# 18.06 Professor Strang Quiz 1 February 26, 1997

1. (36= 4 times 9 points) $A$ is a 3 by 4 matrix and $b$ is a column vector in $\mathbf{R}^3$:

$$A = \begin{bmatrix} 1 & 3 & 2 & 2 \\ 2 & 7 & 6 & 8 \\ 3 & 9 & 6 & 7 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 7 \\ 7 \end{bmatrix}$$

   (a) Reduce $Ax = b$ to echelon form $Ux = c$ and find one solution $x_p$ (if a solution exists).

   (b) Find all solutions to this system $Ax = b$ (if solutions exist). Describe this set of solutions geometrically. Is it a subspace?

   (c) What is the column space of this matrix $A$? Change the entry 7 in the lower right corner to a different number that gives a smaller column space for the new matrix (call it $M$). The new entry is ____________.

   (d) Give a right side $b$ so that your new $Mx = b$ has a solution and a right side $b$ so that $Mx = b$ has no solution.

2. (27= 3 times 9 points) Suppose $A$ is a *square invertible* $n$ by $n$ matrix.

   (a) What is its column space and what is its nullspace?

   (b) Suppose $A$ can be factored into $A = LU$:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 7 & 3 & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}.$$

   Describe the first elimination step in reducing $A$ to $U$. How do you know that $U$ is also invertible?

   (c) Find a specific 3 by 3 invertible matrix $A$ that can *not* be factored into this $LU$ form. What factorization is still possible for your example? (You don't have to find the factors.) How do you know your $A$ is invertible?

3. $A$ is an $m$ by $n$ matrix of rank $r$. Suppose $Ax = b$ has *no solution* for some right sides $b$ and *infinitely many solutions* for some other right sides $b$.

   (a) (9) Decide whether the nullspace of $A$ contains only the zero vector and why.

   (b) (9 points) Decide whether the column space of $A$ is all of $\mathbf{R}^m$ and why.

   (c) (10) For this $A$, find all true relations between the numbers $r$, $m$, and $n$.

   (d) (9 points) Can there be a right side $b$ for which $Ax = b$ has exactly one solution? Why or why not?