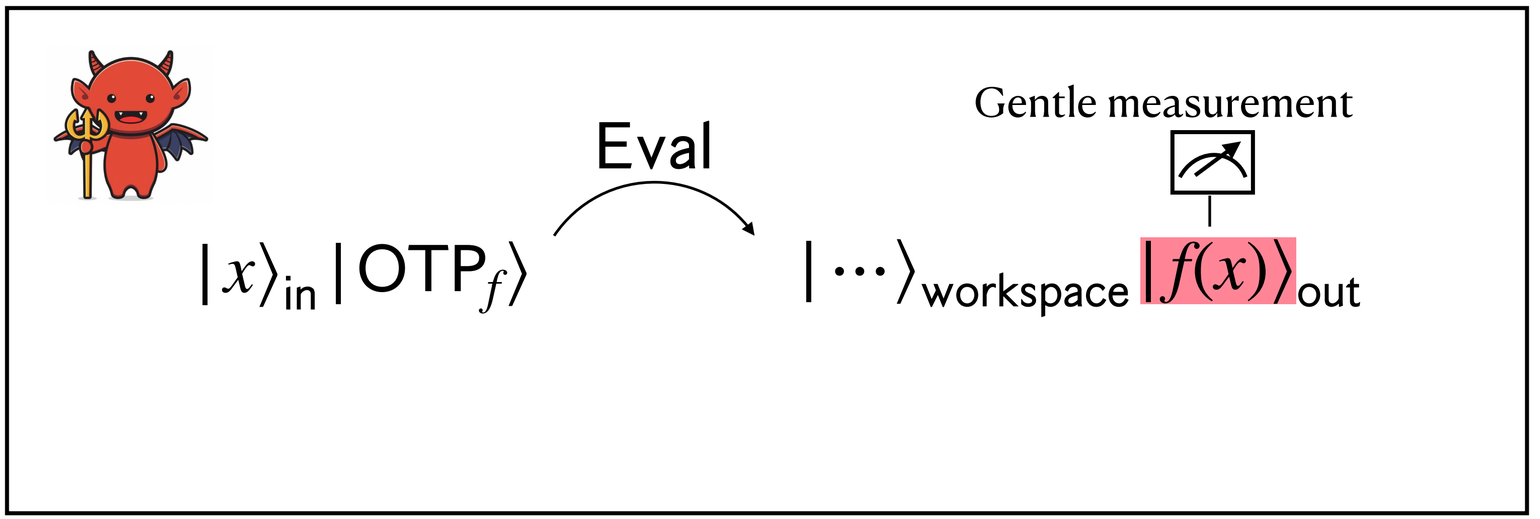

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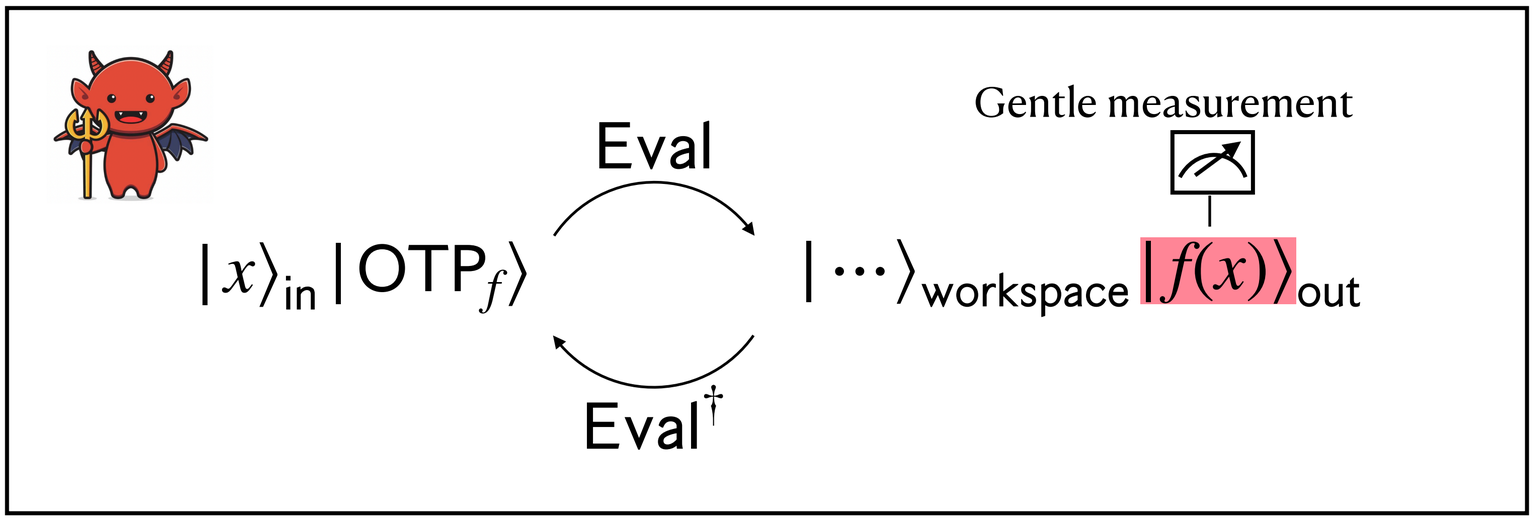

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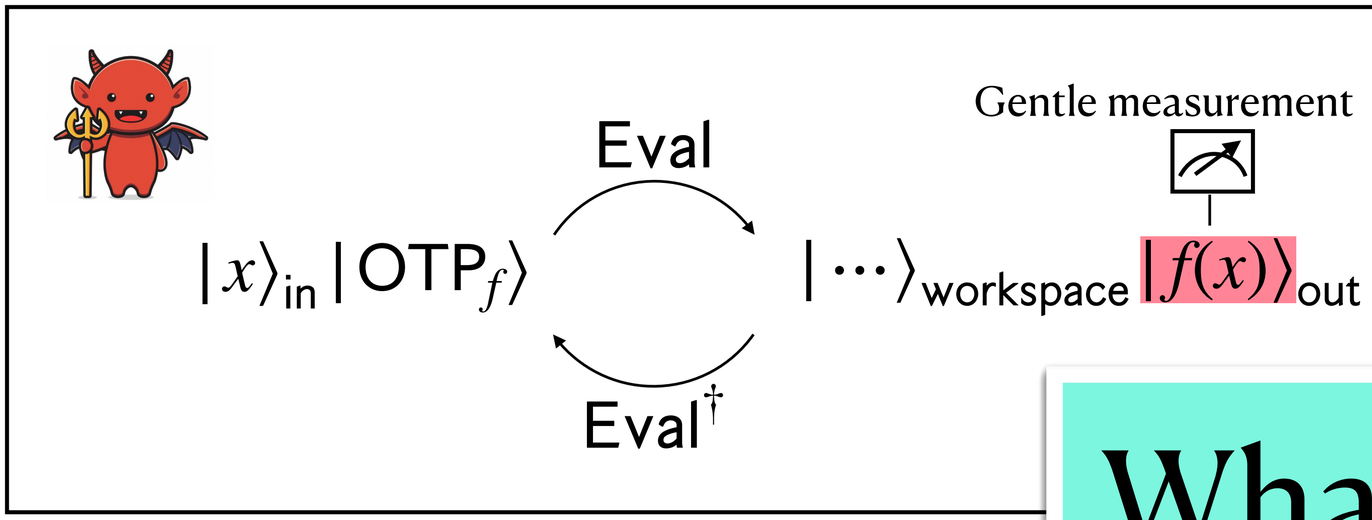

What next? 🤔

Quantum One-Time Programs?

for randomized programs

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Randomized function $f : X \times R \to Y$

- User chooses x but not r

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Eval

$$|x\rangle_{\text{in}} |\text{OTP}_f\rangle \qquad \sum_r |\cdots_r\rangle_{\text{workspace}} |f(x,r)\rangle_{\text{out}}$$

for randomized programs

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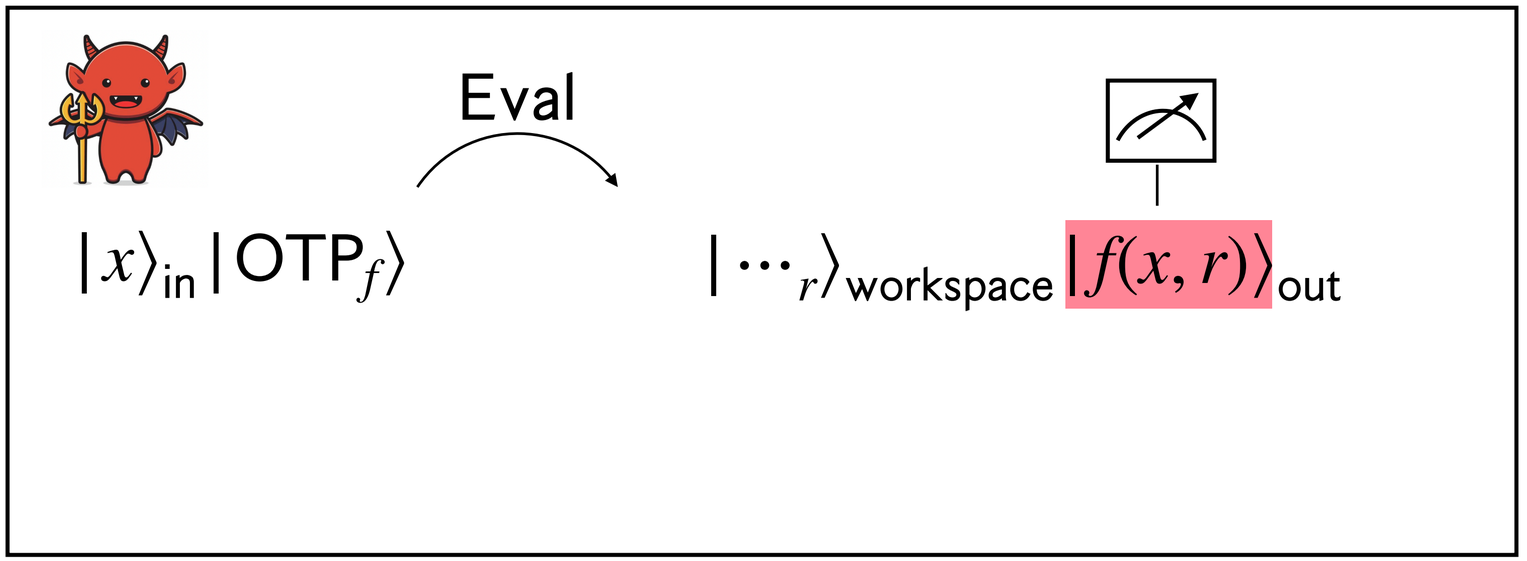

Attack doesn't work! For f with high min-entropy,
measuring output may destroy state and prevent further evaluations

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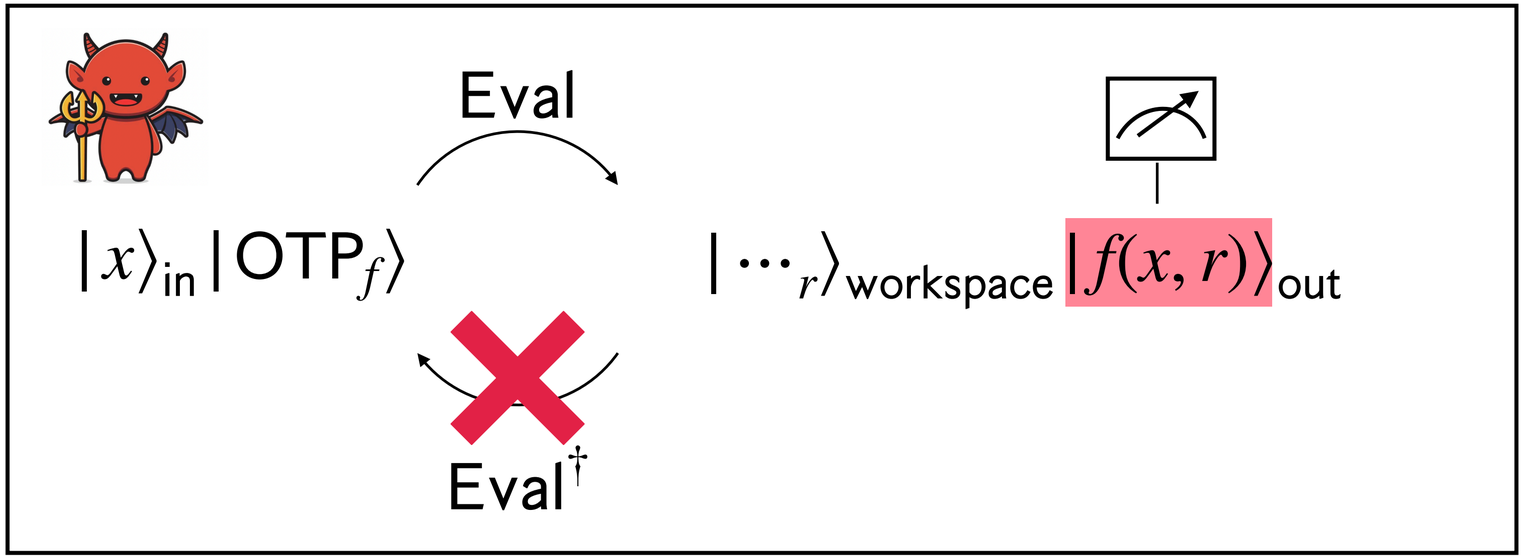

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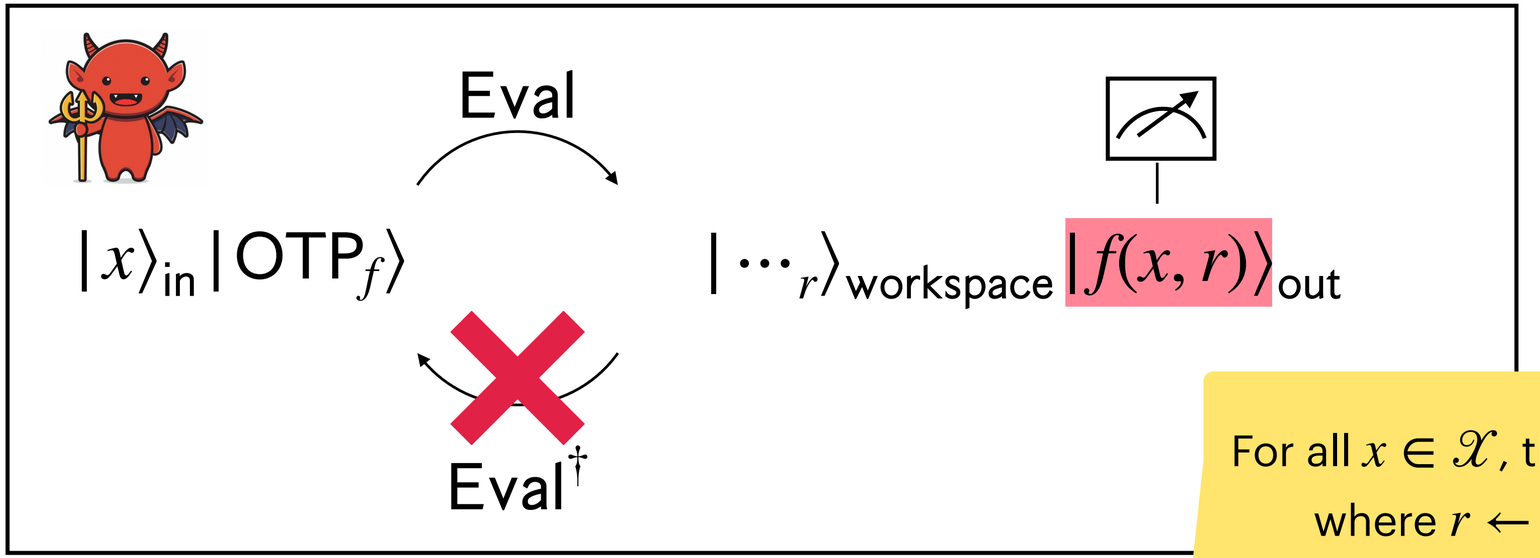

For all $x \in \mathcal{X}$, the distribution of $f(x, r)$ where $r \leftarrow R$ has high entropy

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Signature tokens [Ben-David, Sattath '16]

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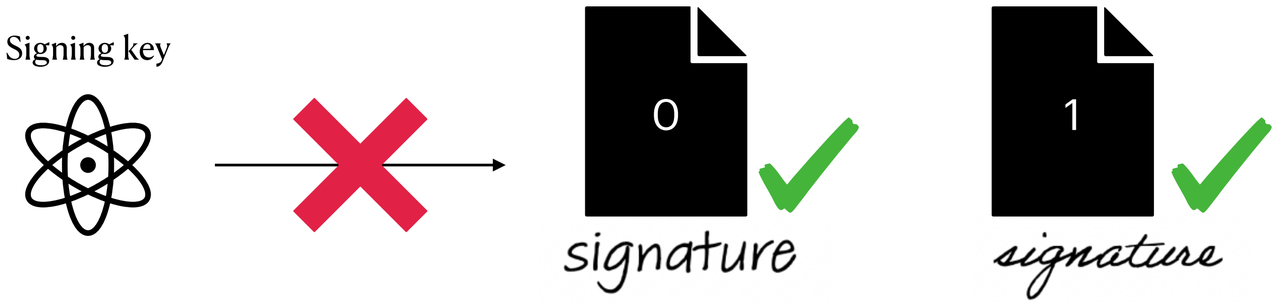

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≈ quantum OTP
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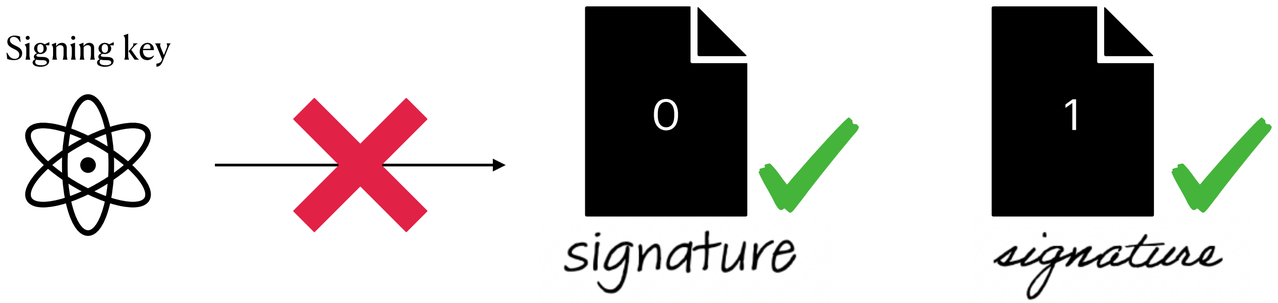

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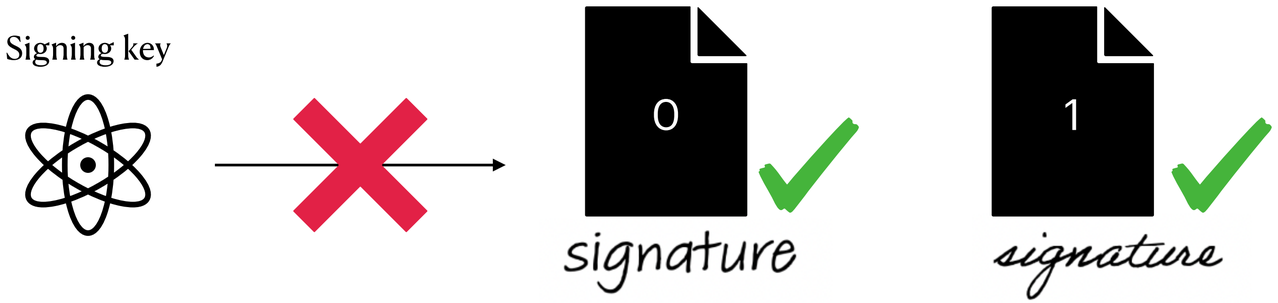

But we cannot be too greedy! 😢

No quantum OTPs that satisfy a *strong simulation security* notion, for any (randomized) programs [Broadbent-Gutoski-Stebila '13]

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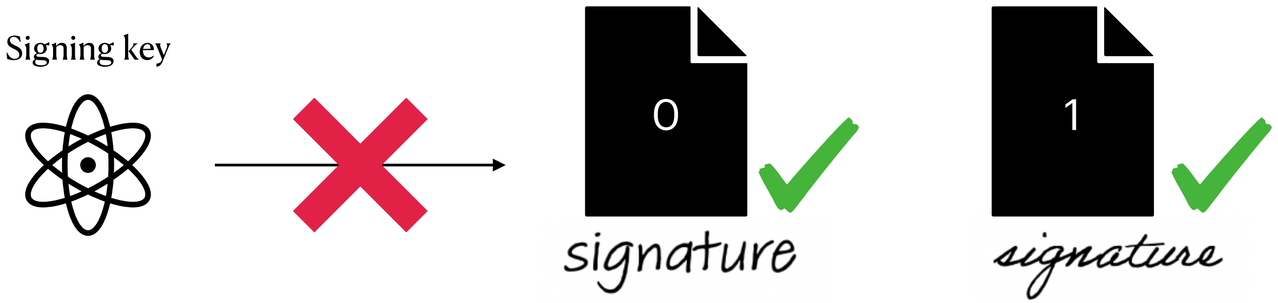

- I will define this soon!
- Signature token does **not** satisfy this strong security definition

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Strong simulation security impossible [BGS13]

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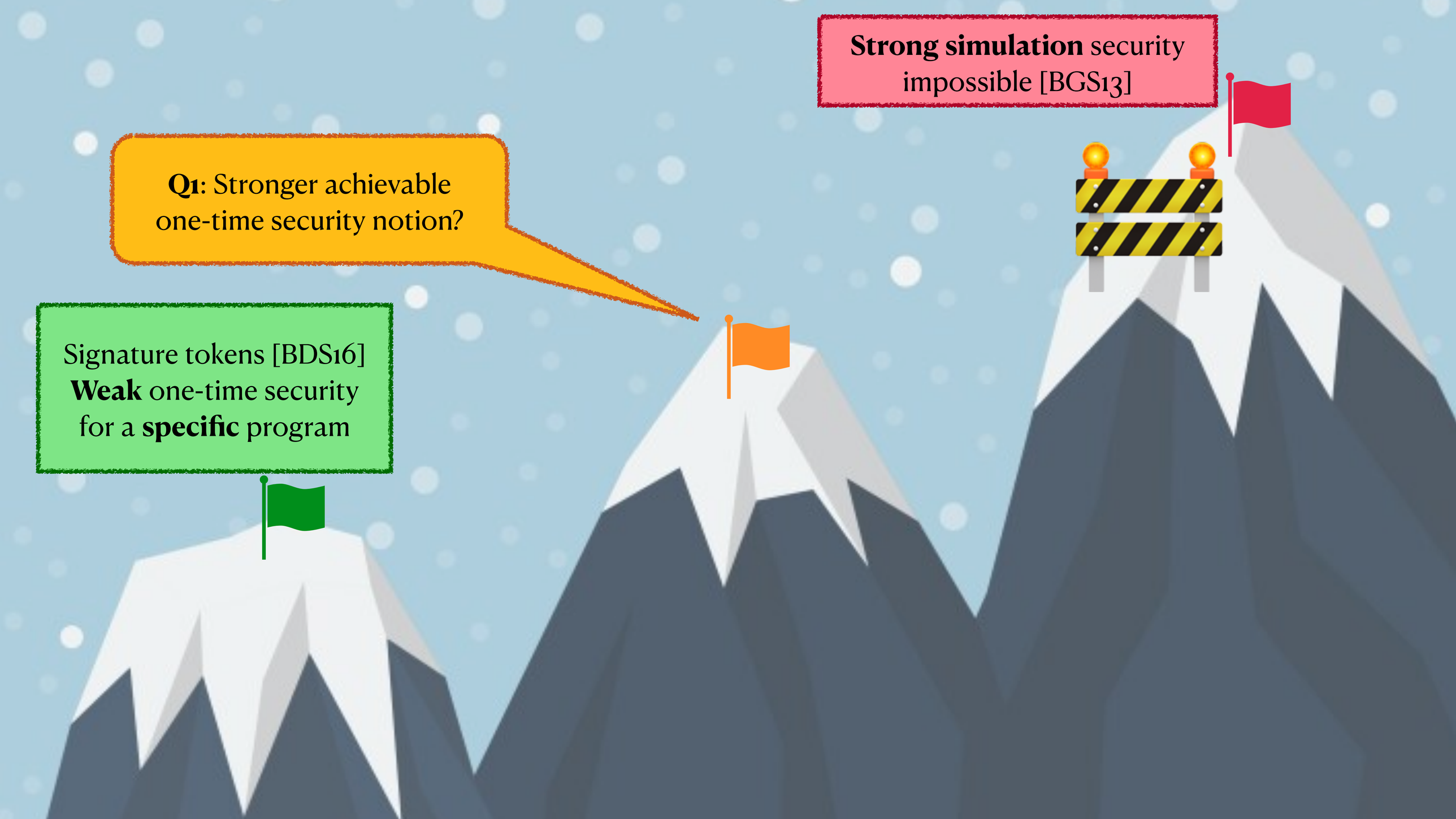
Signature tokens [BDS16]
Weak one-time security
for a **specific** program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Q1: Stronger achievable one-time security notion?
Signature tokens [BDS16] **Weak** one-time security for a **specific** program


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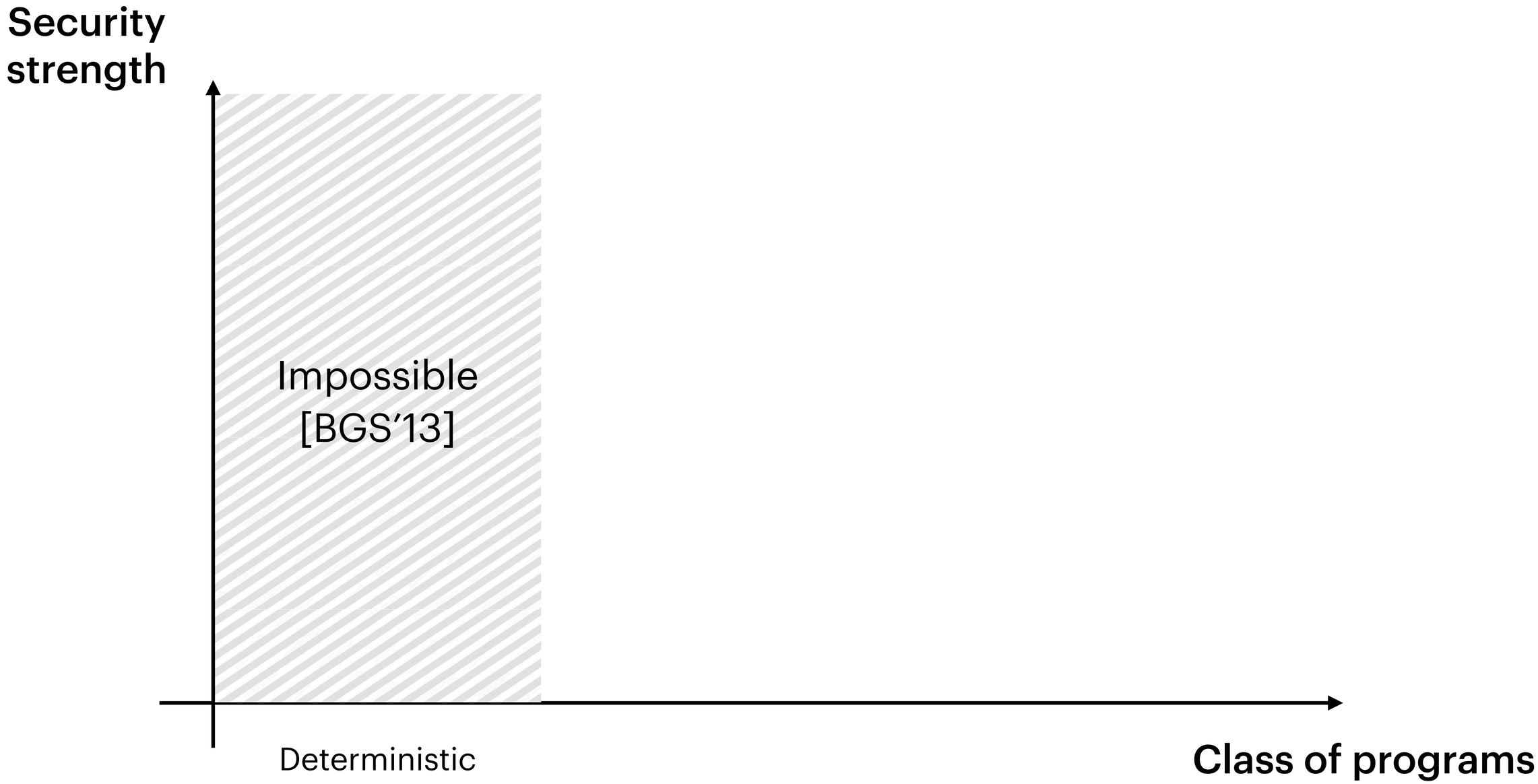
Q1: Stronger achievable one-time security notion?
Q2: What programs can be one-time protected? All high-min entropy programs?
Signature tokens [BDS16] **Weak** one-time security for a **specific** program


Security
strength
Class of programs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Impossible [BGS'13]
Deterministic
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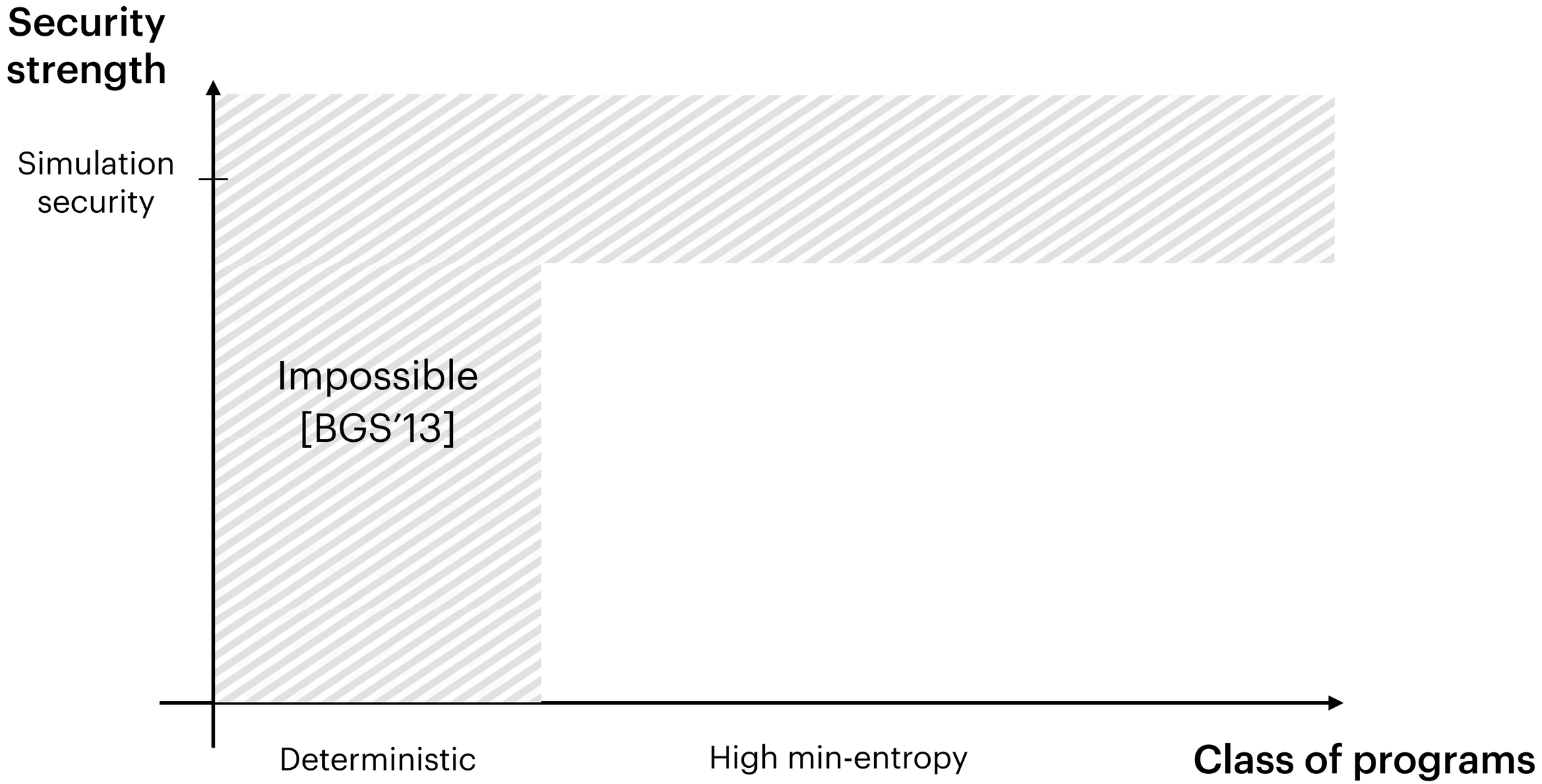
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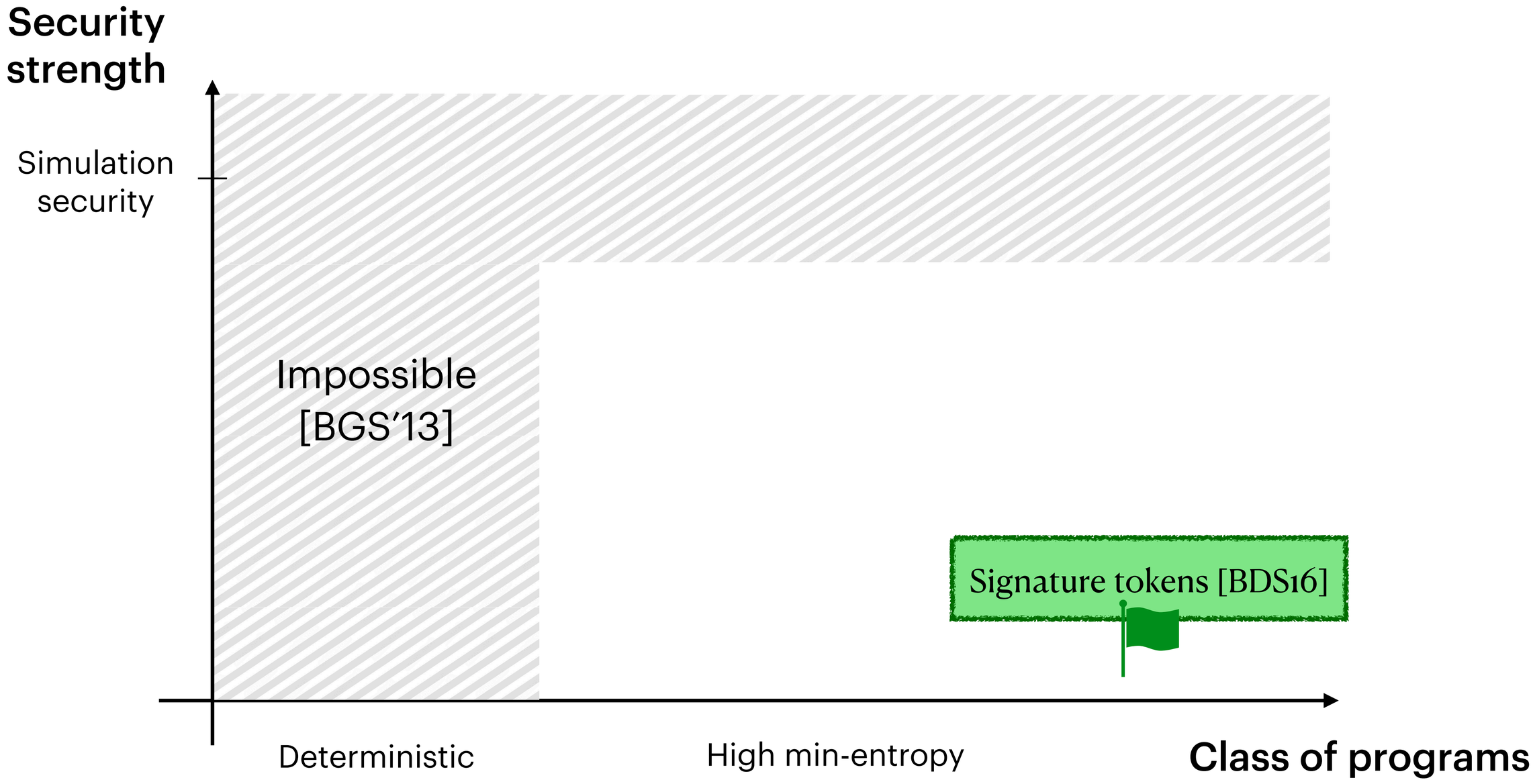
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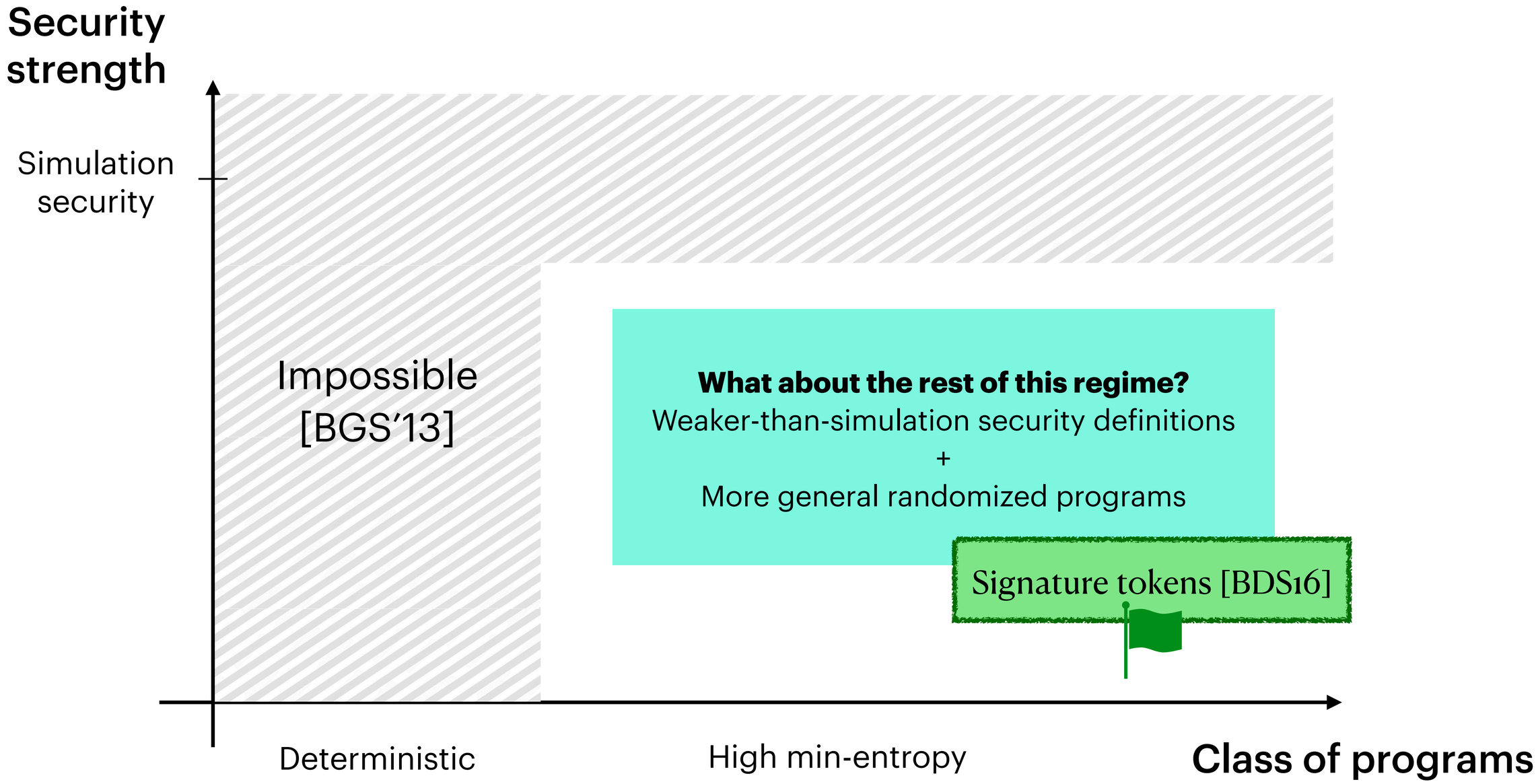
Security strength
Simulation security
Impossible [BGS'13]
What about the rest of this regime?
Weaker-than-simulation security definitions
+
More general randomized programs
Signature tokens [BDS16]
Deterministic
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Our Work

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Definition
Achievable intermediate
one-time security notion

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Concurrent work [Gunn-Movassagh'24] gives similar construction and a simpler but weaker definition of one-time security.

Our Construction

Warm up: Signature Tokens [BDS16]

Background: Subspace states

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$s \in S$ (Measure in computational basis)

OR

$s' \in S^\perp$ (Measure in Hadamard basis)

- Hides subspace S
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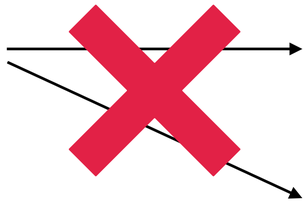

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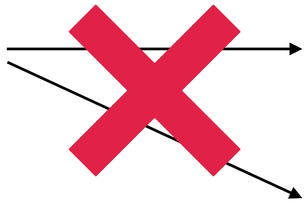
Oracles $O_S, O_{S^\perp}$



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Direct Product Hardness [BDS'16]. Even given membership oracles $O_S, O_{S^\perp}$ hard for query-bounded adv to find both $s, s' \neq 0$ such that $s \in S$ and $s' \in S^\perp$.

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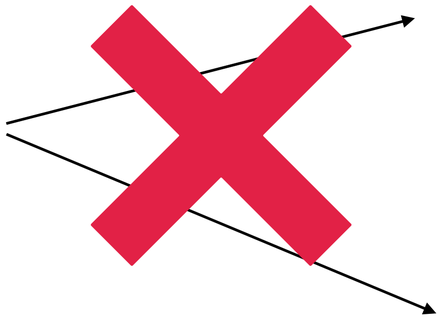

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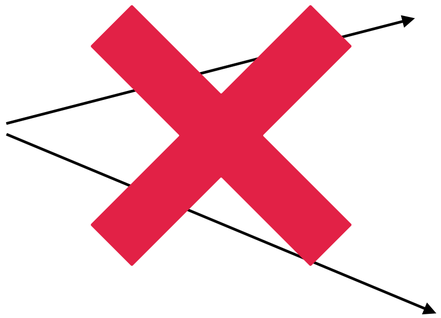

<u>To sign 0:</u>

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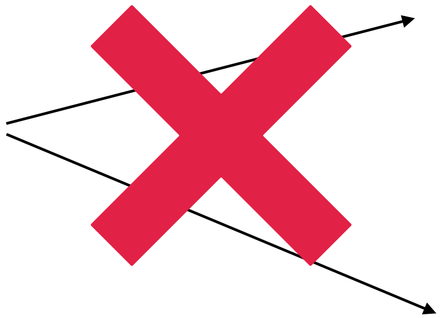

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Warm up: Signature Tokens [BDS16]

Signing key

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Verification key

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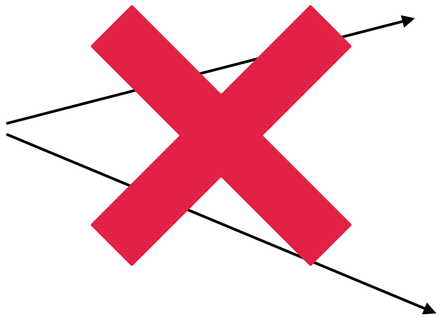

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To verify (m, σ):

1. Measure σ
2. If $m = 0$, check $\sigma \in S$
3. If $m = 1$, check $\sigma \in S^\perp$

Our OTP Construction

(In the classical oracle model)

Goal: OTP for $f : \{0,1\}^n \times R \to Y$. For simplicity, $n = 1$.

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Sample from $f(1,\cdot)$

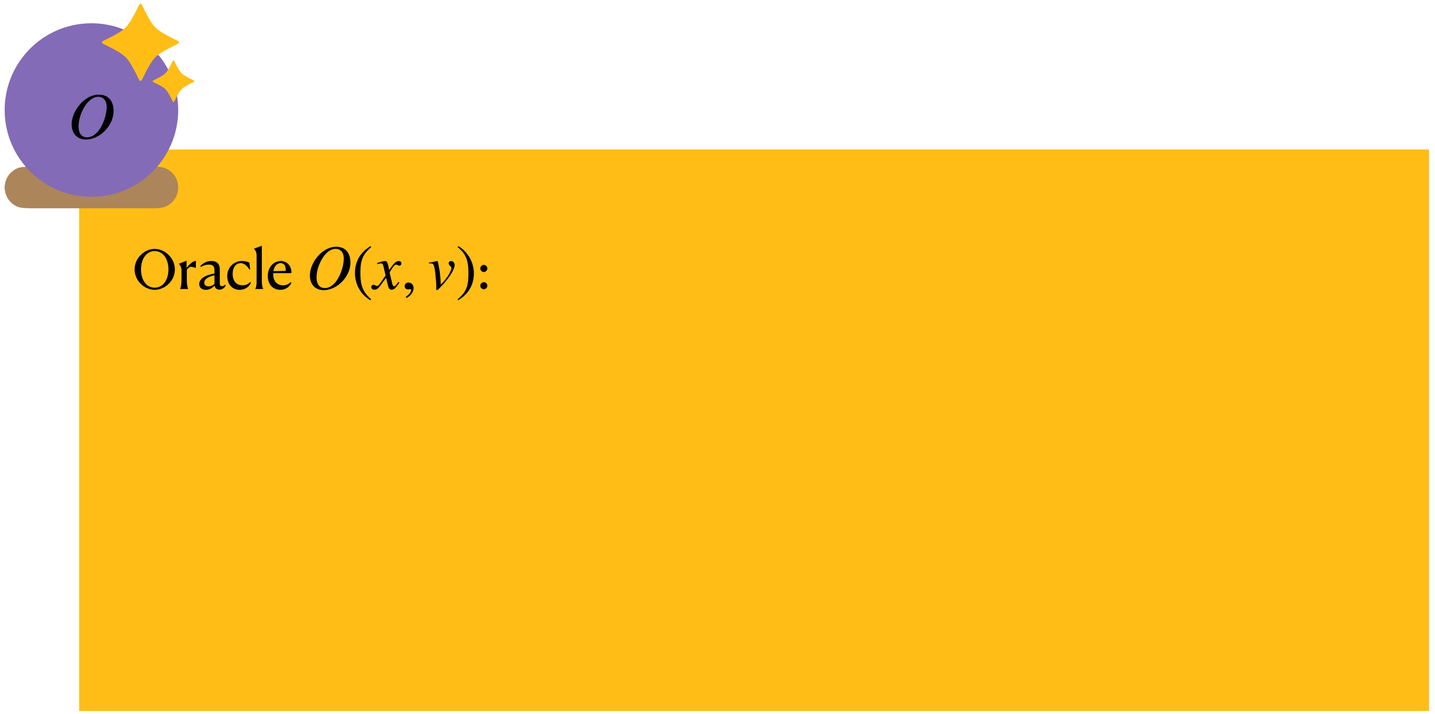
not both

Our OTP Construction

(In the classical oracle model)

Goal: OTP for $f : \{0,1\}^n \times R \to Y$. For simplicity, $n = 1$.

$$|S\rangle := \sum_{s \in S} |s\rangle$$



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If $x = 0$ AND $v \in S$, output $f(0,r)$

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Broken! Measuring output does not destroy $|S\rangle$

Our OTP Construction

(In the classical oracle model)

Goal: OTP for $f: \{0,1\}^n \times R \to Y$. For simplicity, $n = 1$.

$$\sum_{s \in S} |s\rangle$$

1. submit $x = 0$, $|S\rangle$ in superposition
2. measure $f(0,r)$ and then
3. repeat with $|S^{\perp}\rangle$ to get $f(1,r)$

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Goal: OTP for $f : \{0,1\}^n \times R \to Y$. For simplicity, $n = 1$.

$$|S\rangle := \sum_{s \in S} |s\rangle$$

Entangle $|S\rangle$ with randomness r

Measuring output destroys $|S\rangle$

(if f has high min-entropy)

Oracle $O(x, v)$:

If $x = 0$ AND $v \in S$, output ~~$f(0,r)$~~ $f(0, H(v))$

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Else, abort

Random oracle $H : \{0,1\}^{\lambda} \to \mathscr{R}$

Our OTP Construction

Goal: OTP for $f : \{0,1\}^n \times R \to Y$. For simplicity, $n = 1$.

$\mathsf{OTP}_f := (|S\rangle, O)$

Oracle $O(x, v)$:

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Measuring output destroys $|S\rangle$ (if f has high min-entropy)

Random oracle $H : \{0,1\}^{\lambda} \to R$

Our OTP Construction

(In the classical oracle model)

Goal: OTP for $f : \{0,1\}^n \times R \to Y$ into a one-time program. For $n > 1$:

$\mathsf{OTP}_f := \left(|S_1\rangle, \ldots, |S_n\rangle, \right.$  $\left.\right)$

Oracle $O(x, v_1, \ldots, v_n)$:

If $v_i \in S_i^{x_i}$ for all $i \in [n]$: output $f(x, H(v_1, \ldots, v_n))$

Else, abort

$S_i^0 := S_i$

$S_i^1 := S_i^{\perp}$

Random oracle $H : \{0,1\}^{\lambda} \to R$

$\mathsf{OTP}_f := (|S\rangle,$  $)$

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Seems like if f is "**random enough**", this construction is a **good OTP**!

1. What is the strongest possible one-time security this construction satisfies?
2. What are the "random enough" f? All high min-entropy f?

Security Definitions

Operational Definition

Adversary cannot produce two valid outputs

$|\mathrm{OTP}_f\rangle$

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$(x_1, y_1),$
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Adv wins if

$\mathrm{Verify}_f(x_1, y_1) = \mathrm{Accept}$ and

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Operational Definition

Adversary cannot produce two valid outputs

$|\mathrm{OTP}_f\rangle$

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$(x_1, y_1),$
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Need some way to verify that a sample is valid/correct

Adv wins if

$\mathrm{Verify}_f(x_1, y_1) = \mathrm{Accept}$ and

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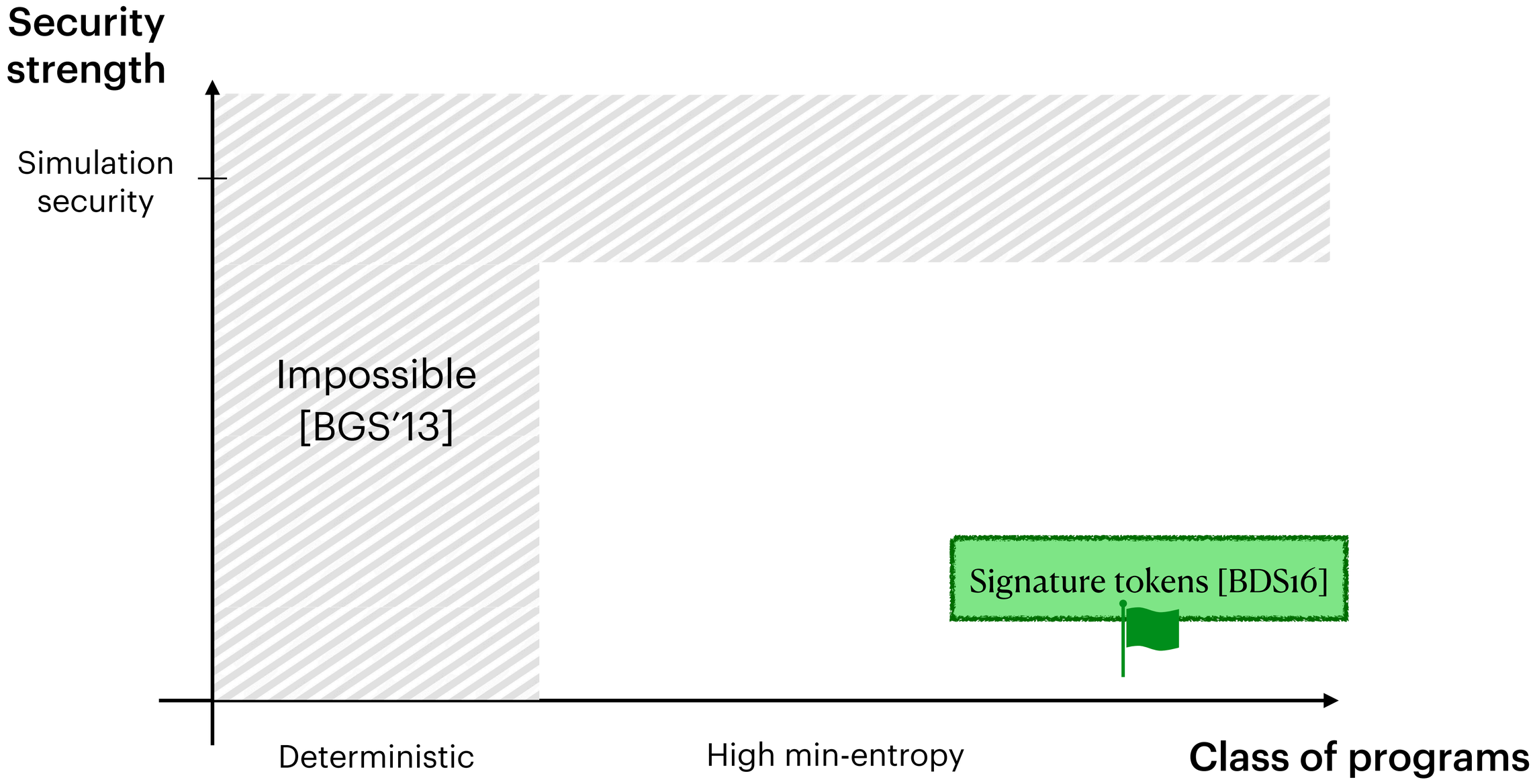
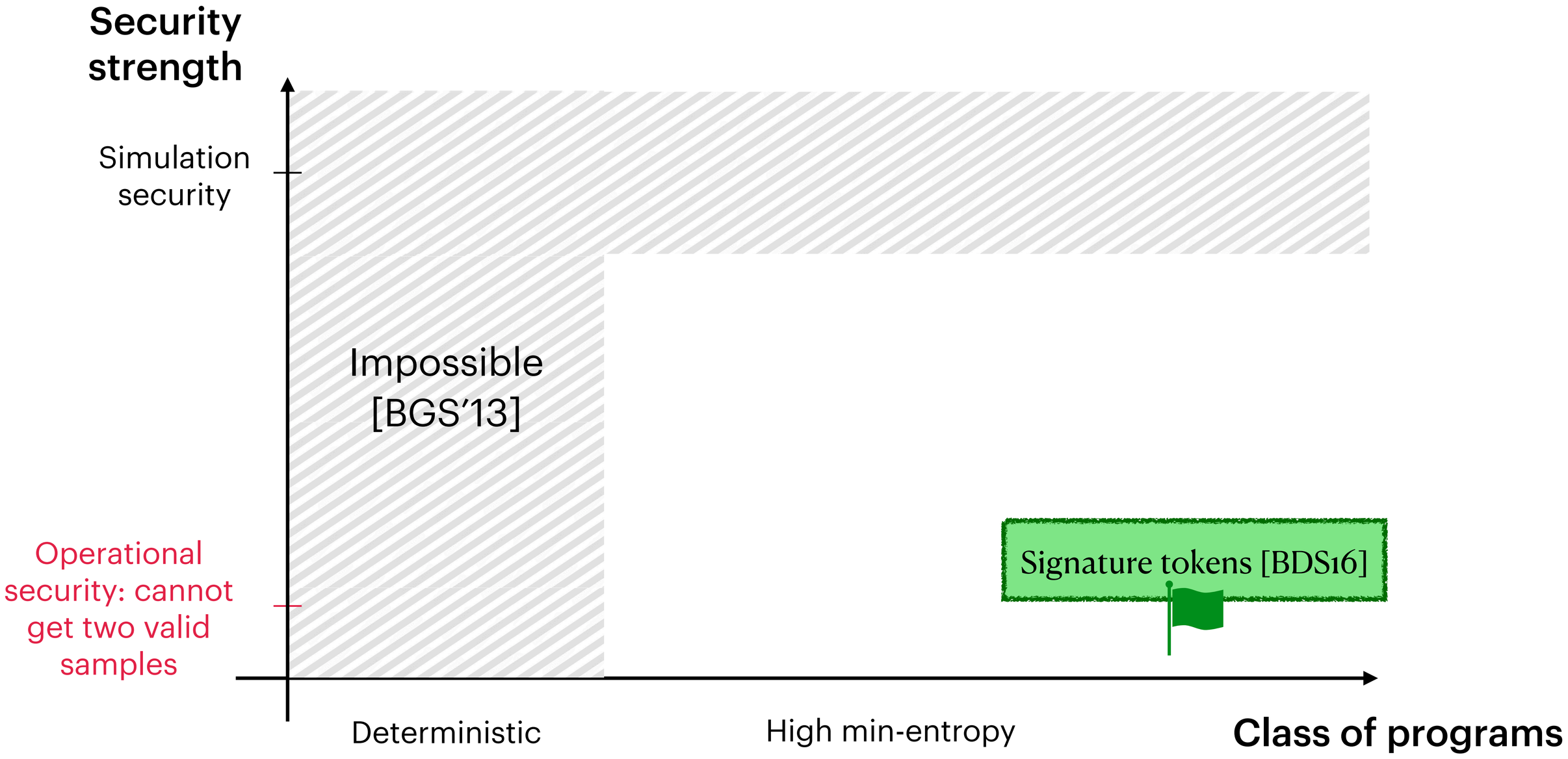

Security strength
Simulation security
Impossible [BGS'13]
Signature tokens [BDS16]
Deterministic
High min-entropy
Class of programs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output 1, output 2

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Simulation security?
"Adversary should learn nothing other than one sample of its choice."

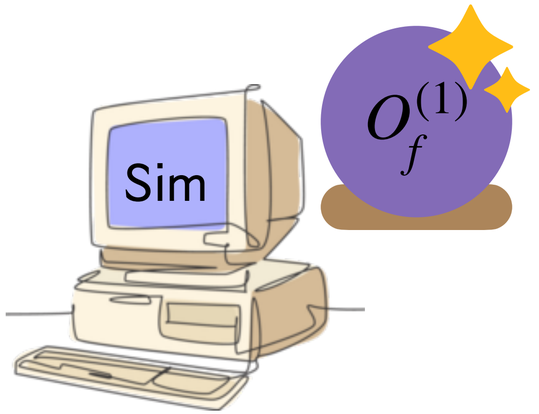
Attempt #1: Simulation-based Definition

[GKR'08, BGS'13]

Real world

$|\mathsf{OTP}_f\rangle$

Ideal world



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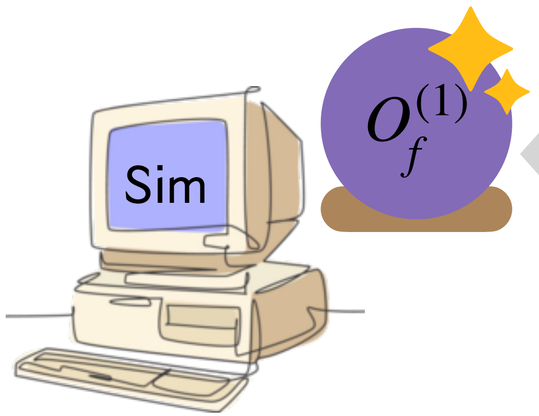

One-time oracle:
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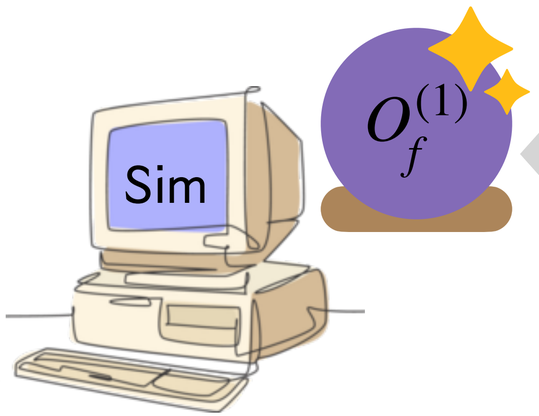

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$\approx_c$

f

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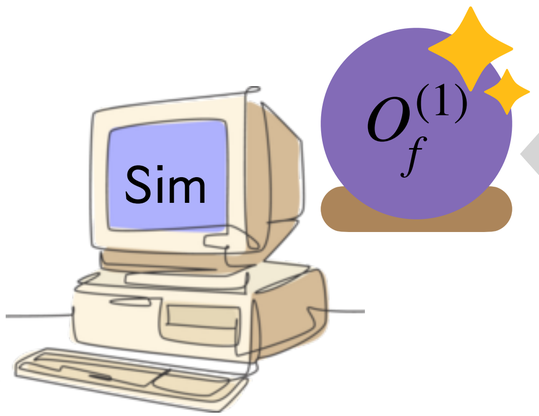

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f

$\forall$ Adv  , $\exists$ Sim  such that $\forall f$, $\forall$ distinguishers 

$$\mathsf{Adv}(|\mathsf{OTP}_f\rangle) \approx_c \mathsf{Sim}^{O_f^{(1)}}$$

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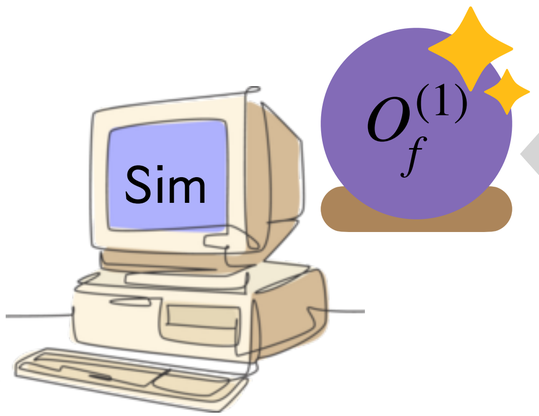

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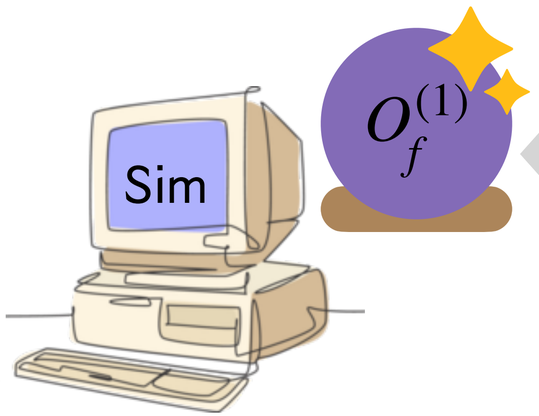

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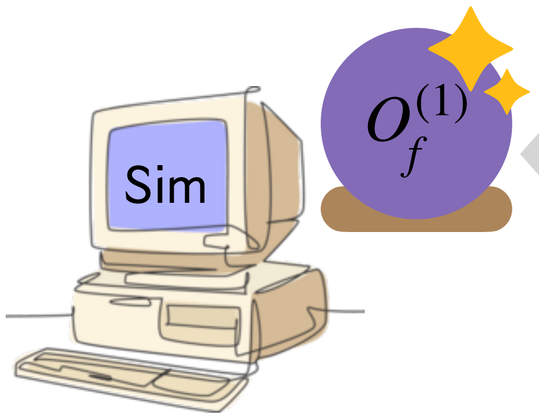

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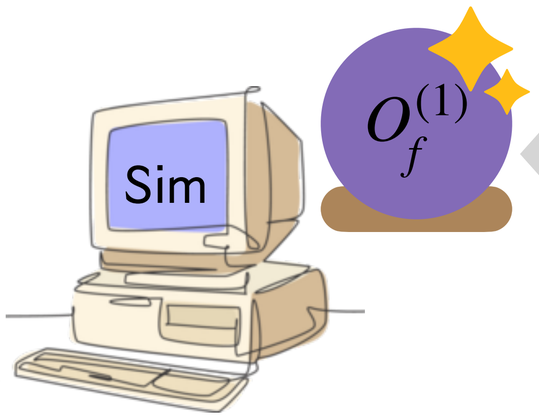

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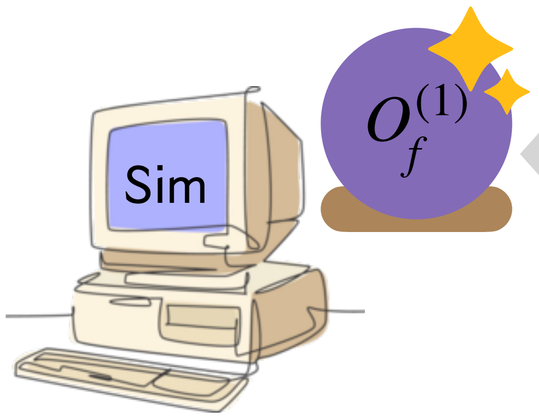

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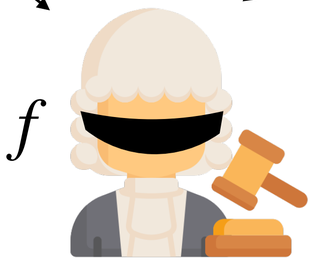

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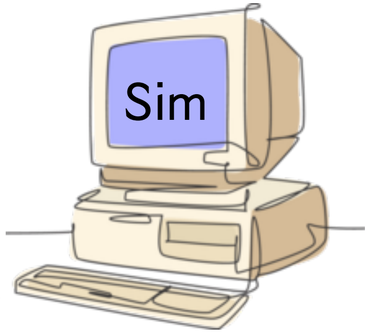

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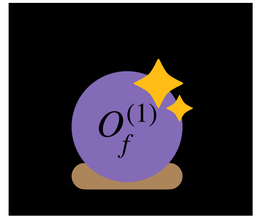

f

Impossible for all (randomized) programs!
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Evaluate on random x

- Sim cannot win in plain model unless it learns the entire program in 1 query.
- Allow Sim to output one-time oracle-aided program?

Attempt #1: Simulation-based Definition

[GKR'08, BGS'13]

Impossible even in oracle model: The un-computation attack

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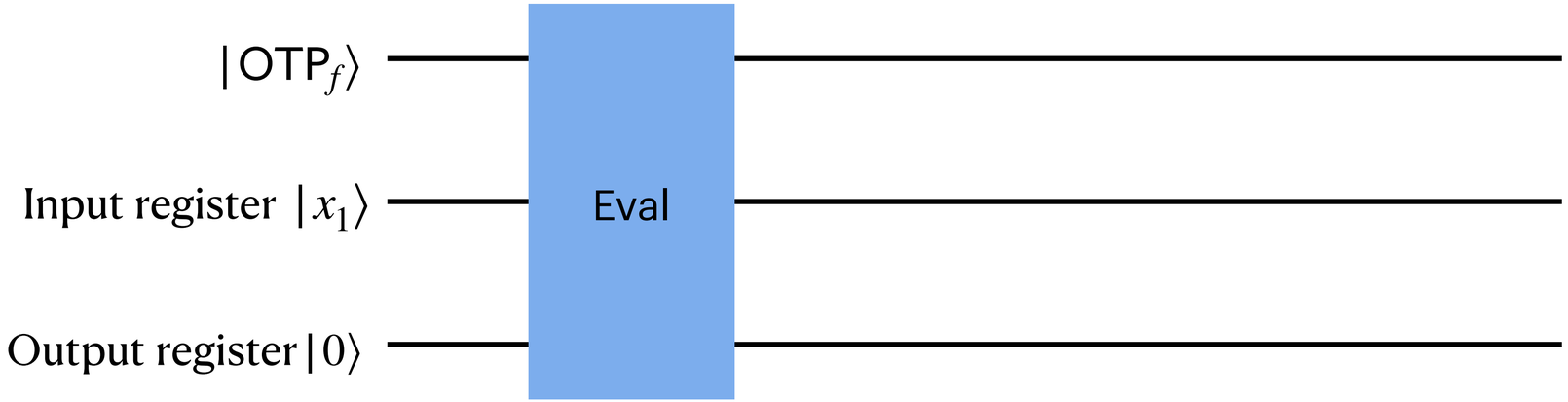

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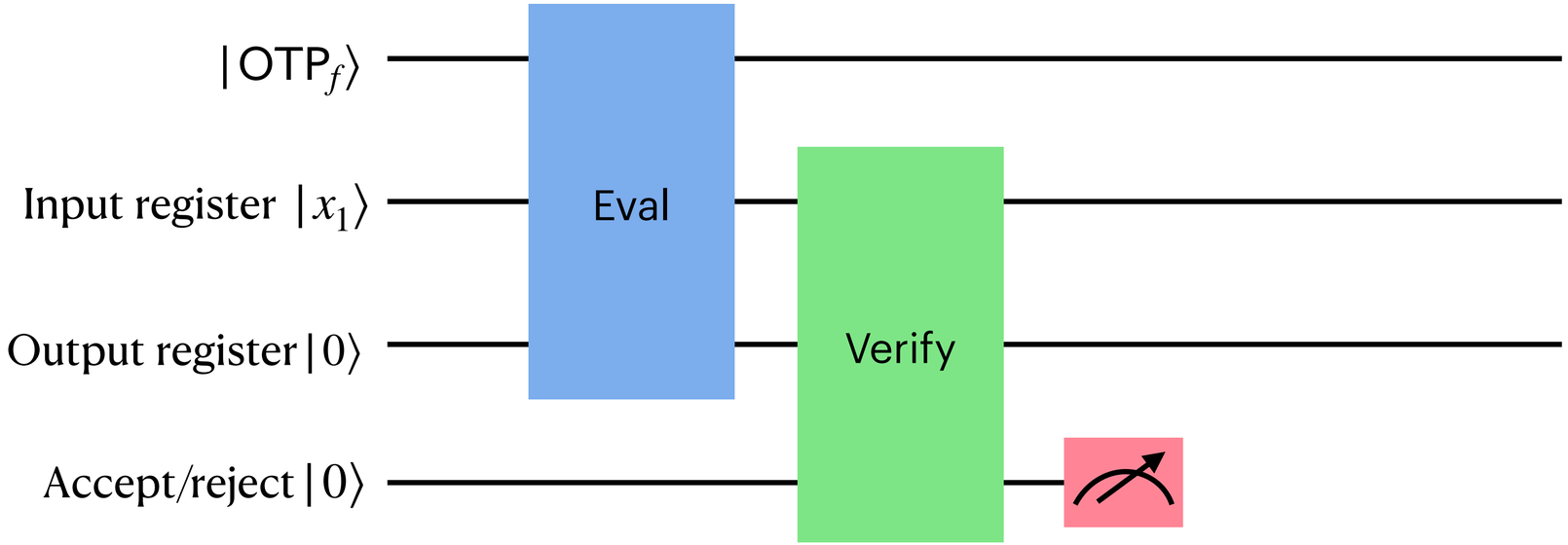

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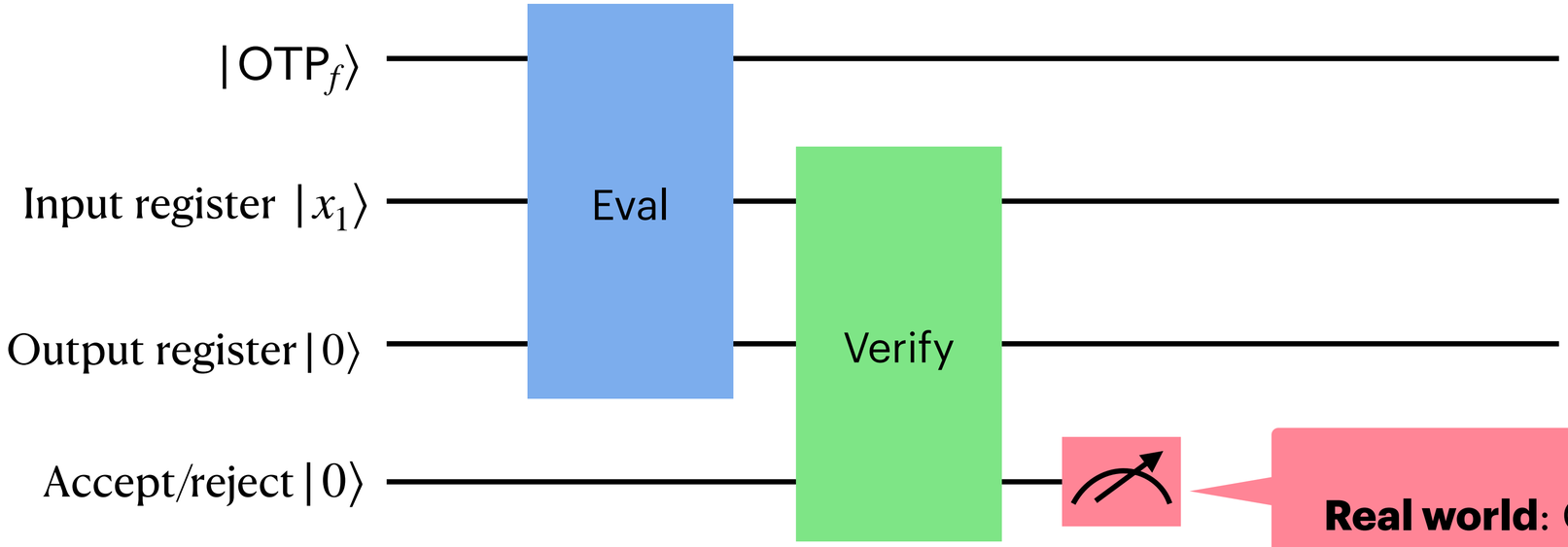

Real world: Correctness of OTP $\Longrightarrow$ gentle measurement, outcome = 1 for both x_1, x_2

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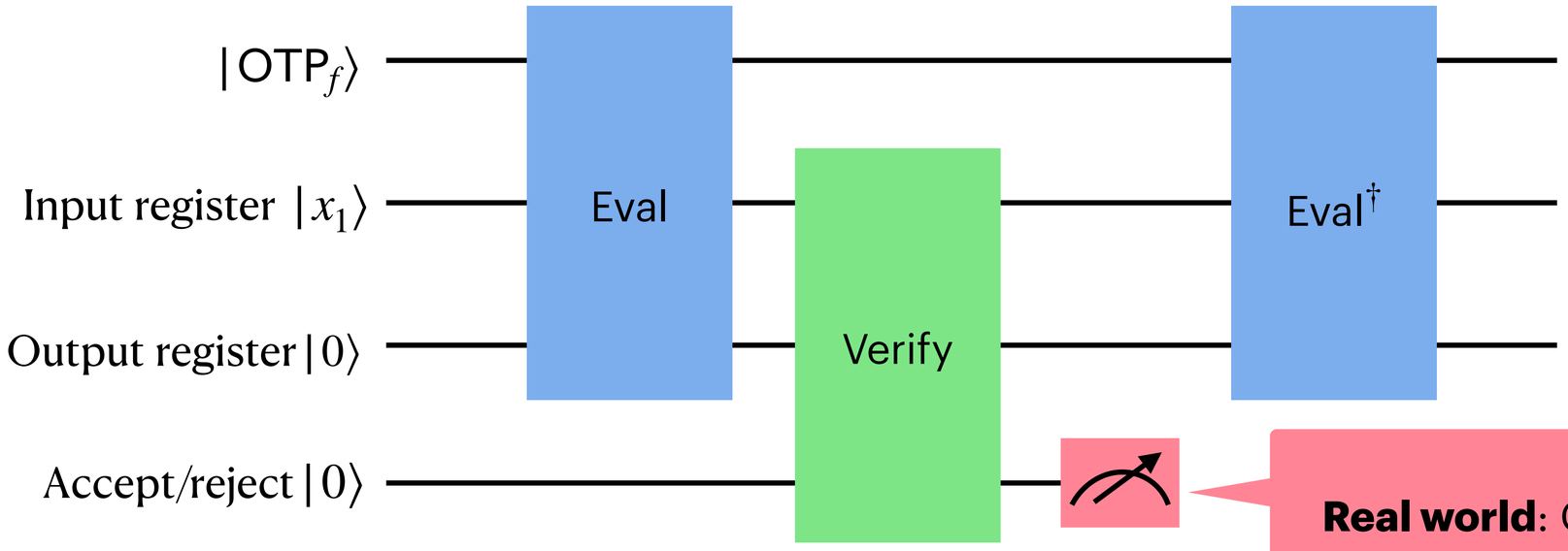

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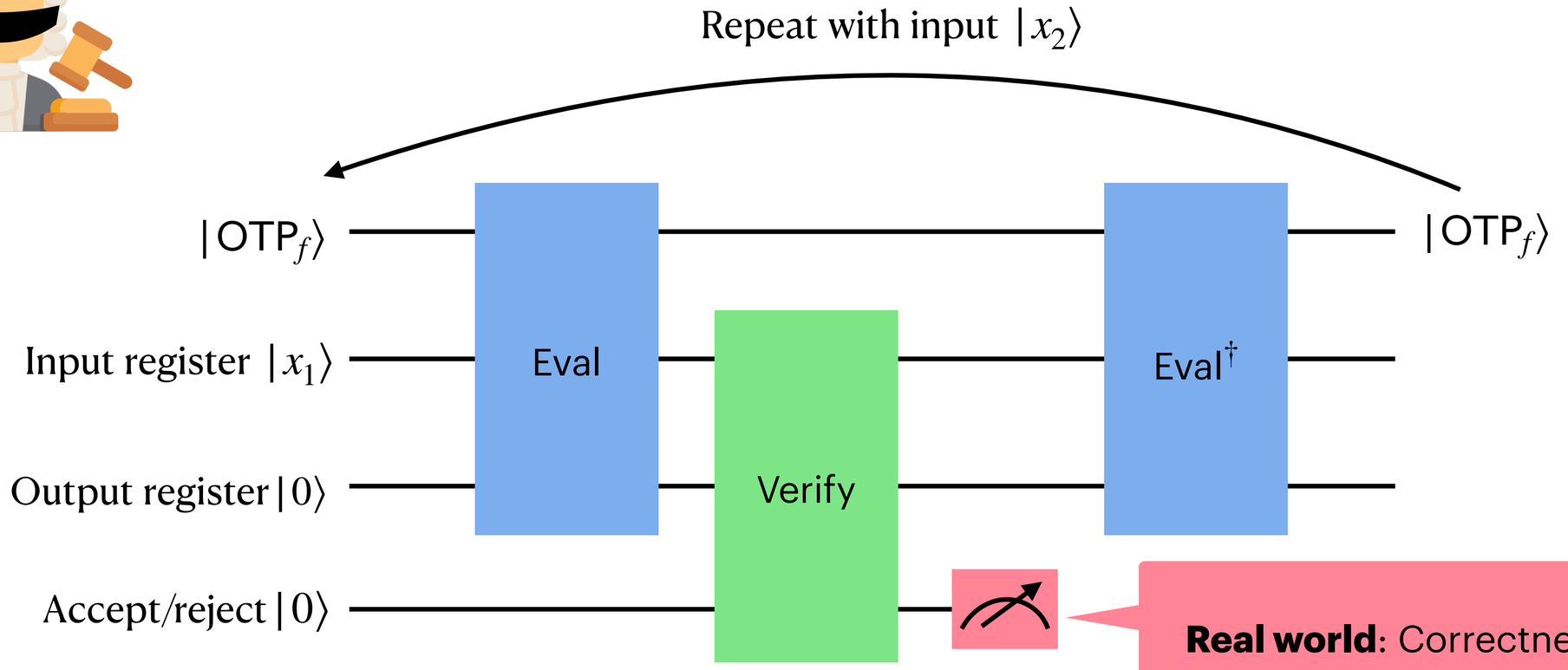

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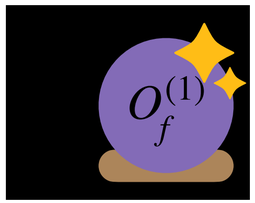

Input register $|x_1\rangle$

Output register $|0\rangle$

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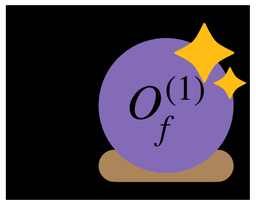

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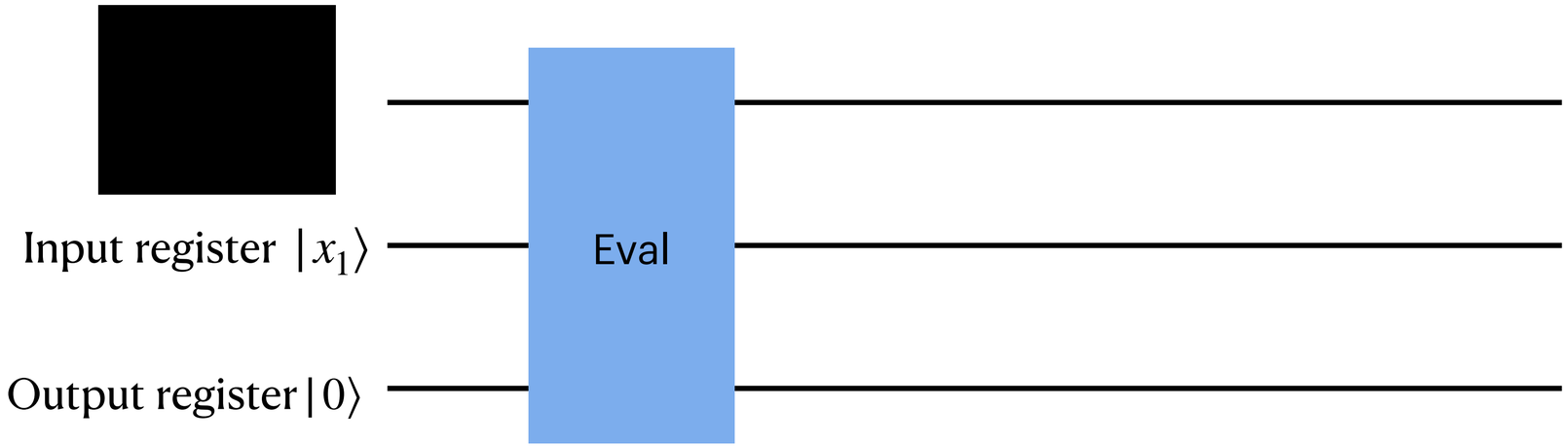

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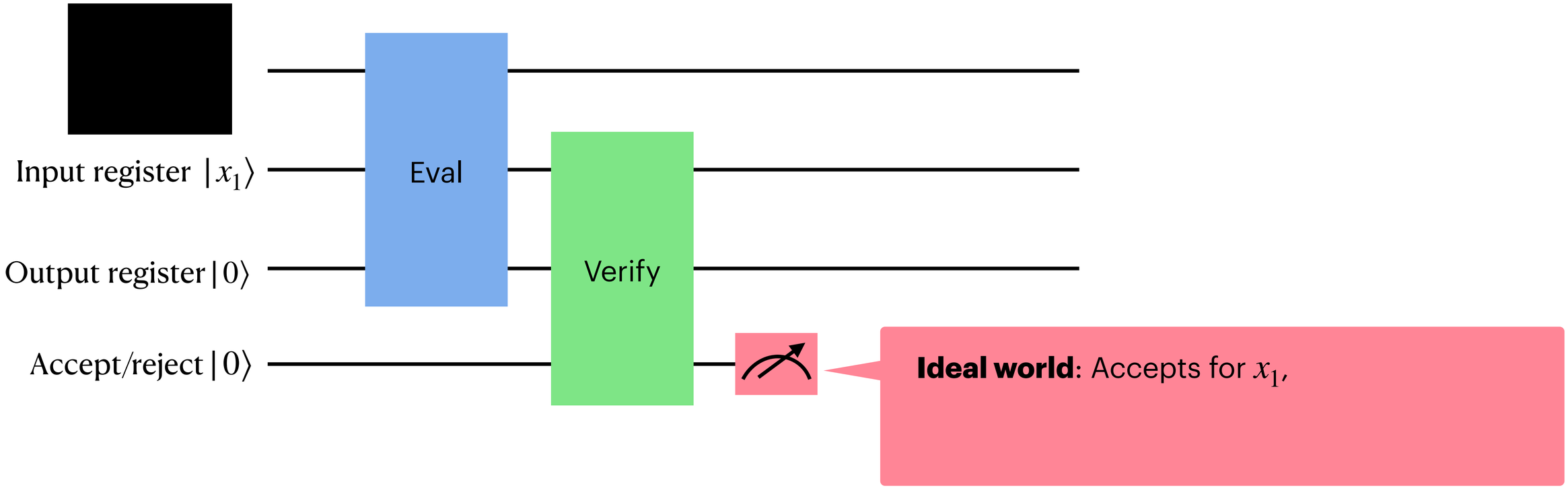

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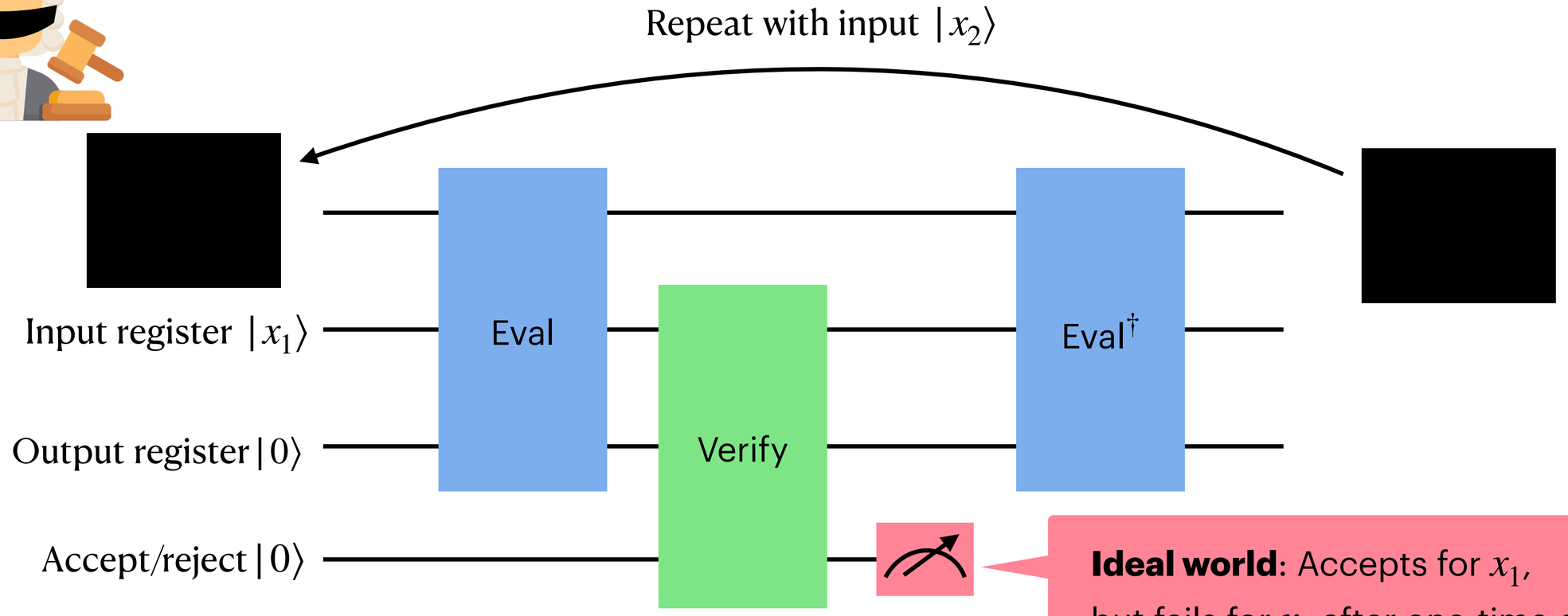
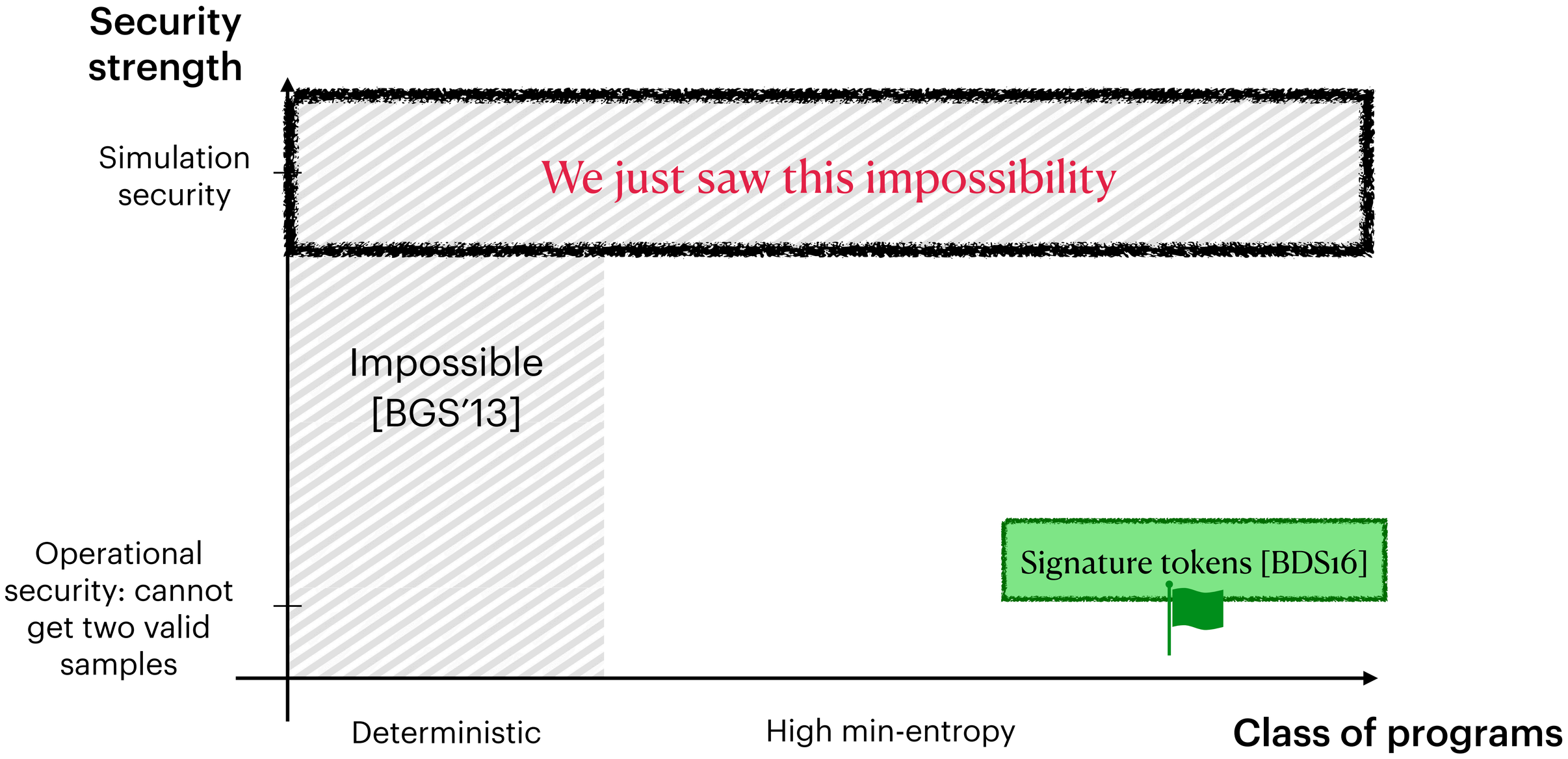


Ideal world: Accepts for x_1,
but fails for x_2 after one-time oracle self-destructs.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[GKR'08, BGS'13]

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$|\mathrm{OTP}_f\rangle$

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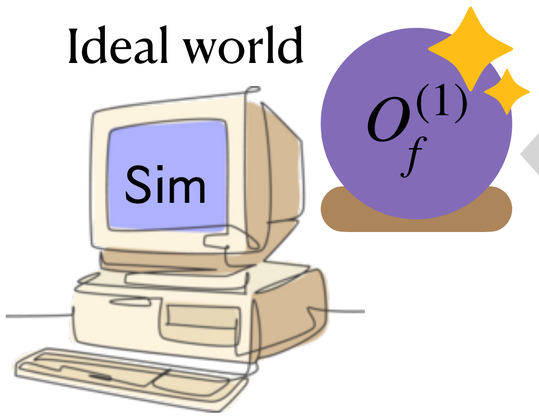

One-time oracle:
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$\approx_c$

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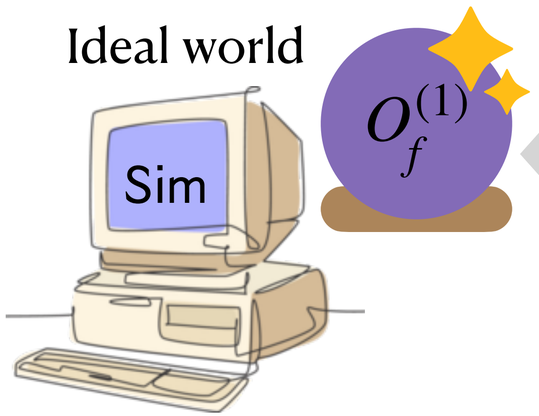

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Unsatisfying impossibility: Adv didn't learn any secrets about the program.

Can we tweak definition slightly to make it achievable?

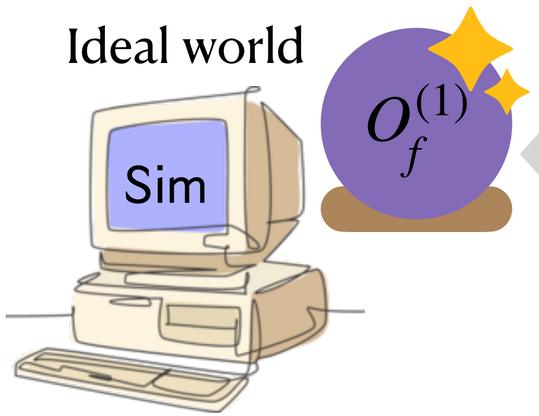
Attempt #2: Restrict adversary to classical output

Rule out "dummy" adversary

Real world

$|\mathrm{OTP}_f\rangle$

Ideal world



One-time oracle: Answers 1 query and turns off.

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Attempt #2: Restrict adversary to classical output

Rule out "dummy" adversary

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$|\mathrm{OTP}_f\rangle$

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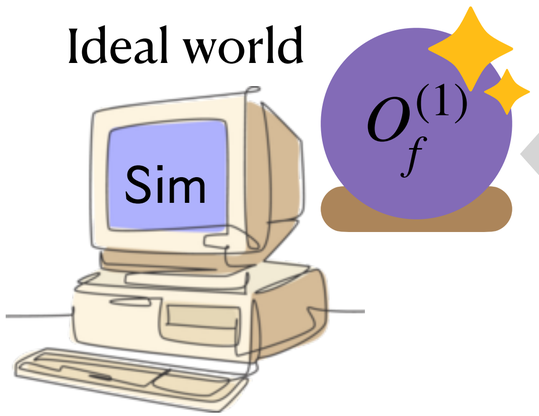

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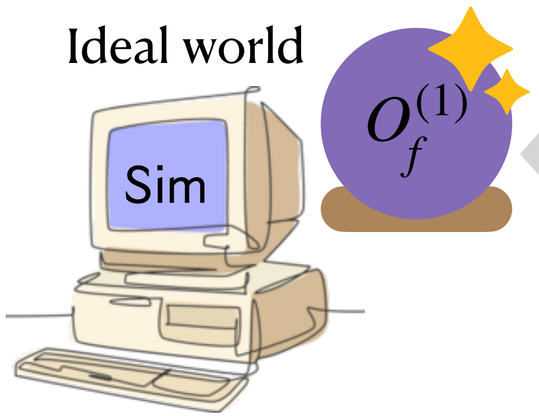

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Impossible, even for high min-entropy programs! [GLRRV'24]

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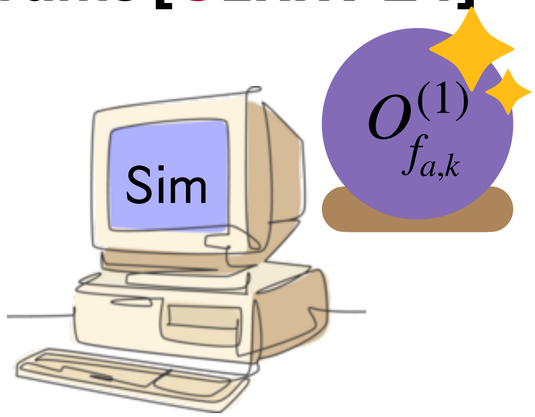

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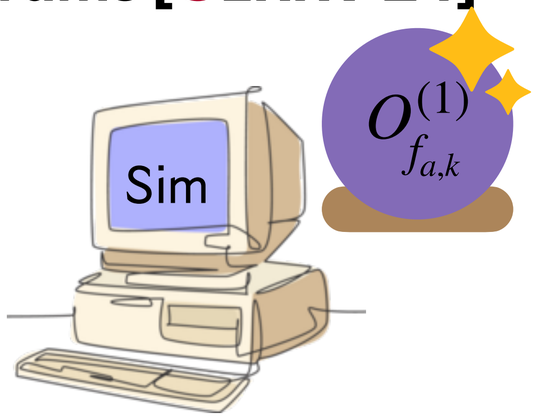

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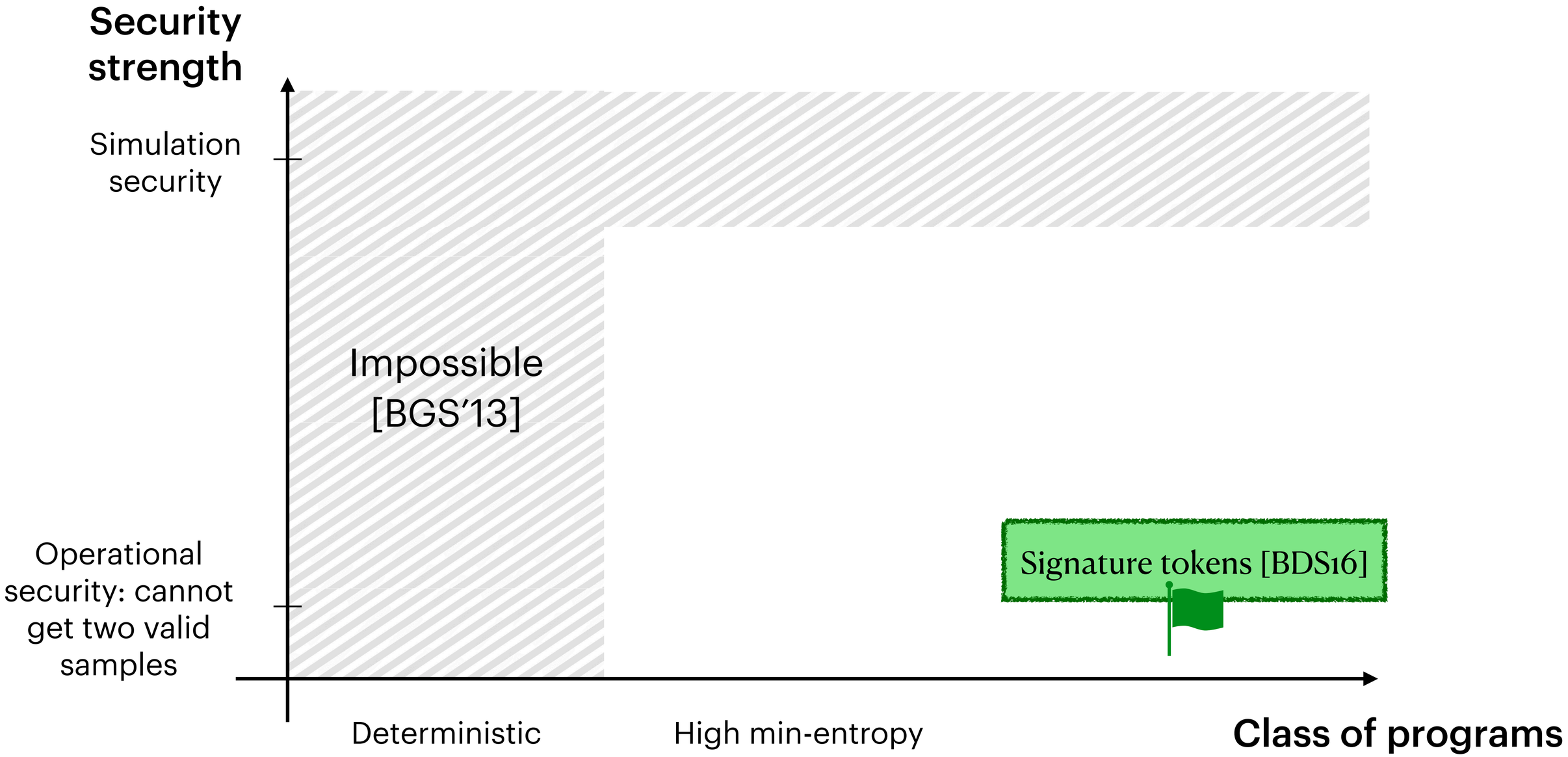
Lesson #1: Restricting adv to classical outputs does not help

Lesson #2: high min-entropy is the wrong notion!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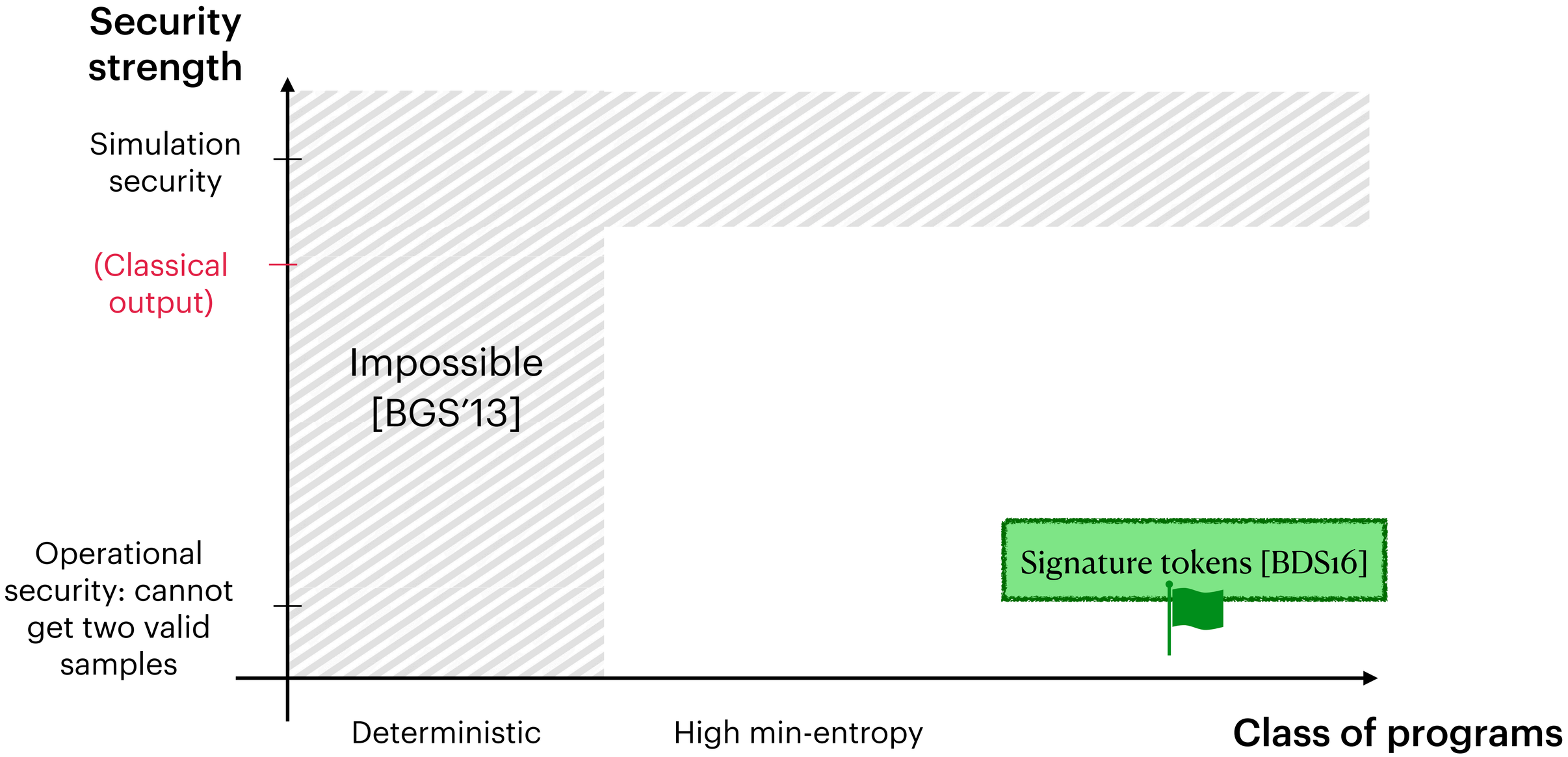
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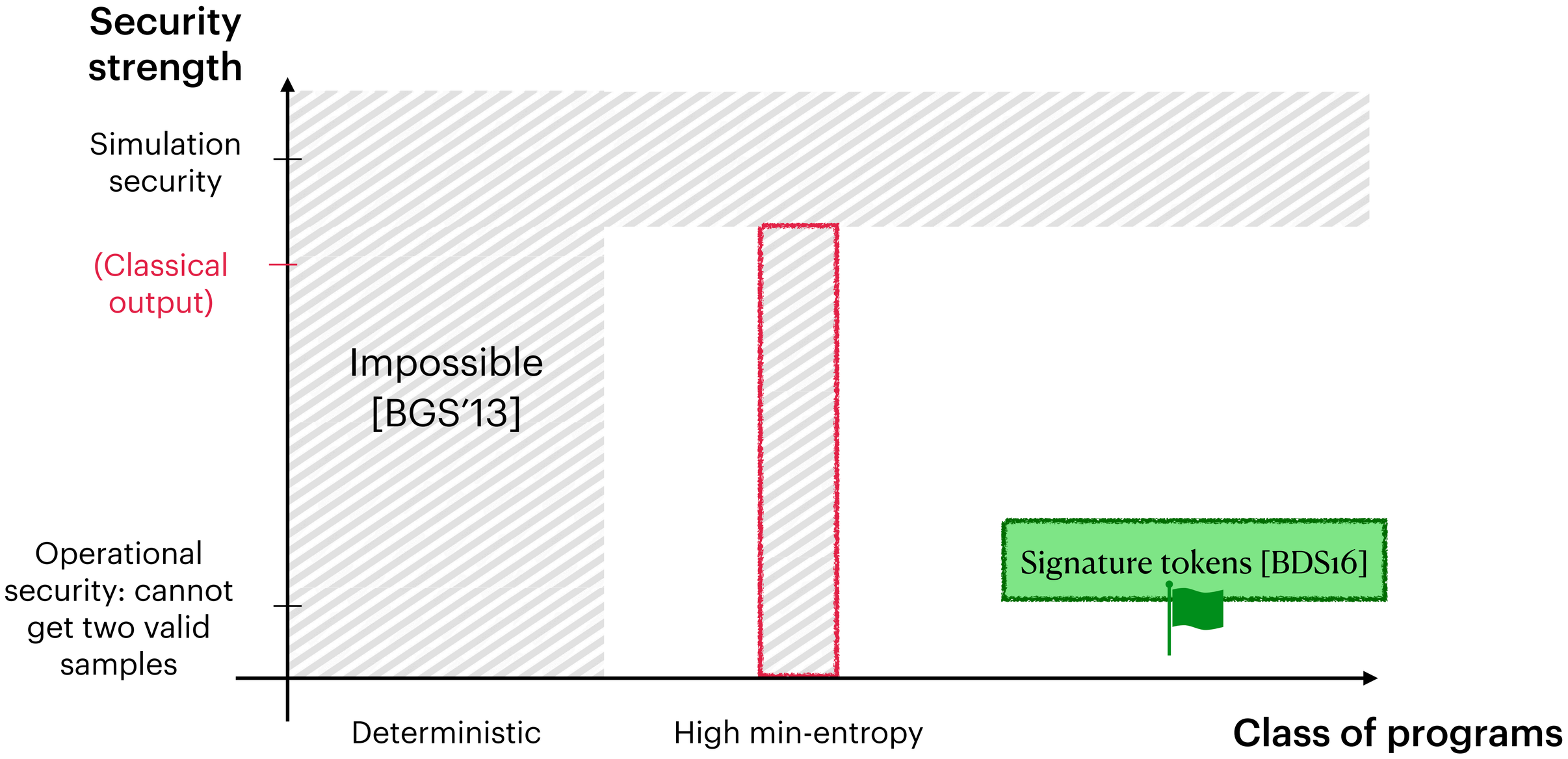
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- So let’s allow Sim to also make gentle measurements & uncompute!

  (As long as it makes only 1 destructive measurement)

Our Definition: Single Effective Query Model

Allow simulator also to make gentle queries and un-computations

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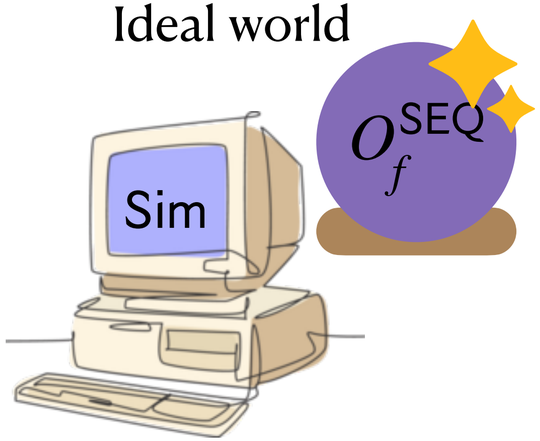

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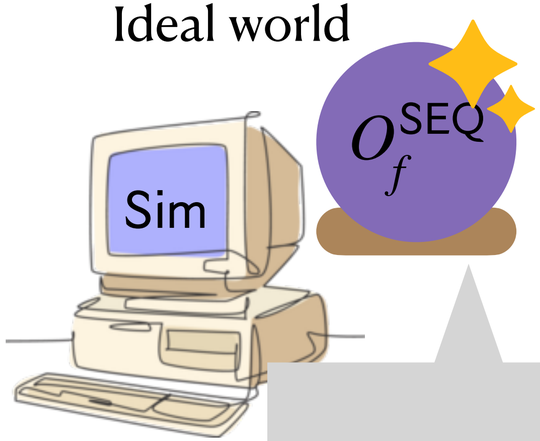

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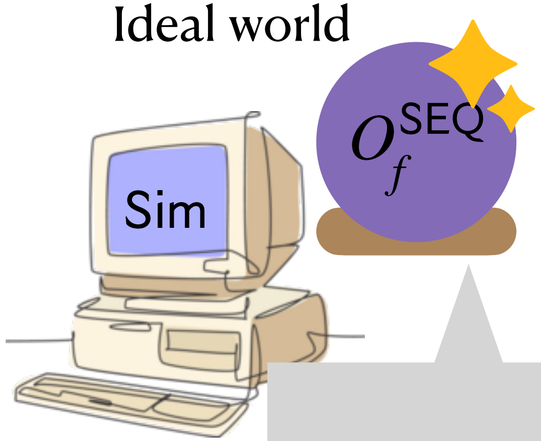

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- SEQ security is meaningless for deterministic programs, just like obfuscation and copy-protection are meaningless for "learnable" programs.

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2. Meaningful: can recover operational one-time security definitions for "random enough" programs

For "random enough" programs,

$$\text{SEQ security} \implies \text{operational security}$$

“Random enough”

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Can define more general learning games.
Eg. One-time PRF security game.

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No hope of ever producing a OTP secure for two-valid-sample game.

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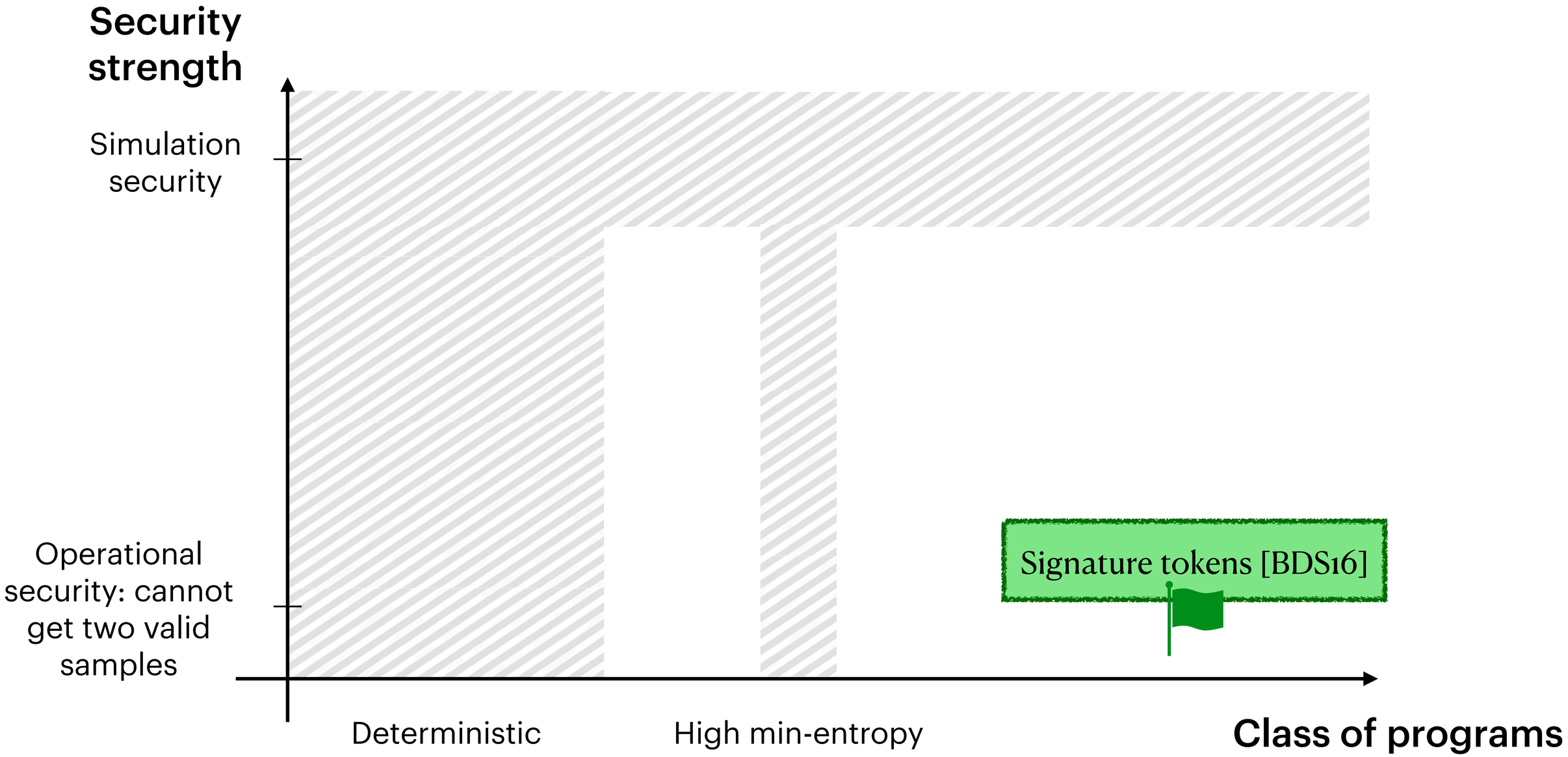
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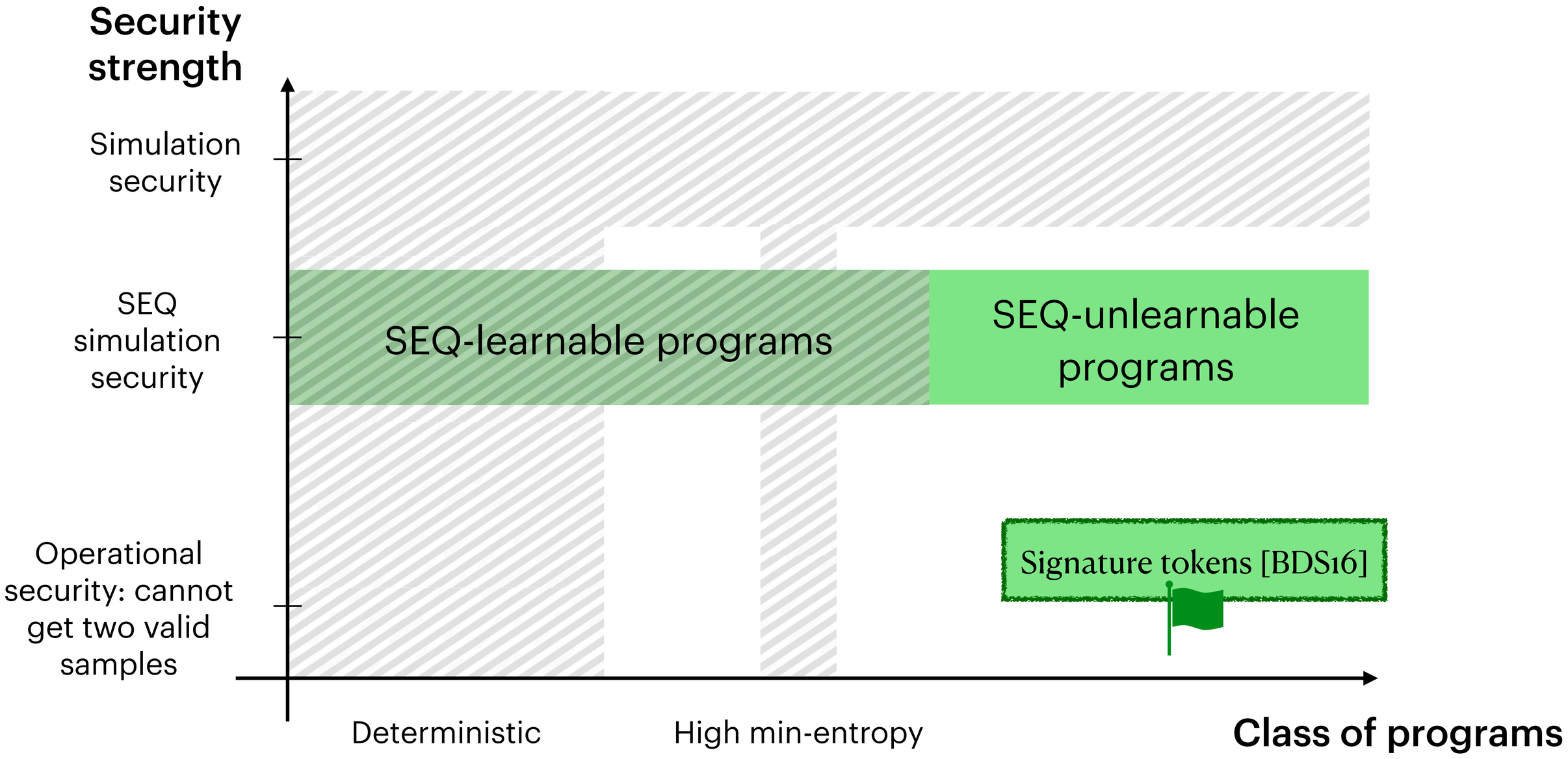
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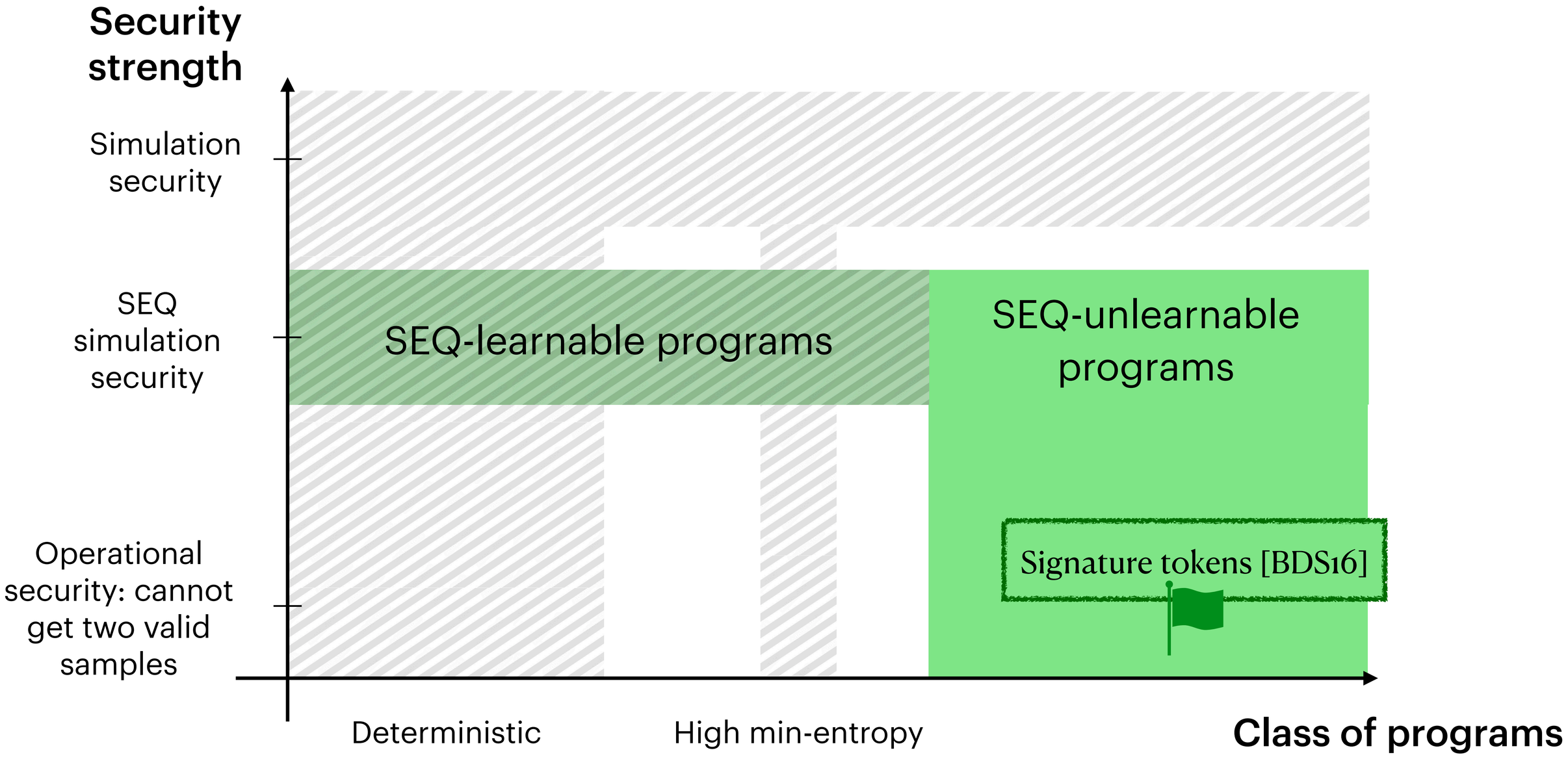
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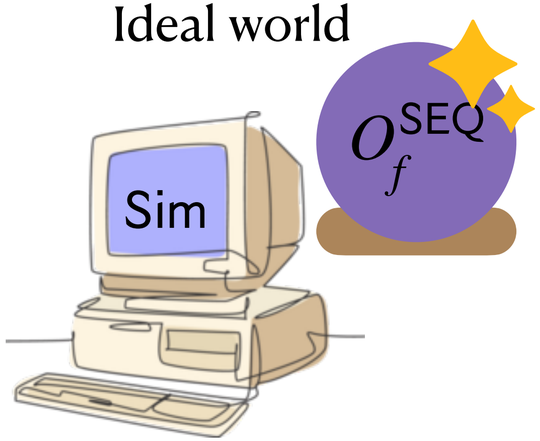

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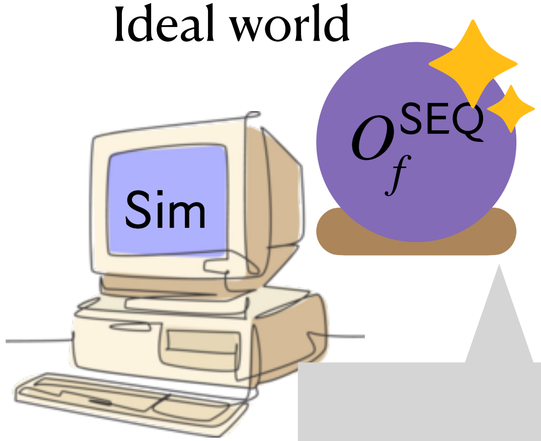

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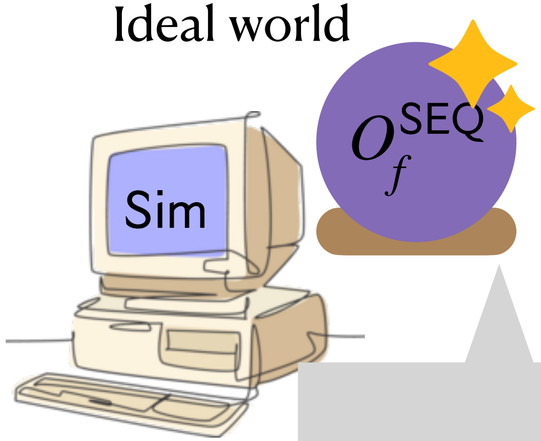

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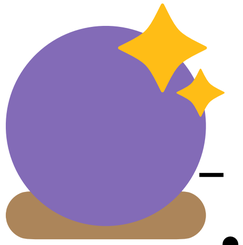

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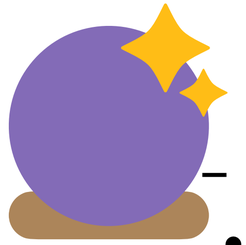

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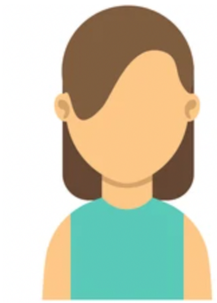

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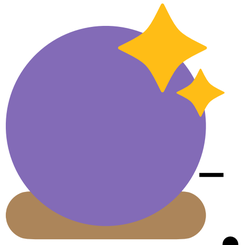

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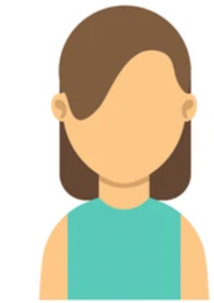

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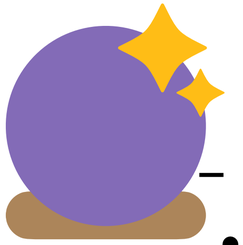

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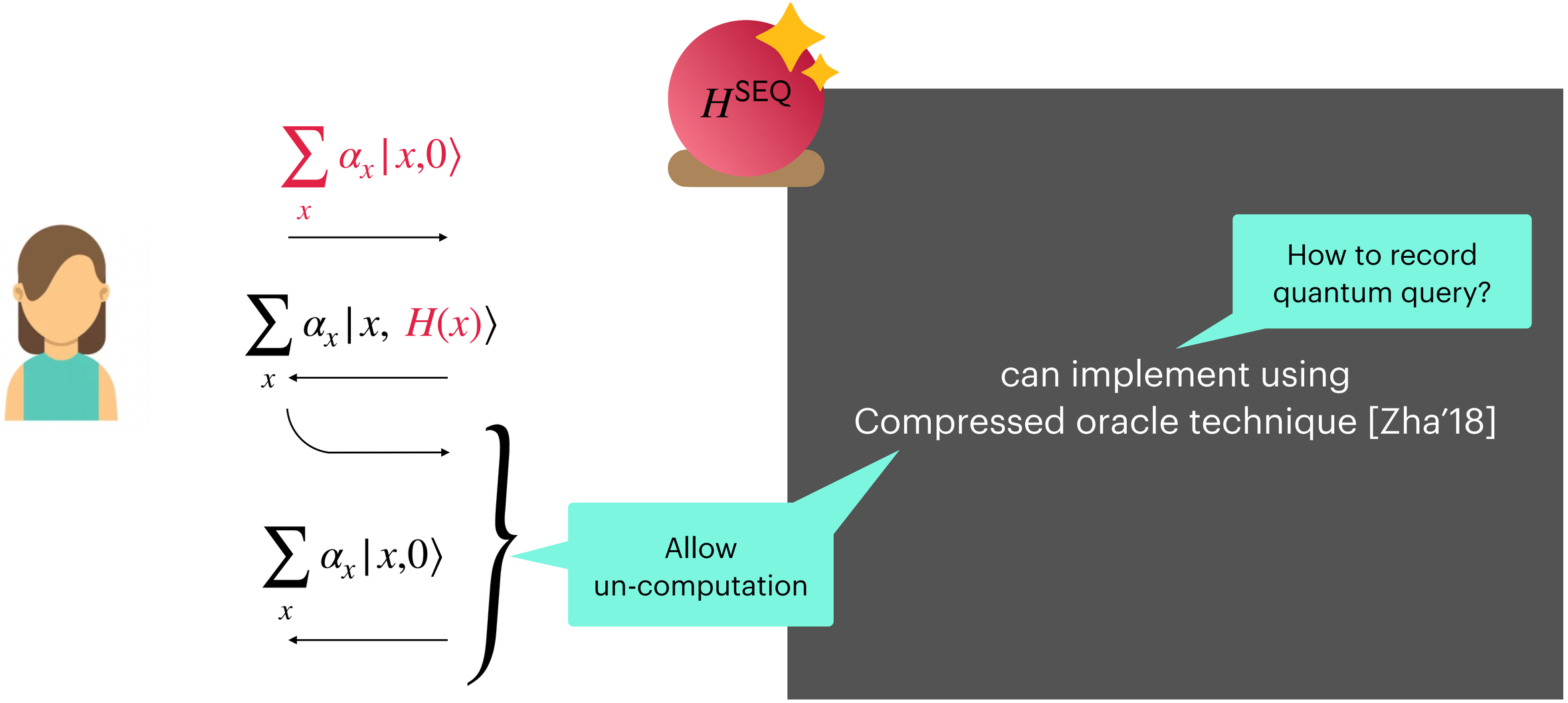
Step 1: Quantum SEQ Random oracle



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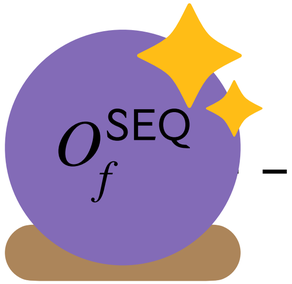

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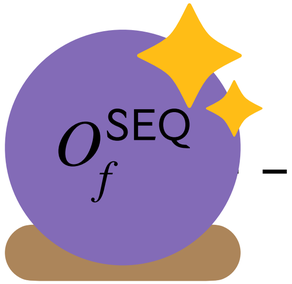

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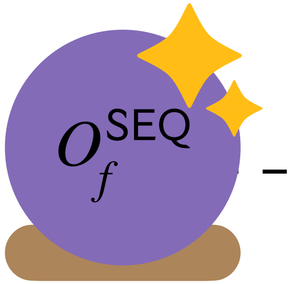

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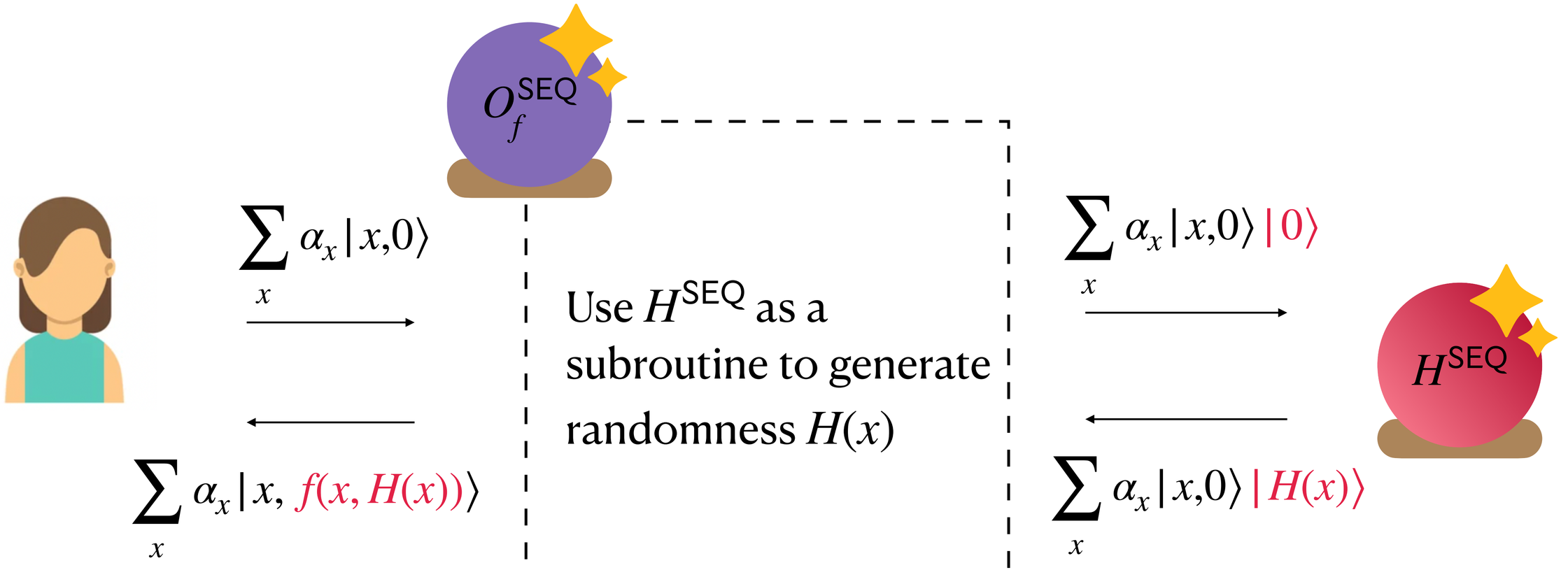

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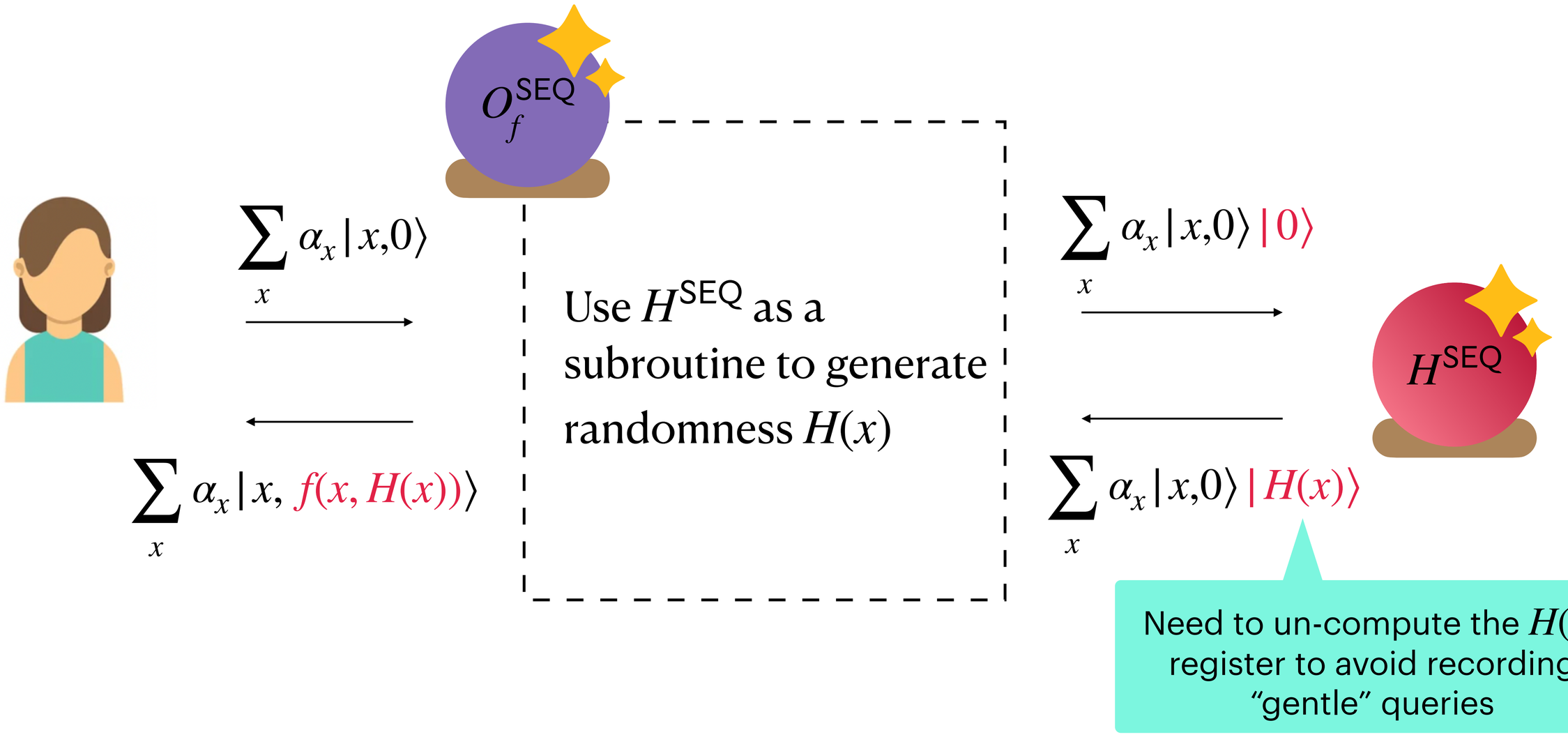
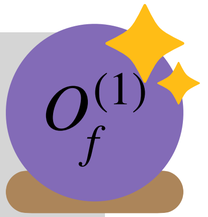


One-time security definitions

Simulation security

Operational security

Attempt #1:
One-time oracle



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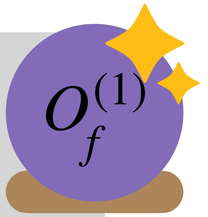
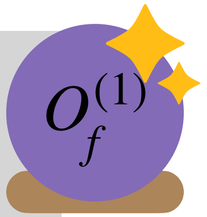


Attempt #2:
Classical output adv



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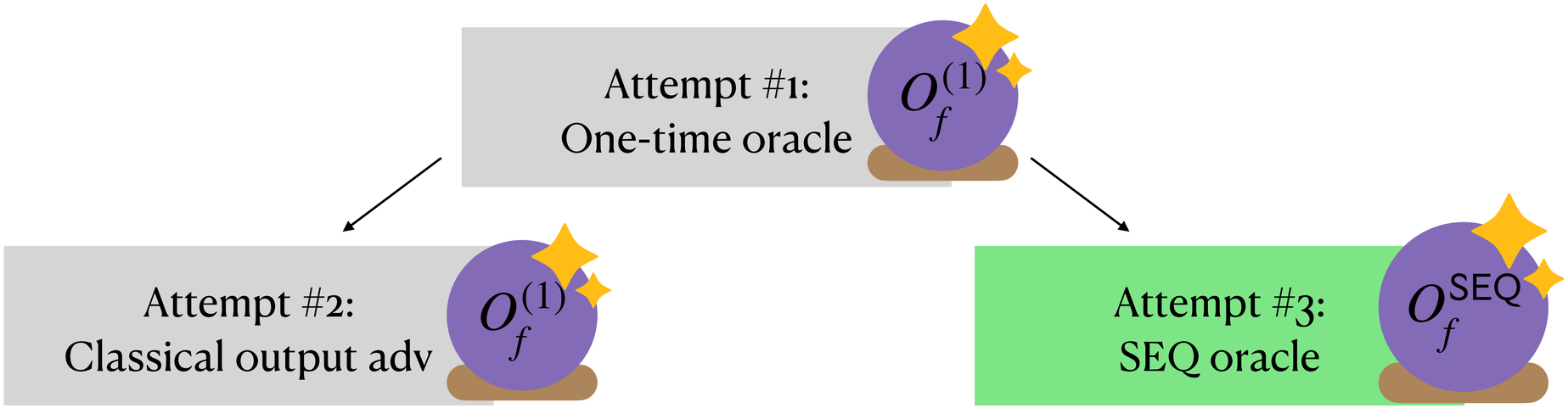

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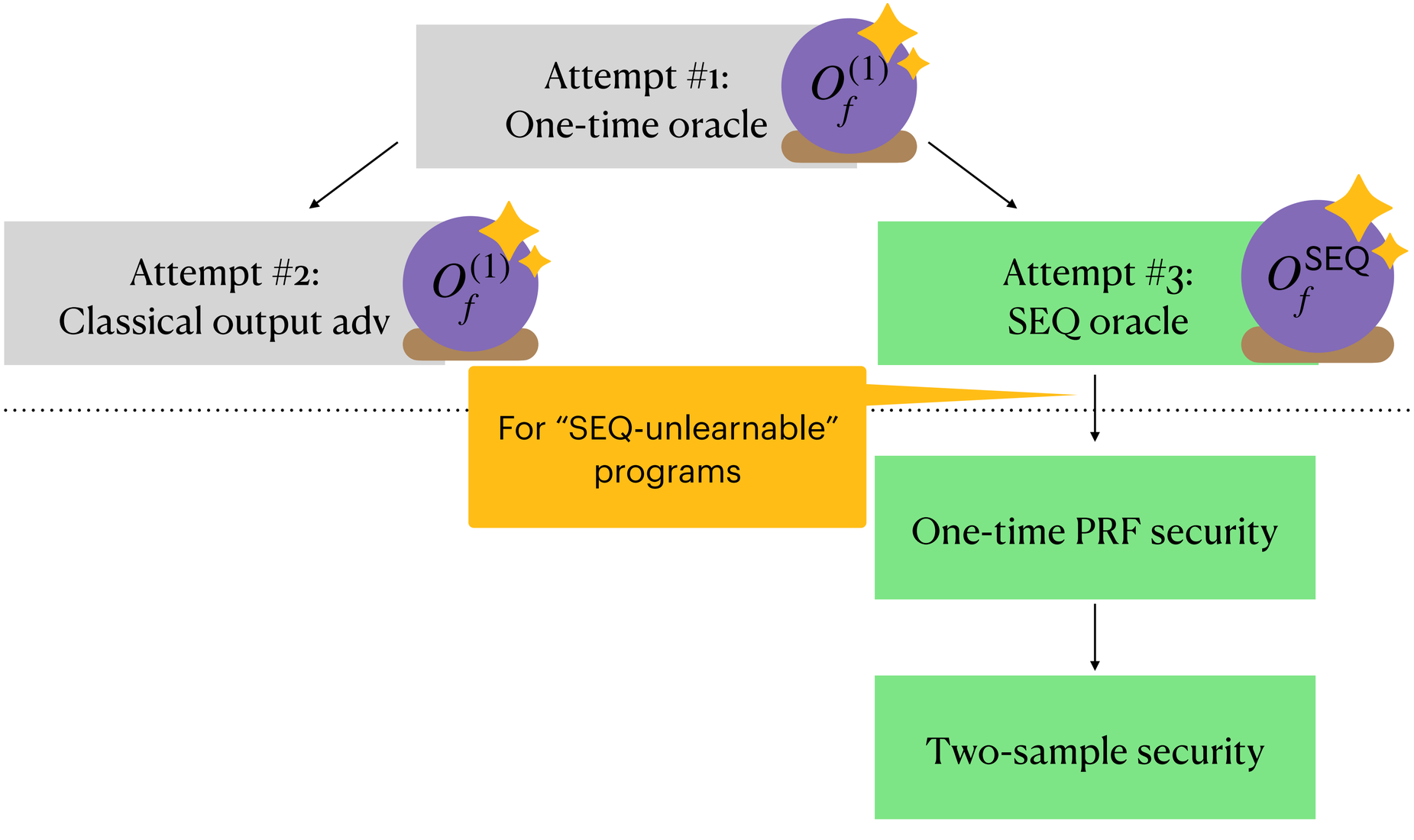

Instantiating OTPs in the Plain Model

Instantiating our OTP the Plain Model

- Use IO to heuristically instantiate oracles

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Theorem [GLRRV'24]

Post-quantum IO + LWE $\Longrightarrow$ OTP for randomized constrained PRFs in plain model

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Theorem [GLRRV'24]

Post-quantum IO + LWE $\Longrightarrow$ OTP for randomized constrained PRFs in plain model

Theorem [GLRRV'24]

$\exists$ SEQ-unlearnable programs that have no OTP in the plain model.

Randomized Constrained PRFs

[KPTZ13, BW13, BGI14]

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Randomized PRF

$$F'_k(x; r) = \left(r\,,\ F_k(x \| r)\right)$$

for PRF F

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- $F_k(x)$ pseudorandom for all $C(x) = 0$

One-time PRF security game

“Two-valid-outputs” game might not be secure enough for some programs, like PRFs

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$|\mathrm{OTP}_{F'_k}\rangle$ ←

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y_1, y_2

Sample PRF key k

Sample $b_1, b_2 \leftarrow \{0,1\}$

If $b_i = 0$ then $y_i := F_k(x_i \| r_i)$

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$\hat{b}_1, \hat{b}_2$

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Adv wins if:

$b_1 = \hat{b}_1$ and $b_2 = \hat{b}_2$

Randomized PRF

$F'_k(x; r) = \left(r\,,\ F_k(x \| r)\right)$

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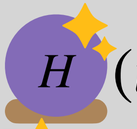
OTP for constrained PRF F'_k

Construction in plain model

Classical oracle construction: $\mathsf{OTP}_f := (|S\rangle,$  $)$

$\underline{O(x,u)}$:

$r \leftarrow$  (u)

If  $(u) = 1$: output $F'_k(x;r)$

Else: abort

OTP for constrained PRF F'_k

Construction in plain model

Plain model

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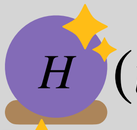
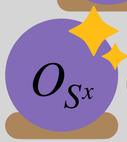
$r \leftarrow$  (u)

If  $(u) = 1$: output $F'_k(x;r)$

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- Constrain PRF F'_k to r whose preimages are valid subspace vectors

*Use coset states instead of subspace states for plain model security proof [CLLZ21, VZ21]

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- Then, adversary must have found $s \in S, s' \in S^\perp$

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Applications

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- **More signature tokens:**
  - Blind unforgeable signatures [AMRS20] $\Longrightarrow$ signature tokens

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- **One-time NIZK proofs**
  - One-time proving token that allows a prover to prove only one statement
- **Quantum money**
  - One-time program for a signature scheme

Open Question #1

Strongest achievable notion of one-time security?

- We gave the SEQ simulation security definition
- Is it the strongest notion achievable?

Open Question #2

More applications for one-time **randomized** programs?

- We suggest a few: more signature tokens, one-time proofs, quantum money
- Flagship application of [Gunn-Movassagh'24] is Generative AI/LLMs
    - Heuristic security, difficult to prove

Open Question #3

Construction for one-time quantum channels?

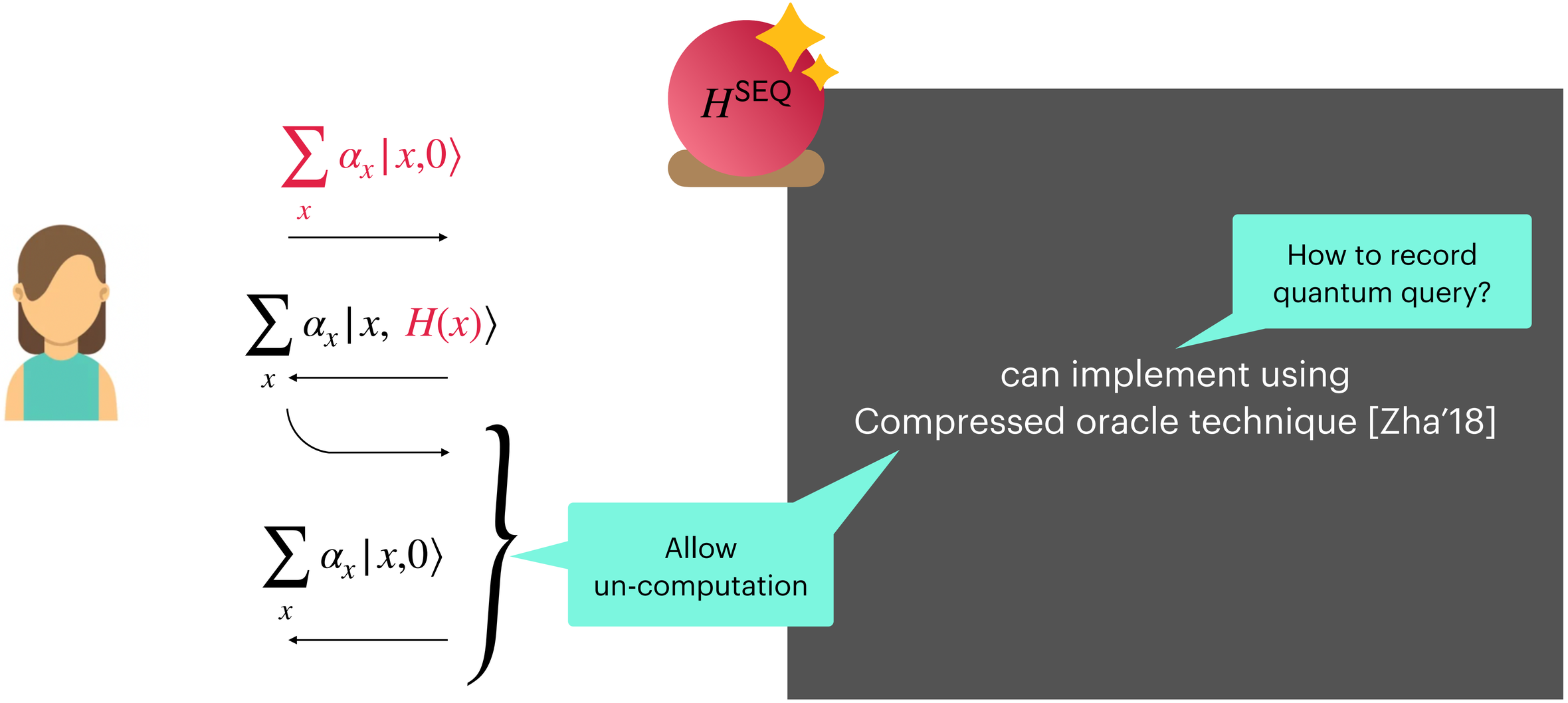
- [BGS13] give a construction assuming one-time memory hardware devices
- Not clear how to generalize our construction
  - We used unclonable subspace states to generate classical randomness
  - How to tamper with inherent quantum randomness?

Thank you!

Bonus slides I

Compressed Oracle Technique

Step 1: Quantum SEQ Random oracle



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Quantum queries to random oracle H

Action on standard basis states: $|x, u\rangle \mapsto |x, u + H(x)\rangle$

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- Writing $H : [N] \to R$ as a truth table,

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  - If database contains x, answer query

#recorded queries: 0 → 1

Uncomputation: #recorded queries: 1 → 0

Disclaimer: (Very) informal description of the SEQ oracle

Single Effective Query (SEQ) Model

Use Zhandry's compressed oracle technique to maintain a database of queries made

- Initialize empty compressed oracle database for RO
- On query $|x, u\rangle$,
  - If database contains no entries, answer query
  - If database contains x, answer query
  - If database contains $x' \neq x$, don't answer query

#recorded queries: 0 → 1

Uncomputation: #recorded queries: 1 → 0

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Disclaimer: (Very) informal description of the SEQ oracle

Bonus slides II

Plain model barrier

Theorem [GLRRV'24]

∃ SEQ-unlearnable programs that have no OTP in the plain model.

Inspired by [ABDS20, AP21]

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Input	Output
x	$f_{sk,\, a,\, b}\,(x\,;\,r)$

Theorem [GLRRV'24]

∃ SEQ-unlearnable programs that have no OTP in the plain model.

Inspired by [ABDS20, AP21]

Input x	**Output** $f_{sk, a, b}(x\,;\, r)$
a	$Enc_{sk}(b;\, r)$

Theorem [GLRRV'24]

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aux

- QFHE public key pk'
- $\mathrm{Enc}_{pk'}(a)$
- Obfuscation $\hat{P}$ of P

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$$P(x) = \begin{cases} (sk, sk') & \text{if } \mathsf{Dec}_{sk}(\mathsf{Dec}_{sk'}(x)) = b \\ 0 & \text{otherwise} \end{cases}$$

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Plain model $|\mathsf{OTP}_f\rangle$ + aux

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↓ **QFHE evaluation** on $\mathsf{Enc}_{pk'}(a)$

$\mathsf{Enc}_{sk'}(\mathsf{Enc}_{sk}(b))$

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$\mathsf{Enc}_{sk'}(\mathsf{Enc}_{sk}(b)) \longrightarrow \hat{P} \longrightarrow (sk, sk')$

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↓

(a, b)

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Plain model $|\mathsf{OTP}_f\rangle$ + aux

Plain model OTP allows you to do non-black box things like QFHE evaluation

QFHE evaluation on $\mathsf{Enc}_{pk'}(a)$

$$\mathsf{Enc}_{sk'}(\mathsf{Enc}_{sk}(b)) \longrightarrow \hat{P} \longrightarrow (sk, sk') \longrightarrow (a, b)$$

aux

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