18.06 Professor Strang Quiz 1 March 10, 1999

Your name is: ______________________________ **Grading** 1 2 3 4

Please circle your recitation:

1)	Mon	2–3	2-131	S. Kleiman	5)	Tues	12–1	2-131	S. Kleiman
2)	Mon	3–4	2-131	S. Hollander	6)	Tues	1–2	2-131	S. Kleiman
3)	Tues	11–12	2-132	S. Howson	7)	Tues	2–3	2-132	S. Howson
4)	Tues	12–1	2-132	S. Howson					

1 (32 pts.) The 3 by 3 matrix A is

$$A = \begin{bmatrix} c & c & 1 \\ c & c & 2 \\ 3 & 6 & 9 \end{bmatrix}.$$

(a) Which values of c lead to each of these possibilities?

1. $A = LU$: three pivots without row exchanges
2. $PA = LU$: three pivots after row exchanges
3. A is singular: less than three pivots. *(Continued)*

(b) For each c, what is the rank of A?

(c) For each c, describe exactly the nullspace of A.

(d) For each c, give a basis for the column space of A.

2 (21 pts.) A is m by n. *Suppose $Ax = b$ has at least one solution for every b.*

(a) The rank of A is ______________.

(b) Describe all vectors in the nullspace of A^T.

(c) The equation $A^T y = c$ has (0 or 1)(1 or ∞)(0 or ∞)(1) solution for every c.

3 (16 pts.) Suppose u, v, w are a basis for a subspace of R^4, and these are the columns of a matrix A.

(a) How do you know that $A^T y = 0$ has a solution $y \neq 0$?

(b) How do you know that $Ax = 0$ has only the solution $x = 0$?

4 (31 pts.) (a) To find the first column of A^{-1} (3 by 3), what system $Ax = b$ would you solve?

(b) Find the first column of A^{-1} (if it exists) for

$$A = \begin{bmatrix} a & 3 & 2 \\ 1 & 3 & 0 \\ 1 & b & 0 \end{bmatrix}.$$

(c) For each a and b, find the rank of this matrix A and say why.

(d) For each a and b, find a basis for the column space of A.