18.06 Professor Strang Quiz 3 May 5, 1999

Your name is: ______________________________

Grading 1 2 3 4

Please circle your recitation:

1)	Mon	2–3	2-131	S. Kleiman
2)	Mon	3–4	2-131	S. Hollander
3)	Tues	11–12	2-132	S. Howson
4)	Tues	12–1	2-132	S. Howson
5)	Tues	12–1	2-131	S. Kleiman
6)	Tues	1–2	2-131	S. Kleiman
7)	Tues	2–3	2-132	S. Howson

1 (27 pts.) Suppose $A = \begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix}$.

(a) Find the matrices $\wedge$ and S in the diagonalization formula $S^{-1}AS = \wedge$.

(b) Find the matrix A^k (all four entries of the k^{th} power of A).

(c) Find the limit as $k \to \infty$ of $u_k = A^k u_0$ if $u_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

2 (25 pts.) Suppose a 3 by 3 real symmetric matrix A has eigenvalues $4, 4, 0$.

(a) Find the determinant from the eigenvalues of $(2A - I)^{-1}$.

(b) True or false or not enough information:

This matrix A has 3 independent eigenvectors and can be diagonalized.

(c) True or false or not enough information:

The function $x^T Ax$ is never negative for any vector x.

(d) True or false or not enough information:

The matrix $\frac{1}{4}A$ is a Markov matrix.

(e) If A has orthonormal eigenvectors q_1, q_2, q_3 with $\lambda = 4, 4, 0$, find a formula for A in terms of q_1, q_2, q_3 using diagonalization.

3 (24 pts.) Suppose that A is an invertible 3 by 3 matrix.

(a) Show me how to prove that $x^T A^T A x$ is *always positive* if x is not the zero vector. Why will this fail if A is not invertible?

(b) Show me how to prove that $A^T A$ is *similar* to AA^T. Does it follow that these matrices have the same eigenvalues and eigenvectors?

(c) If the SVD is written in the usual form $A = U\Sigma V^T$, what is the matrix $A^T A$ (reduced to the simplest form)?

4 (24 pts.) (a) Find the eigenvalues of $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Then find the eigenvalues of $A = \begin{bmatrix} a & 1 \\ 1 & a \end{bmatrix}$ and $R = \begin{bmatrix} b & -1 \\ 1 & b \end{bmatrix}$. The numbers a and b are real.

(b) Under what condition on "a" do all solutions of $du/dt = Au$ approach zero as $t \to \infty$?

(c) Under what conditions on "b" do all solutions of $dv/dt = Rv$ approach zero as $t \to \infty$?

(d) Under what condition on "a" is the matrix A positive definite?