18.06 Problem Set 6

Due Wednesday, April 11, 2007 at **4:00 p.m.** in 2-106

Problem 1 *Wednesday 4/4*

Do problem 9 of section 6.1 in your book.

Problem 2 *Wednesday 4/4*

Do problem 28 of section 6.1 in your book.

Problem 3 *Wednesday 4/4*

Do problem 33 of section 6.1 in your book.

Problem 4 *Wednesday 4/4*

Let A be a fixed $n \times n$ matrix. We would like to find a matrix B such that $AB = BA$. This is the same as solving $AB - BA =$ zero matrix. It turns out that this is a system of n^2 equations on the entries of B (which are unknown). Since all these equations are linear, we can associate this system to a matrix M. Find an eigenvector of this matrix M with its corresponding eigenvalue.

Problem 5 *Monday 4/9*

Do problem 7 of section 6.2 in your book.

Problem 6 *Monday 4/9*

Do problem 10 of section 6.2 in your book.

Problem 7 *Monday 4/9*

Do problems 15 and 16 of section 6.2 in your book.

Problem 8 *Monday 4/9*

Do problem 22 of section 6.2 in your book.

Problem 9 *Monday 4/9*

Do problem 28 of section 6.2 in your book.

Problem 10 *Monday 4/9*

(a) Give an example of a 3×3 matrix $A \neq 0$ such that $A^2 \neq 0$ but $A^3 = 0$. Four your A find all the eigenvalues and the eigenvectors.
(b) Now, let B be a diagonalizable matrix such that there exists some positive integer k such that $B^k = 0$. Prove that $B = 0$.

(c) Does part (b) contradict part (a)? Explain your answer.