18.06 Professor Strang Quiz 2 April 4, 1997

1. **(a)** The system $Ax = b$ has a solution if and only if b is orthogonal to what subspace?

(b) Find the determinant of the 4×4 matrix A whose entries are $a_{ij} =$ smaller of i^2 and j^2.

(c) What is the relation between the determinant of A and the pivots? *Why is this true*?

2. At $t = 1, 2, 3$ we are given values b_1, b_2, b_3. The idea is to fit the best straight line $b = C + Dt$ to those three points.

(a) What three equations in two unknowns will have a solution if the three points lie exactly on a line?

(b) Under what condition $m\, b_1 + n\, b_2 + p\, b_3 = 0$ (**find** m, n, p) will the three points lie on a line? (You could use elimination or your answer to Question 1(a).)

(c) Find the best line $\bar{C} + \bar{D}t$ if the values are $(b_1, b_2, b_3) = (0, 0, 1)$.

(d) What 3×3 matrix P projects every vector onto the plane containing the column vectors $(1, 1, 1)$ and $(1, 2, 3)$?

3. **(a)** Suppose q_1, q_2, q_3 are orthonormal vectors in $\mathbb{R}^6$. Under *what condition on the vector* v will there be a fourth orthonormal vector q_4 that is a combination of v, q_1, q_2, q_3?

(b) Give a formula for that fourth orthonormal vector q_4.

(c) Suppose $q_1, \ldots, q_n$ is an orthonormal basis for $\mathbf{R}^n$. Define the $n \times n$ matrix $A = q_1 q_1^T + \cdots + q_n q_n^T$. What does Aq_1 equal? What does Aq_i equal? What is A?