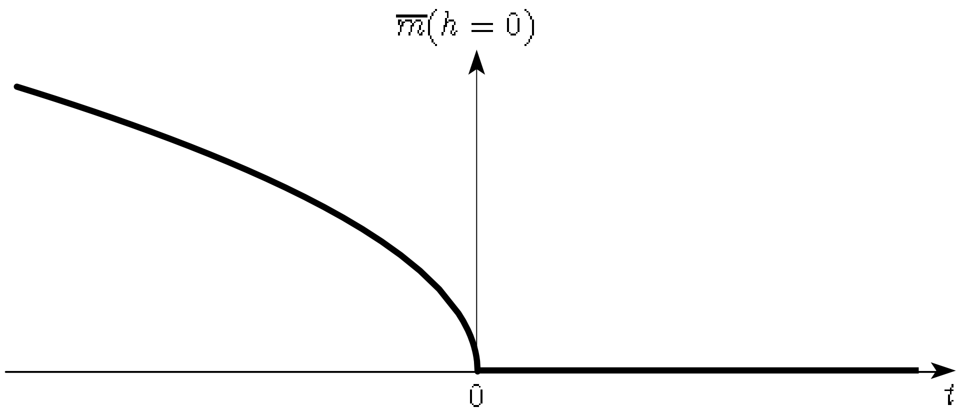
Using Eq.(II.15), we can quantify the singular behaviors predicted by the saddle point evaluations of the free energy in eqs.(II.11)–(II.13).

• **Magnetization:** In zero field, Eq.(II.13) reduces to $\partial\Psi/\partial m = t\overline{m} + 4u\overline{m}^3 = \overline{m}(t + 4u\overline{m}^2) = 0$, and we obtain

$$\overline{m}(h=0) = \begin{cases} 0 & \text{for } t > 0, \\ \sqrt{\frac{-t}{4u}} = \sqrt{\frac{a}{4u}}\,(T_c - T)^{1/2} & \text{for } t < 0. \end{cases} \tag{II.16}$$

For $t < 0$, the non-magnetized solution $\overline{m} = 0$ is a maximum of $\Psi(m)$, and there is a spontaneous magnetization than vanishes with a universal exponent of $\beta = 1/2$. critical exponent β The overall amplitude is non-universal and material dependent.



II.4. The saddle point spontaneous magnetization vanishes with a square–root singularity.

Along the critical isotherm (the dashed line if Fig.II.4), with $t = 0$, Eq.(II.13) gives

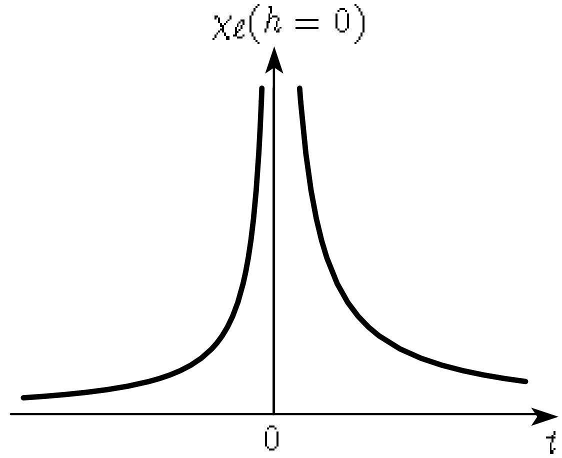
$$\overline{m}(t=0) = \left(\frac{h}{4u}\right)^{1/3}, \tag{II.17}$$

i.e. $h \propto \overline{m}^{\delta}$, with an exponent $\delta = 3$.critical exponent δ

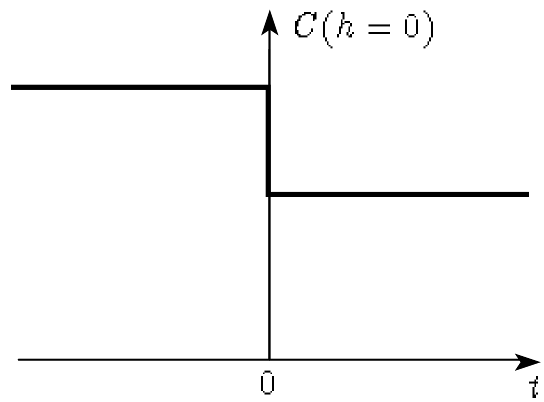
• **Susceptibility:** The magnetization is aligned to the external field, $\vec{m} = \overline{m}(h)\hat{h}$, its magnitude given by the solution to $t\overline{m} + 4u\overline{m}^3 = h$. The changes in the magnitude are governed by the *longitudinal susceptibility* χ_ℓ, whose inverse is easily obtained as

$$\chi_\ell^{-1} = \left.\frac{\partial h}{\partial \overline{m}}\right|_{h=0} = t + 12u\overline{m}^2 = \begin{cases} t & \text{for } t > 0, \text{ and } h = 0, \\ -2t & \text{for } t < 0, \text{ and } h = 0. \end{cases} \tag{II.18}$$

On approaching the critical point from either side, the zero field susceptibility diverges as $\chi_\pm \sim A_\pm |t|^{-\gamma_\pm}$, with $\gamma_+ = \gamma_- = 1$. critical exponent γ Although the amplitudes



II.5. The zero field longitudinal susceptibility diverges at $t = 0$.



II.6. The saddle point approximation predicts a discontinuous heat capacity.

$A_\pm$ are material dependent, Eq.(II.18) predicts that their ratio is universal, given by $A_+/A_- = 2$.amplitude ratio (We shall shortly encounter the *transverse susceptibility* χ_t, which describes the change in magnetization in response to a field perpendicular to it. For $h = 0$, χ_t is always infinite in the magnetized phase.)

- **Heat capacity:** The free–energy for $h = 0$ is given

$$\beta F = \beta F_0 + V\Psi\left(\overline{m}\right) = \beta F_0 + V \begin{cases} 0 & \text{for } t > 0, \\ -\dfrac{t^2}{16u} & \text{for } t < 0. \end{cases} \tag{II.19}$$

Since $t = a(T - T_c) + \cdots$, to leading order in $(T - T_c)$, we have $\partial/\partial T \sim a\partial/\partial t$. Using similar approximations in the vicinity of the critical point, we find the behavior of the heat capacity at zero field as

$$C(h=0) = -T\frac{\partial^2 F}{\partial T^2} \approx -T_c a^2 \frac{\partial^2}{\partial t^2}\left(k_B T_c \beta F\right) = C_0 + V k_B a^2 T_c^2 \times \begin{cases} 0 & \text{for } t > 0, \\ \dfrac{1}{8u} & \text{for } t < 0. \end{cases} \tag{II.20}$$

The saddle point method thus predicts a discontinuity, rather than a divergence, in the heat capacity. If we insist on describing this singularity as a power law $t^{-\alpha}$, we have to choose the exponent $\alpha = 0$.critical exponent α