Active Spinners in a Circular Confinement: an Incompressible Active Fluid Model

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Active spinner systems and active chiral fluids exhibit odd viscosity. These features lead to phase separations and unforeseen physical properties. In this work, we aim to investigate the hydrodynamical properties of active incompressible fluids and examine an appropriate solution that matches the observations on the collective behavior of starfish embryos on the surface of a passive fluid.

Active matter studies have been growing in the last decade. Active matter systems set up unforeseen physical phenomena such as emergent collective behavior [13], motility-induced phase separation [3], odd mobility [9] and elasticity[10] and viscosity. The origin of the odd viscosity has been shown due to the intrinsic self-spinning of microscopic constituents in a system [5].

In the modeling active matter system, microscopic and macroscopic approaches are present. Simulations and experiments are fundamental in the microscopic level analysis since general behavior cannot be attained from equations alone. The outcome of such a study is that co-rotating active spinning particles exert an attractive force on each other with appropriate parameters [1]. This force leads to phase separations in dense populations [6, 8, 11, 12, 15]. The general tendency of the active spinner with attractive force is collecting and (or) rotating together locally or globally. At the macroscopic level, hydro-dynamical equations govern the dynamics of a swarm of active particles in the mean-field perspective. This perspective has shown to be accurate since the internal propulsion mechanism has no effect on the pattern formation or phases of the system [2]. Similar to active spinners, hydrodynamics of active chiral fluids indicate odd viscosity [4].

The addition of a confinement may induce exotic phases of active matter. A detailed phase space analysis implies phases such that particles rotate collectively in the vicinity of the boundary for some parameter choices. In some phases, they are flocking or oscillating but they tend to accumulate near the boundary [7]. In addition to these observations, under a circular boundary in the co-rotating frame, several vortexes surround the center.

This work aims to investigate a denser active spinner population, where the population density across the system is almost constant. Providing the experimental results of self-spinning starfish embryos, we will examine hydro-dynamical equations for active chiral fluids and combine the results to explain the collective behavior of starfish embryos.

EXPERIMENT AND DATA ANALYSIS

The experimental setup consists of starfish embryos in a water tank under a circular confinement. Their dynamical behavior is recorded and the raw data is processed with an ML segmentation algorithm to detect the positions of each embryo. The processed data includes 500 frames of positions of embryos. An example frame is shown in Fig.1. The system exhibits a constant density profile overall.

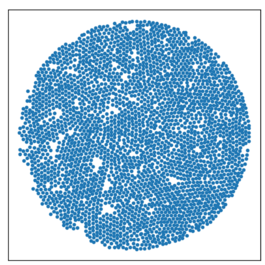

FIG. 1. A caption from the processed frame.

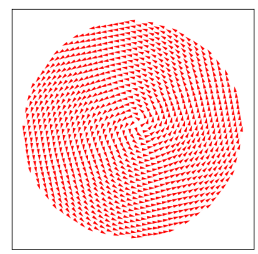

FIG. 2. The velocity field averaged over all 500 frames with spatial smoothing

To get a smoother flow field, we generated a square grid with a grid length of $d \approx 0.055R$, where R is the radius of the circular confinement. After setting up the grid, the flow field at the location of grids is calculated by averaging the velocities of the embryos in the vicinity of a grid with distance $r_g = 0.28R$. The recordings indicate a collective rotation around the center of the confinement as in Fig.2. However, this is far from rigid body rotation and it is observed in the rotation curve of the system in Fig.3.

METHODS

Modeling the self-spinning embryos as an incompressible active fluid requires solving hydrodynamical equations of momentum conservation. The two-dimensional hydrodynamic theory has been utilized in some previous studies [14]. The conservation of momentum leads to the

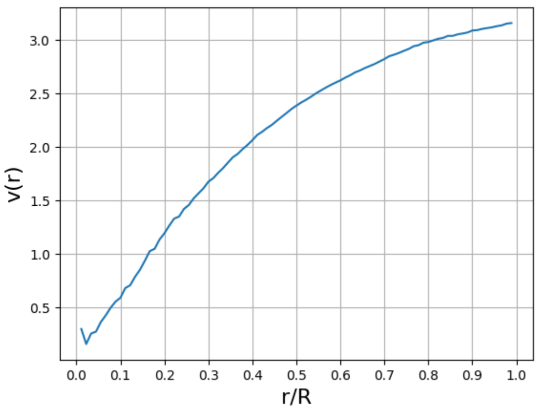

FIG. 3. The magnitude of the velocity(flow) field averaged over all 500 frames vs. the distance from the center.

equation

$$\rho(\partial_t + \mathbf{v}.\nabla)v_i = \partial_j \sigma_{ij} - \Gamma v_i \quad (1)$$

, where Γ is the friction coefficient and σ_{ij} is the stress tensor. The stress tensor has an additional term due to odd viscosity and the whole expression for that is

$$\sigma_{ij} = -p\delta_{ij} + \eta(\partial_i v_j + \partial_j v_i) + \epsilon_{ij}\eta_R(2\omega - \Omega) + \eta_o(\partial_i \epsilon_{jk} v_k + \epsilon_{ik}\partial_k v_j) \quad (2)$$

, where $I\omega(r,t)$ is the local angular momentum density and $\Omega = (\nabla \times \mathbf{v}).\hat{\mathbf{z}}$ is the vorticity of velocity field $\mathbf{v}$. The first represents the external pressure and the second term is due to shear stress. The third term is the rotational stress due to self-spinning and the last one is the odd viscosity.

To solve the steady-state flow field in polar coordinates we shall make some assumptions to simplify the equation.

1. *Azimuthal symmetry ($\partial_\theta v_\theta = 0$:* The velocity field and angular momentum density are constant across a circular ring. There is no condition to break this symmetry and the experiments also exhibit azimuthal symmetry.

2. *Zero mean radial velocity ($v_r = 0$):* This assumption is compatible with the experiments and can be also observed in the averaged velocity field in Fig.2.

3. *Frictional wall ($\sigma_{\theta r}(R) = -\Gamma_s v_\theta(R)$):* This is the most suitable choice. This condition leads to a boundary condition $\partial_r v_\theta(R) = \frac{\eta}{\eta - \eta_R}\left(-\Gamma_s + v_\theta(R)\frac{\eta_R}{\eta R}\right)$, if the flow is counter-clockwise.

These assumptions simplify Eq. 1 and lead to the steady-state solution as

$$(\eta+\eta_R)\partial_r^2 v_\theta + \frac{\eta_R}{r}\partial_r v_\theta - \left(\Gamma + \frac{\eta_R}{r^2}\right)v_\theta - 2\eta_R\partial_r\omega = 0 \quad (3)$$

The odd viscosity term does not appear in this equation and thus, the rotation curves are independent from odd viscosity. To solve the flow field an equation for w is necessary and the conservation of angular momentum leads to

$$I(\partial_t + \mathbf{v}.\nabla)\omega = -\Gamma_R\omega - 2\eta_R(2\omega - \Omega) + D\nabla^2\omega \quad (4)$$

, where Γ_r rotational frictional parameter and D translational diffusivity constant. With the previous assumptions the equation above is modified as,

$$-(4\eta_R + \Gamma_R)\omega + 2\eta_R\left(\frac{v_\theta}{r} + \partial_r v_\theta\right) + D\left(\partial_r^2\omega + \frac{\partial_r\omega}{r}\right) = 0 \quad (5)$$

The steady-state solution ($\partial_t v_\theta = 0$) depends on the viscosity and friction parameters. The parameters may be estimated using experimental tools: however, we can also fit parameters to obtain the experimental rotation curve. The coupled differential equations 3 and 5 can be solved using appropriate boundary conditions. However, the set of plausible boundary conditions is not trivial. The obvious choices are the zero flow field and rotation density in the center. We can also argue von Neumann boundary condition $v'_\theta(R) = 0$. The last boundary condition can be extracted from the experimental data, which is $v_\theta(R) \approx 3$.

$$v_\theta(0) = 0,\ \omega(0) = 0,\ v'_\theta(R) = 0,\ v_\theta(R) = 3 \quad (6)$$

The curve fitting procedure indicates that the shear viscosity η and the friction coefficients Γ_r, Γ are not strongly related to the observation. In other words, the behavior (rotation curve) is stable to changes in these parameters. However, the existence of rotational viscosity plays a crucial role. If its value is set to zero the flow field exhibits a rigid body rotation. The fitted theoretical line and the experimentally observed mean flow field have a decent agreement as shown in Fig.4.

CONCLUSION

In this report, an active spinner system is modeled as an active incompressible fluid as in the mean-field perspective. The general behavior of models is consistent with the experimental data of a swarm of starfish embryos. In the experiments, the mean flow field is a clockwise rotation where radial flow is negligible. The rotation curve is sub-linear and indicates non-rigid rotation. The hydrodynamics of active incompressible fluid with spin-

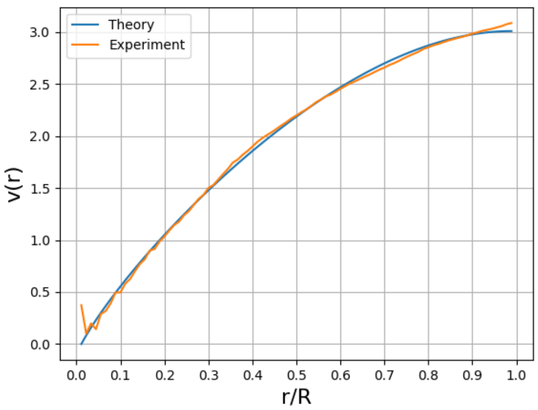

FIG. 4. The magnitude of the velocity(flow) field averaged over all 500 frames vs. the distance from the center and theoretical expectation with fitted parameters

ning particles exhibit both rotational and shear viscosity. The existence of rotational viscosity leads to a sub-linear rotation curve which agrees with the experiment.

The agreement with the theory and experiments can be reevaluated by measuring the parameters experimentally rather than finding the best set to fit experimental observation. The validity and plausibility of the boundary conditions and assumptions are open to discussion and require further investigation. Odd viscosity has no effect in this model due to the choice of assumptions. Possible different settings may suggest different exotic behavior of active matter systems and with the hydrodynamic theory, they may be examined with the mean-field perspective.

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[1] Juan Luis Aragones, Joshua Paul Steimel, and Alfredo Alexander-Katz. Elasticity-induced force reversal between active spinning particles in dense passive media. *Nature communications*, 7(1):11325, 2016.

[2] Tommaso Brotto, Jean-Baptiste Caussin, Eric Lauga, and Denis Bartolo. Hydrodynamics of confined active fluids. *Physical review letters*, 110(3):038101, 2013.

[3] Michael E Cates and Julien Tailleur. Motility-induced phase separation. *Annu. Rev. Condens. Matter Phys.*, 6(1):219–244, 2015.

[4] Ming Han, Michel Fruchart, Colin Scheibner, Suriyanarayanan Vaikuntanathan, Juan J De Pablo, and Vincenzo Vitelli. Fluctuating hydrodynamics of chiral active fluids. *Nature Physics*, 17(11):1260–1269, 2021.

[5] Tomer Markovich and Tom C Lubensky. Odd viscosity in active matter: microscopic origin and 3d effects. *Physical Review Letters*, 127(4):048001, 2021.

[6] Helena Massana-Cid, Demian Levis, Raúl Josué Hernández Hernández, Ignacio Pagonabarraga, and Pietro Tierno. Arrested phase separation in chiral fluids of colloidal spinners. *Physical Review Research*, 3(4):L042021, 2021.

[7] Archit Negi, Kazusa Beppu, and Yusuke T Maeda. Geometry-induced dynamics of confined chiral active matter. *Physical Review Research*, 5(2):023196, 2023.

[8] Nguyen HP Nguyen, Daphne Klotsa, Michael Engel, and Sharon C Glotzer. Emergent collective phenomena in a mixture of hard shapes through active rotation. *Physical review letters*, 112(7):075701, 2014.

[9] Anthony R Poggioli and David T Limmer. Odd mobility of a passive tracer in a chiral active fluid. *Physical Review Letters*, 130(15):158201, 2023.

[10] Colin Scheibner, Anton Souslov, Debarghya Banerjee, Piotr Surówka, William TM Irvine, and Vincenzo Vitelli. Odd elasticity. *Nature Physics*, 16(4):475–480, 2020.

[11] Zaiyi Shen, Alois Würger, and Juho S Lintuvuori. Hydrodynamic self-assembly of active colloids: chiral spinners and dynamic crystals. *Soft matter*, 15(7):1508–1521, 2019.

[12] Benjamin C Van Zuiden, Jayson Paulose, William TM Irvine, Denis Bartolo, and Vincenzo Vitelli. Spatiotemporal order and emergent edge currents in active spinner materials. *Proceedings of the national academy of sciences*, 113(46):12919–12924, 2016.

[13] Tamás Vicsek, András Czirók, Eshel Ben-Jacob, Inon Cohen, and Ofer Shochet. Novel type of phase transition in a system of self-driven particles. *Physical review letters*, 75(6):1226, 1995.

[14] Qing Yang, Hongwei Zhu, Peng Liu, Rui Liu, Qingfan Shi, Ke Chen, Ning Zheng, Fangfu Ye, and Mingcheng Yang. Topologically protected transport of cargo in a chiral active fluid aided by odd-viscosity-enhanced depletion interactions. *Physical Review Letters*, 126(19):198001, 2021.

[15] Bo Zhang, Andrey Sokolov, and Alexey Snezhko. Reconfigurable emergent patterns in active chiral fluids. *Nature communications*, 11(1):4401, 2020.