18.06 Professor Strang Quiz 1 Solutions March 10, 1999

1 (a) 1. (no c)
2. (all $c \neq 0$)
3. $c = 0$

(b) rank 3 $c \neq 0$
rank 2 $c = 0$

(c) $N(A) = \{0\}$ if $c \neq 0$

$N(A) =$ all multiples of $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ if $c = 0$.

(d) $c \neq 0$ Give any basis for R^3

$c = 0$ one basis is $\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$

2 (a) m

(b) Only the zero vector.

(c) (0 or 1) solutions.

3 The matrix A is 4 by 3. A^T is 3 by 4.

(a) Every system $A^T y = 0$ with more unknowns than equations has a nonzero solution. (By the way, y will be a vector *perpendicular* to the 3-dimensional hyperplane.)

(b) A has independent columns, since u, v, w form a basis.

4 (a) Solve $Ax = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ for the first column of A^{-1}.

(b) $\begin{bmatrix} a & 3 & 2 \\ 1 & 3 & 0 \\ 1 & b & 0 \end{bmatrix} \begin{bmatrix} \\ x \\ \\ \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ gives $x = \begin{bmatrix} 0 \\ 0 \\ 1/2 \end{bmatrix}$ by inspection.

(c) If $b = 3$ then rank$(A) = 2$ (Two equal rows, regardless of a)
If $b \neq 3$ then rank$(A) = 3$ (Three independent rows, regardless of a)

(d) If $b = 3$ then one basis is $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

If $b \neq 3$ then choose any basis for R^3.