Spin Glass: Edwards-Anderson Model and Mean-Field Approximation

Nobel Prizes Relevant to Statistical Physics

- 1962: Lev Landau Superfluids
- 1968: Lars Onsager 2D Ising Model
- 1977: Philip Anderson Localization, Spin Glasses
- 1982: Kenneth Wilson Renormalization Group
- 1991: Pierre-Gilles de Gennes Polymers, Liquid Crystals
- 2016: Kosterlitz and Thouless Topological Defects
- 2021: Giorgio Parisi Replica Symmetry Breaking
- 2024: John Hopfield Neural Networks

Experimental Observations in Spin Glasses

- No remnant magnetization at zero field
- Cusp in low-field susceptibility
- History dependence (field-cooled vs. zero-field-cooled)
- Slow dynamics and aging (memory effects)

Origin of Interaction

Integrating out mobile electrons in Cu or Au leads to the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between magnetic elements.

Frustration

- The changing sign of interactions causes frustration.
- Many low-energy states exist.
- Magnetization m is not a good order parameter.

Edwards-Anderson Model

$$H = -\sum_{\langle i,j \rangle} J_{ij}\sigma_i\sigma_j$$

- **Quenched randomness:**
$$\langle J_{ij} \rangle = 0, \quad \langle J_{ij}^2 \rangle = J^2$$
- Quenched average:
$$\overline{O} = \int \mathcal{D}J P(J) O(J)$$
- Edwards-Anderson order parameter:
$$q = \overline{\langle \sigma_i \rangle^2}$$

Mean-Field Approximation

Each spin feels an effective local field:

$$h_i = h_0 + \sum_j J_{ij}\langle \sigma_j \rangle$$

Assume $h_i \sim \mathcal{N}(0, J^2 zq)$ with z the number of neighbors. Then:

$$q = \int dh\, P(h)\, \tanh^2(\beta h), \quad m = \int dh\, P(h)\, \tanh(\beta h)$$

Zero Field Limit

When $h_0 = 0$, then $m = 0$. We have:

$$q = \int \frac{dh}{\sqrt{2\pi J^2 zq}} \exp\left(-\frac{h^2}{2J^2 zq}\right) \tanh^2(\beta h)$$

Let $h = x\sqrt{J^2 zq}$. For small q:

$$\tanh(\beta h) \approx \beta h - \frac{1}{3}(\beta h)^3 + \cdots$$

This gives:

$$q \approx \beta^2 J^2 z q - \frac{2}{3}\beta^4 J^4 z^2 q^2 + \cdots \Rightarrow q = 0 \text{ for } T > T_c$$

$$T_c = J\sqrt{z}$$

For $T < T_c$, expand near transition:

$$q \sim \left(1 - \frac{T}{T_c}\right)$$

Susceptibility

$$\chi = \left.\frac{\partial m}{\partial h_0}\right|_{h_0=0} = \beta(1-q)$$

At $T = T_c$: $q = 0 \Rightarrow \chi = \beta$
As $T \to 0$: $q \to 1 \Rightarrow \chi \to 0$

Figures